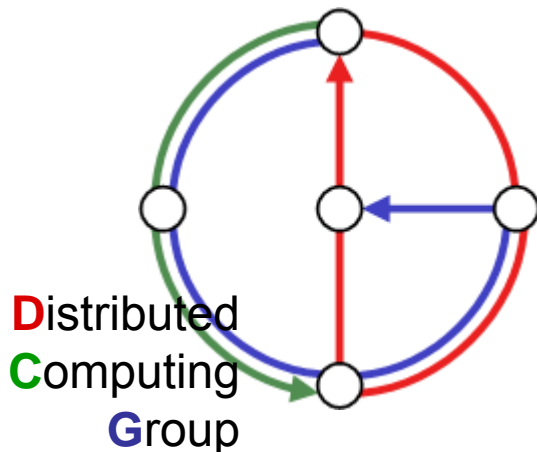


# Word of Mouth: Rumor Dissemination in Social Networks

*Jan Kostka*

*Yvonne Anne Oswald*

*Roger Wattenhofer*



**ETH**

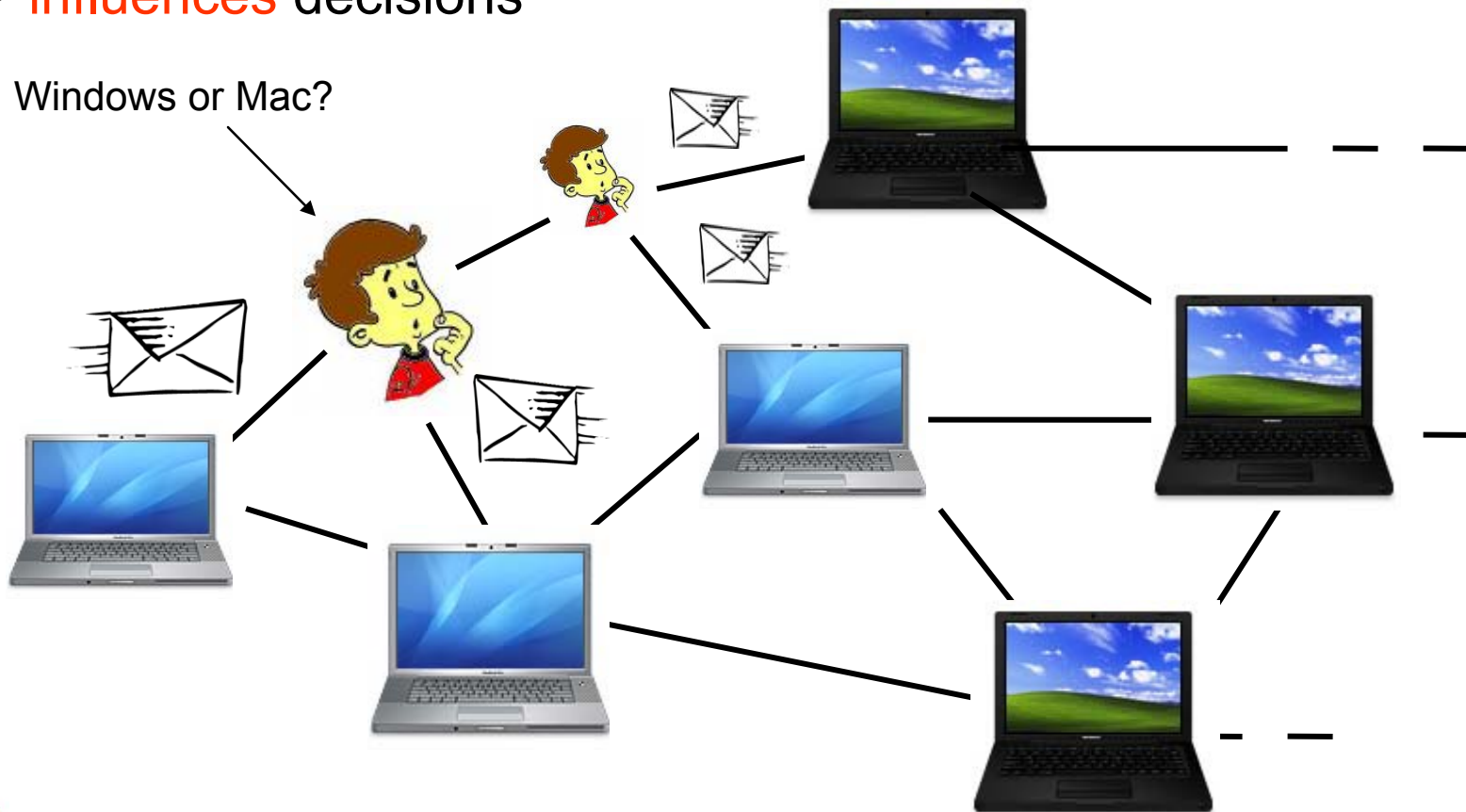
Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

# Introduction

social networks everywhere: facebook, co-authors, email ....

=> effects **dissemination** of information

=> **influences** decisions



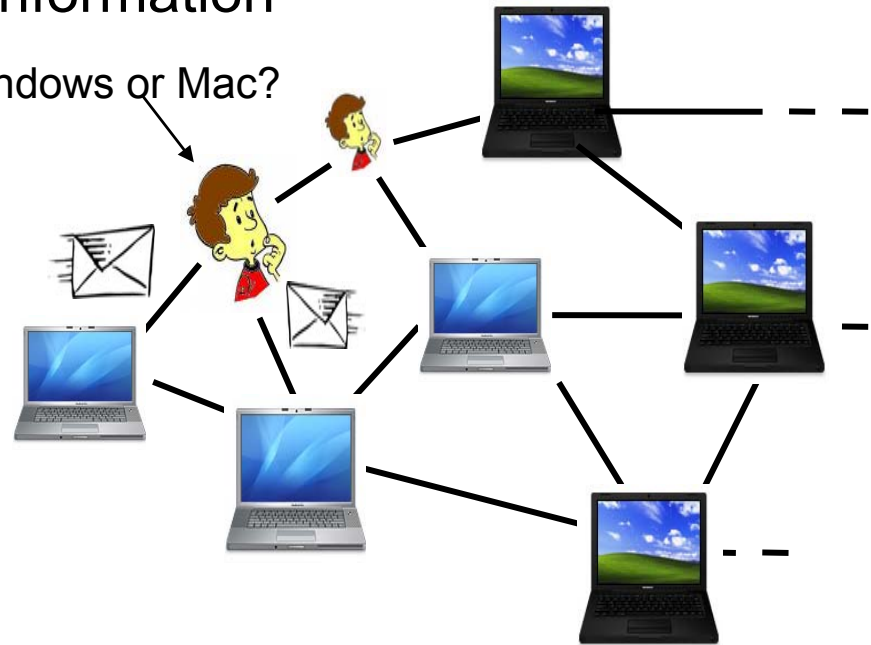
# Introduction

social networks everywhere: facebook, co-authors, email ....

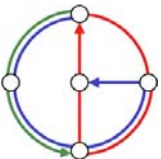
=> effects **dissemination** of information

=> **influences** decisions Windows or Mac?

- viral marketing
- competing theses, theories
- virus vs immunisation



GOAL: select **optimal** initiator set,  
to convince as many nodes as possible



# Related Work

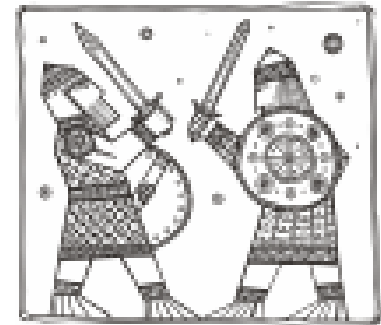
## 1 rumour

- epidemics, physical processes:  
sophisticated propagation models + simulation
- Kempe et al. [KDD03] :  
selecting optimal initiators is NP-hard  
greedy hill climbing algorithm:  $(1-1/e)$ -approximation

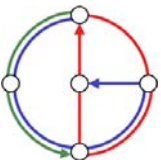


## 2 competing rumours

- Bharati et al.[WINE07], Carnes et al.[ICEC07]  
2nd player: selecting optimal initiators is NP-hard  
hill climbing works as well



What about the 1st player?



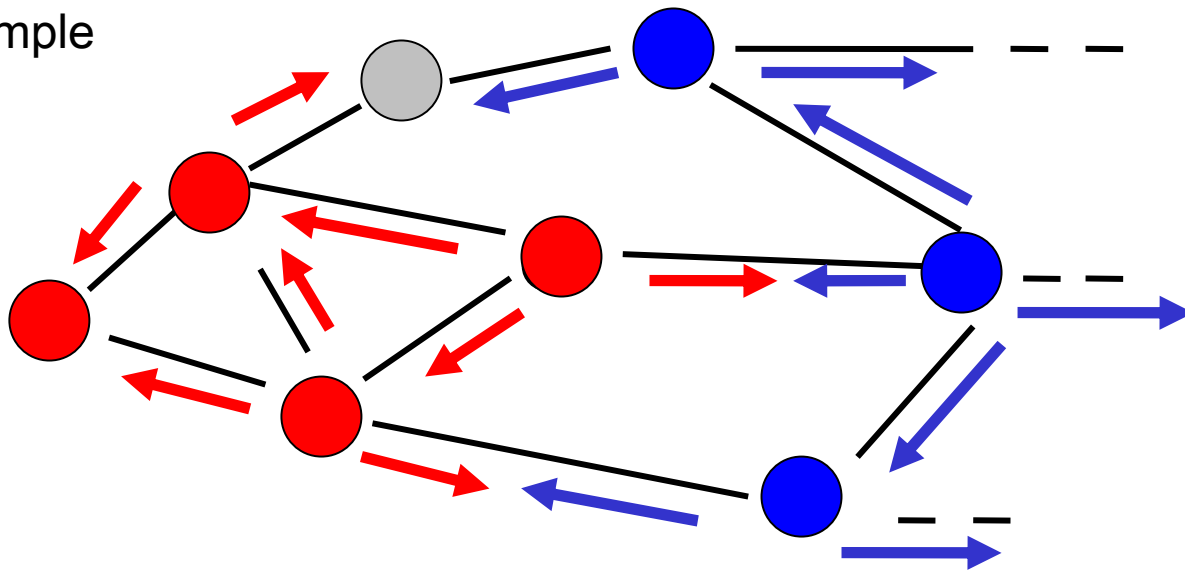
# Basic Model



- **strategy:** select set of nodes to initiate the rumour
- **rumour propagation:**
  - accept first rumour encountered
  - forward rumour to all adjacent nodes

Payoff:  
# convinced nodes

Example



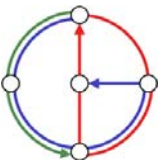
1<sup>st</sup> player:

4 nodes

2<sup>nd</sup> player:

3 nodes

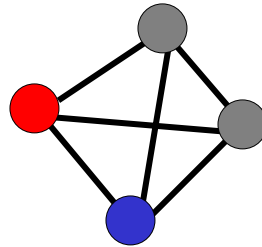
**variations:** more players, payoff definition, propagation model (cascade, threshold, ...), weighted or directed edges, ...



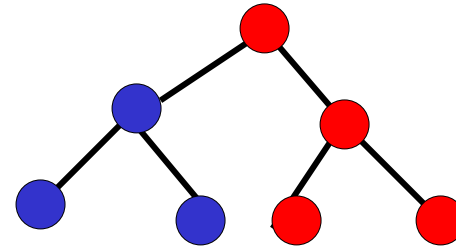
# Warm-Up: 1 vs 1



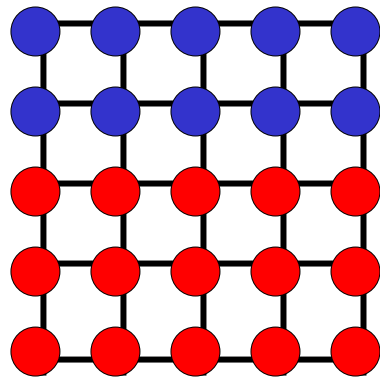
- Complete graph



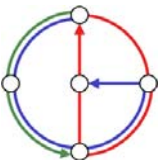
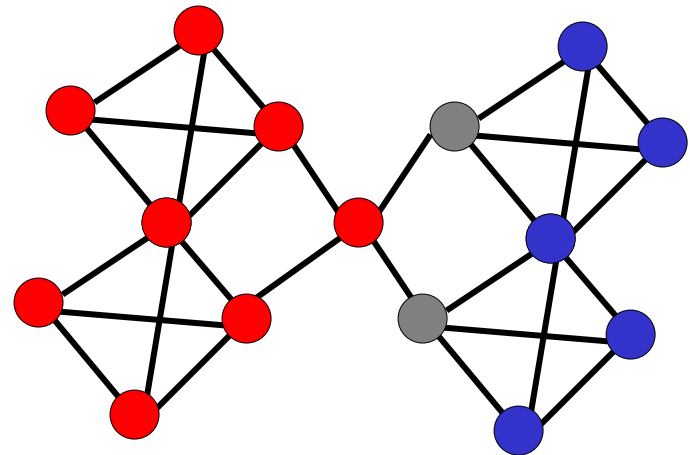
- Trees



- Grid



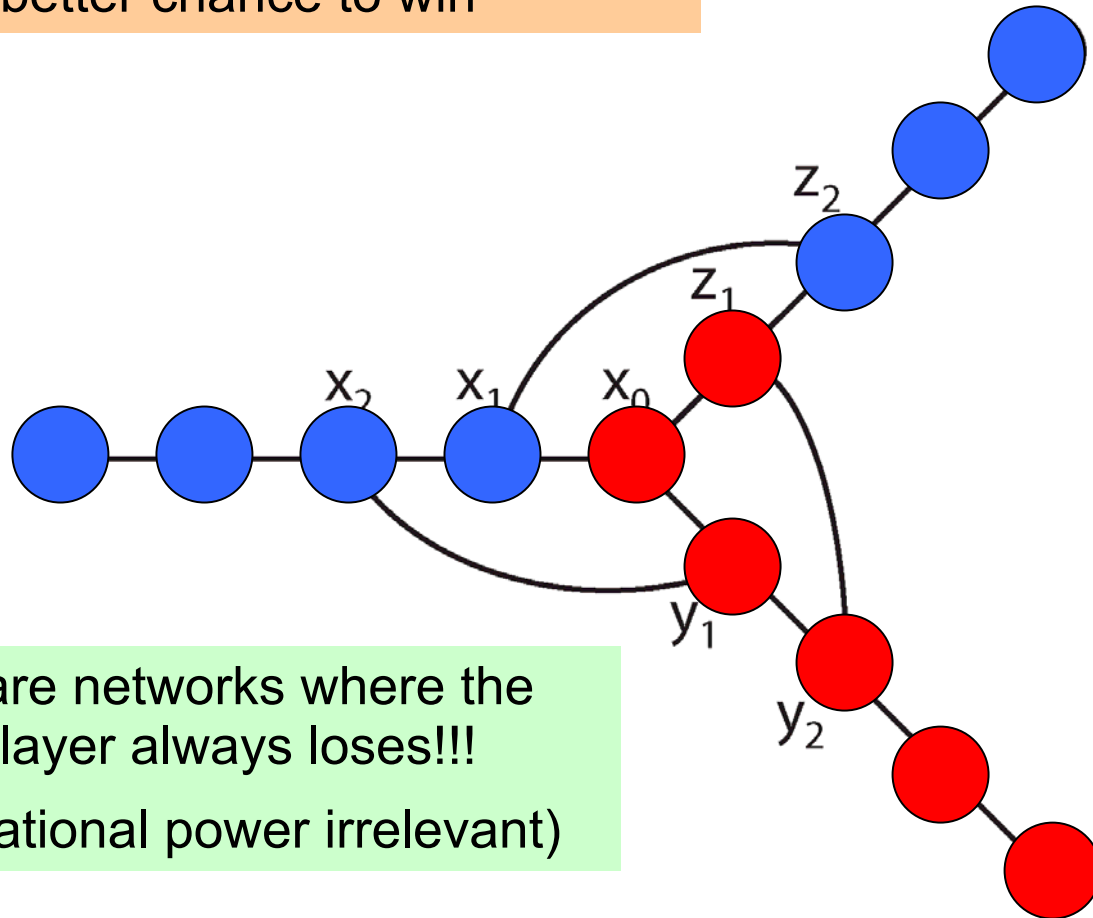
- Bottleneck



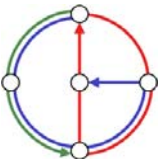
# He who laughs last, laughs best?



**Intuition:** 1st player has more choice  
⇒ better chance to win



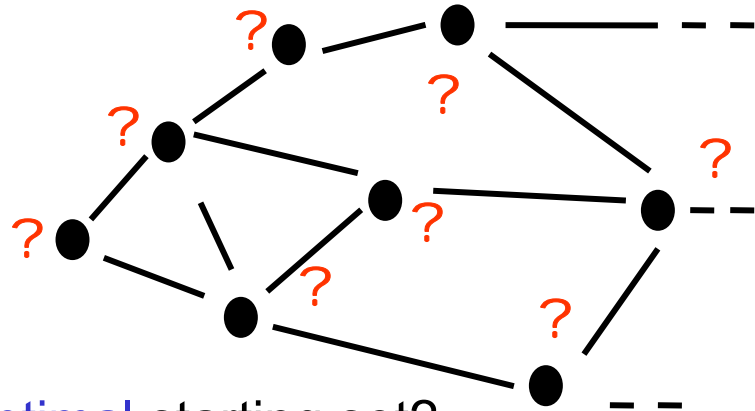
There are networks where the  
1st player always loses!!!  
(computational power irrelevant)



# How hard is it to compute the optimal strategy?

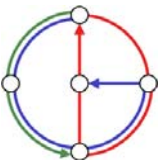
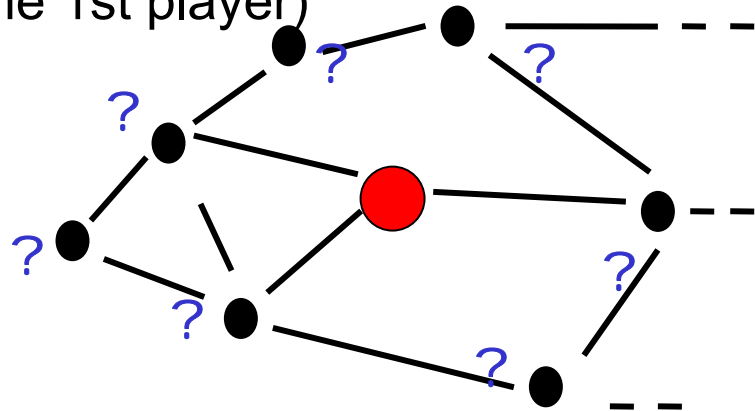
## Centroid Problem

1st player: how do I choose the **optimal** starting set?  
(knowing how many nodes the second player can select)



## Medianoid Problem

2nd player: how do I choose my **optimal** starting set?  
(knowing the nodes selected by the 1st player)



# NP-hardness of Medianoid Problem



**Theorem.** *The (r|p)-medianoid problem is NP-hard.*

**Proof:**

Reduce Dominating Set (DS) problem to (r|1)-medianoid problem.

**Idea:** show that  
 $\exists Y_r$  s.t. 2nd player wins  
at least  $|V|+r$  nodes

$\Leftrightarrow$

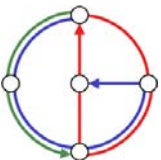
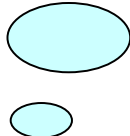
$\exists$  DS with  $r$  nodes

1st player chooses  $x_1$   
2nd player selects  $Y_r$

payoff 2nd player:  
# nodes closer to  $Y_r$   
than to  $X_p$ .

$G(V,E)$   
 $\cup \{v, c\}$

**DS**



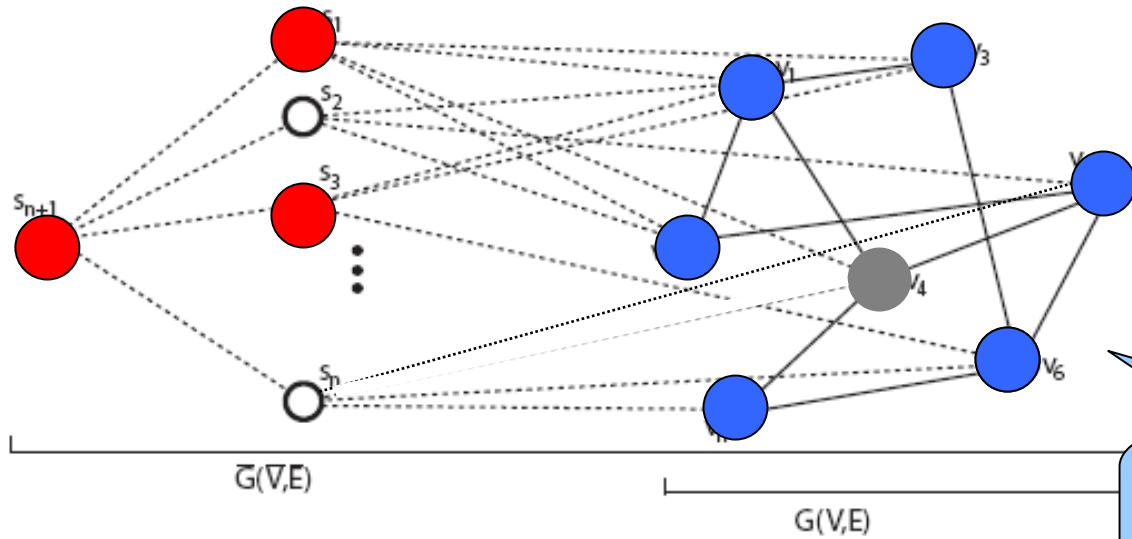
# NP-hardness of Medianoid Problem



**Theorem.** *The  $(r|p)$ -medianoid problem is NP-hard.*

**Proof:**

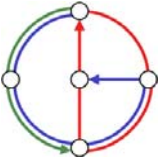
Reduce Dominating Set (DS) problem to  $(r|1)$ -medianoid problem.



1st player chooses  $x_1$   
2nd player selects  $Y_r$

payoff 2nd player:  
# nodes closer to  $Y_r$   
than to  $X_p$ .

**$Y_r \neq DS$**



# NP-hardness of Centroid Problem

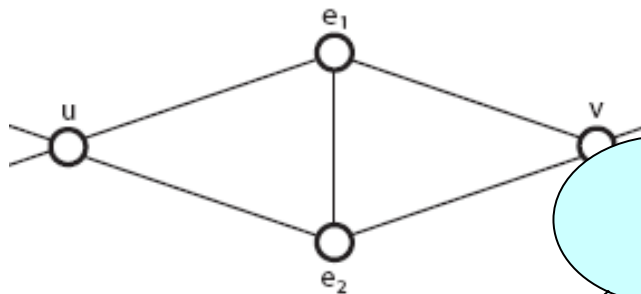


**Theorem.** *The  $(r|p)$ -centroid problem is NP-hard.*

## **Proof:**

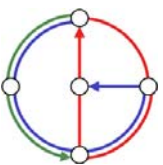
Reduce Vertex Cover (VC) problem to  $(1|p)$ -centroid problem.

Given graph  $G(V,E)$ , replace each edge with “diamond structure”



**Idea:** show that  
 $\exists X_p$  s.t. less than 2  
nodes closer to  $Y_1(X_p)$   
 $\Leftrightarrow$   
 $\exists$  VC with  $p$  nodes  
if  $x_i$  on every diamond, we are ok  
if no  $x_i$  on diamond, contradiction

1st player chooses  $X_p$   
2nd player selects  $Y_1(X_p)$



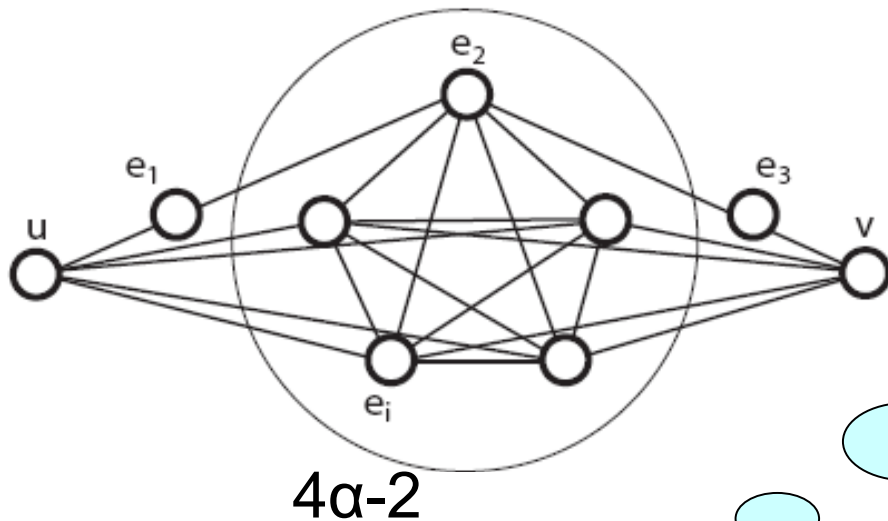
# NP-hardness of **Approximating the Centroid** Problem

**Theorem.** Computing an  $\alpha$ -approximation of the  $(r|p)$ -centroid problem is NP-hard.

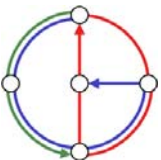
## **Proof:**

Reduce Vertex Cover (VC) problem to  $(1|p)$ -centroid problem.

Given graph  $G(V,E)$ , replace each edge with “clique structure”

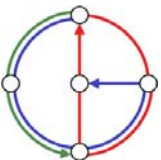
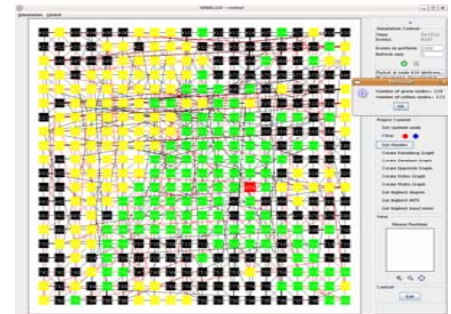


same idea, a little bit more complicated



# More findings...

- relationship **Condorcet vertex** – centroid
- characterize weaknesses of **heuristics** for centroid
  - small radius
  - high degrees
  - midpoint of spanning tree
- (not in paper) **simulation of strategies** in random graphs:  
Kleinberg, Watts, Epstein model



The End!



**THANK  
YOU!**

Questions? Comments?

