TRUTHFUL MECHANISMS FOR GENERALIZED UTILITARIAN PROBLEMS

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Abstract In this paper we investigate extensions of the well-known Vickrey-Clarke-Groves (VCG) mechanisms to problems whose objective function is not utilitarian and whose agents' utilities are not quasi-linear. We provide a generalization of utilitarian problems, termed *consistent* problems, and prove that every consistent problem admits a *truthful mechanism*. These mechanisms, termed *VCG-consistent* (VCGc) mechanisms, can be seen as a natural extension of VCG mechanisms for utilitarian problems.

We then investigate extensions/restrictions of consistent problems. This yields three classes of problems for which (i) VCGc mechanisms are the only truthful mechanisms, (ii) no truthful VCGc mechanism exists, and (iii) no truthful mechanism exists, respectively. Showing that a given problem is in one of these three classes is straightforward, thus yielding a simple way to see whether VCGc mechanisms are appropriate or not.

Finally, we apply our results to a number of basic non-utilitarian problems.

Keywords: Algorithmic Mechanism Design, Algorithms for the Internet, Game Theory

*Supported by the European Project IST-2001-33135, Critical Resource Sharing for Cooperation in Complex Systems (CRESCCO). Work partially done while visiting ETH Zentrum.
†Supported by the Research Project GRID.IT, partially funded by the Italian Ministry of Education, University and Research. Work partially done while visiting ETH Zentrum.

1. Introduction

In the Internet a multitude of heterogeneous entities (e.g., providers, autonomous systems, universities, private companies, etc.) offer, use, and even compete with each other for resources. As the Internet is emerging as the platform for distributed computing, new solutions should take into account the new aspects deriving from a multi-agent system in which agents cannot be assumed to be either *honest/obedient* (i.e., to follow the protocol) or *adversarial* (i.e., to "play against"). Indeed, the entities involved in the computation are driven by different goals (e.g., minimizing *their own* costs) and they may act *selfishly*. In this case, agents cannot be assumed to follow the protocol, though they respond to *incentives* (e.g., a payment received to compensate the costs).

In essence, game theory is the study of what happens when independent agents act selfishly (for a more extensive discussion of applications of game theoretic tools and micro economics to the Internet we refer the reader to [8, 16]). Mechanism design asks how one can design systems so that agents' selfish behavior results in the desired system-wide goals. In a nutshell, each agent i has a function $u_i(\cdot)$ which expresses her *utility* derived from the system outcome. For instance, if the system computes a solution X and provides agent i with a payment P_i, then the corresponding utility is equal to $u_i(X, P_i, t_i)$, where t_i is another parameter called the *type* of agent i. The difficulty here is that the type t_i is *not known to the system* and is also part of the input required to construct the desired solution X^* (e.g., t_i represents the speed of a router and the system goal is to forward packets optimally). This piece of information is known to agent i which may report a different (false) type $r_i \neq t_i$ in order to improve her utility: on input r_i, the system computes a solution X' and provides a payment P' such that $u_i(X', P'_i, t_i) > u_i(X^*, P_i, t_i)$, where P_i is the payment that i would have received if reporting t_i. Notice that the computed solution X' is not the desired one since the input provided to the underlying algorithm is not the correct one. Therefore, one should design a suitable payment rule $p(\cdot)$ such that, for every possible t_i, agent i cannot improve her utility by reporting $r_i \neq t_i$. Moreover, this should hold also when some other agent j does not act rationally and reports $r_j \neq t_j$. The combination of an algorithm $\mathcal{A}$ and a payment rule $p(\cdot)$ that guarantees this property, for every agent, is called *truthful mechanism* with dominant strategies.

A central question in (algorithmic) mechanism design is the study of which system goals are achievable via truthful mechanisms, that is, if algorithm $\mathcal{A}$ is required to produce a certain output (e.g., an optimal solution for a combinatorial optimization problem) does a payment function $p(\cdot)$ exist such that the resulting mechanism $(\mathcal{A}, p)$ is truthful?

A large body of the existing literature focuses on the class of problems in which the utilities are *quasi-linear*, that is, agent i's utility factors into $u_i(X, P_i, t_i) = v_i(X, t_i) + P_i$, where $v_i(X, t_i)$ represents the valuations of agent i of solution X. For such problems, the celebrated Vickrey-Clarke-Groves (VCG) mechanisms [4, 10, 18] guarantee the truthfulness under the hypothesis that the algorithm $\mathcal{A}$ maximizes the function $\sum_i v_i(X, t_i)$. VCG mechanisms have been successfully applied to a multitude of optimization problems involving selfish agents with applications to networking [1, 2, 7, 14, 17] and electronic commerce [5, 11]. All these works assume that the problem is *utilitarian*, that is, the utility functions are quasi-linear and the objective maximization function can be written as the sum above. Moreover, even though Nisan and Ronen [13] first focused on problems whose objective function is not utilitarian, their n-approximation mechanism is nothing but a VCG mechanism for a related utilitarian problem. Actually, VCG mechanisms remain the only general technique to design truthful mechanisms (see Sect. 1 for a discussion on previous related work).

Unfortunately, there are problems for which (i) the objective function is not the sum of the agents' valuations and/or (ii) the utility function is not quasi-linear. Consider the following basic problem (see Sect. 3.2 for a more detailed description). In a communication network, each link e can successfully transmit a message with probability $q_e \in (0,1)$. We want to select a *most reliable path*, i.e., a path between two given nodes which maximizes the probability that none of its links fails. Links are owned by selfish agents which are asked to report a (possibly uncorrect) probability $q'_e \in (0,1)$. We provide to a chosen link e a payment specified by a function $p_e(\cdot)$ if and only if link e performed the transmission correctly. Each agent tries to maximizes the *expected* amount of money received.[1] Hence, *both* the objective and the utility functions can be expressed by means of the common "operator" '$\cdot$'. The Most Reliable Path (MRP) problem just described can be easily reduced to a utilitarian problem by considering the logarithms of both the optimization function and of the utility functions, thus implying the existence of a truthful mechanism. It is then natural to ask whether this is just "good chance", or this problem (and others) has some "similarities" with the class of utilitarian problems.

In this paper we address this question by defining a class of problems, termed *consistent* problems (see Sect. 2), which admit truthful mechanisms. The main advantages of our approach are that: (i) it provides an answer to the following question: which mathematical properties guarantee the existence of truthful mechanisms? Moreover, for a given problem, it is easy to see whether it satisfies these properties (while reducing the problem to a utilitarian one may not be as simple as for the MRP); (ii) it provides a more intuitive interpretation of the payments, e.g., for problems like the MRP described above.

We define *VCGc* mechanisms as a natural extension of the VCG mechanisms and show that they are truthful for consistent problems. We then consider possible extensions of our result and provide both positive and negative answers depending on which property we add/drop from the definition of consistent problems. In particular, we identify four classes of problems:

$\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{only}}$. This class is a natural restriction of consistent problems. In particular, VCGc mechanisms are the only truthful mechanisms for problems in this class (Theorem 8).

$\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{vp}}$. This is a subclass of consistent problems. We prove that every problem in this class admits a truthful VCGc mechanism which also satisfies the *voluntary participation* condition (Theorem 10).

$\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$. This is a class of non-consistent problems in which the set of feasible solutions *depends* on the private part of the input, thus not satisfying one of the constraints of the definition of consistent problem (see Constraint (1) in Def. 4). We show that every VCGc mechanism for such a problem is not truthful (Theorem 15).

$\mathsf{C}_{\mathsf{none}}$. This is a subclass of $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$ whose problems do not admit truthful mechanisms (Theorem 20). As this class in non-empty (see Sect. 4.1), our assumption on the set of feasible solutions is necessary (indeed, removing this assumption would give a *superclass* of $\mathsf{C}_{\mathsf{none}}$).

Other problems to which we apply our results are α-RENT TASK SCHEDULING (see Sect. 3.2) and KNAPSACK (see Sect. 4.1).

α-RENT TASK SCHEDULING is a variant of the TASK SCHEDULING problem considered in [13] obtained by modifying the (quasi-linear) utility functions. The resulting problem is consistent, though straightforward reductions to a utilitarian problem do not seem to exist. This shows that the non-existence of an exact mechanism in [13] is due to the "combination" of quasi-linear utilities with a non additive objective function (i.e., the makespan). Finally, the problem does not admit a truthful mechanism satisfying voluntary participation, thus implying that $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{vp}}$ is a proper subclass of consistent problems.

Concerning KNAPSACK, we consider three variants of this problem depending on which part of the input is held by the agents (namely, the item profits, the item sizes, or both). The corresponding versions belong to $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{vp}}$, $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$ and $\mathsf{C}_{\mathsf{none}}$, respectively. This basic problem has applications to scheduling, resource allocation and to a problem of web advertising [6].

Further related work. Green and Laffont [9] showed that for certain utilitarian problems VCG mechanisms are the only truthful mechanisms. Nisan and Ronen [14] considered the approximability of NP-hard optimization problems via *VCG-based* mechanisms: these mechanisms are obtained from VCG ones by replacing an optimal algorithm $\mathcal{A}$ with a (polynomial-time) non-optimal one $\mathcal{A}'$. Archer and Tardos [3] considered so called *one-parameter* agents: here the valuation functions factor as $v_i(X, t_i) = w_i(X) \cdot t_i$. The authors provided a technique which allows to obtain truthful mechanisms (A, p) whenever A satisfies a "monotonicity" property. To the best of our knowledge this is the only technique other than the VCG one. All above mentioned results apply to the case of quasi-linear utility functions only.

Organization of the paper. We present some basic definitions and notation in Sect. 2. In Sect. 3 we provide the definition of consistent problem, VCGc mechanisms and prove our main positive results. Sect. 3.1 deals with the voluntary participation condition, while Sect. 3.2 contains some applications of our positive results. Finally, we prove the negative results in Sect. 4 where we also apply these results to some of the above mentioned problems. Conclusions and open problems are in Sect. 5. Due to lack of space some details concerning the problems formulation and some proofs are omitted (see also [12]).

2. Preliminaries

Informally, in a mechanism design problem one can imagine that the input $\mathcal{I} = (I_P, I)$ is split into a public and into a private part held by k agents. Public valuation and utility functions express the agents' preferences and how each agent "responds" to incentives. We next provide a formal setting. Without loss of generality, we present the definition for maximization problems.

Given any vector $I = \langle y_1, \ldots, y_k \rangle \in \Theta_1 \times \ldots \times \Theta_k$, let $I_{-i} = \langle y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_k \rangle$ and $\langle I_{-i}, x_i \rangle = \langle y_1, \ldots, y_{i-1}, x_i, y_{i+1}, \ldots, y_k \rangle$. Moreover, if $\mathcal{I} = (I_P, I)$, we let $\langle \mathcal{I}_{-i}, x_i \rangle = (I_P, \langle I_{-i}, x_i \rangle)$.

DEFINITION 1 *A* Mechanism Design Maximization (MDMax) *problem is specified as follows:*

- Private instance. *Each agent a_i has available a* private input type $t_i \in \Theta_i$, *where Θ_i denotes the type space of agent a_i which is public knowledge. Given the part of the instance I_P which is* public knowledge, $\mathcal{I}_T = (I_P, I_T)$ *is the* private (or true) instance *specified by the true agents' types $I_T = \langle t_1, \ldots, t_k \rangle$.*
- Reported instance. *Each agent a_i makes public a reported type $r_i \in \Theta_i$; then, for $I_R = \langle r_1, \ldots, r_k \rangle$, the* reported instance $\mathcal{I}_R = (I_P, I_R)$ *is the input provided to the algorithm.*

  In the following, we will often write $\mathcal{I} = (I_P, I)$, for a vector $I = (y_1, \ldots, y_k) \in \Theta_1 \times \ldots \times \Theta_k$ to denote any possible input of the algorithm (i.e., any "reportable" instance) as opposed to $\mathcal{I}_T$ and $\mathcal{I}_R$ representing the specific private and reported instances, respectively.
- Feasible solutions. *Given any instance $\mathcal{I} = (I_P, I)$, $\Phi(\mathcal{I})$ denotes the set of feasible solutions, and $\overline{\Phi}(I_P) = \bigcup_{I' \in \Theta_1 \times \ldots \times \Theta_k} \Phi(I_P, I')$. The set of feasible solutions does not depend on the private part of the input, i.e.,*

$$\forall I_P \;\; \forall I \in \Theta_1 \times \ldots \times \Theta_k, \quad \Phi(I_P, I) = \overline{\Phi}(I_P). \tag{1}$$

- Objective function. *A function $\mu(X, \mathcal{I})$ expresses the measure of a solution X, given any instance $\mathcal{I}$.*
- Valuation functions. *For every agent a_i, a function $v_i(X, t_i)$ expresses the* valuation *of a_i of a solution X, given any value $t_i \in \Theta_i$. The function $v_i(\cdot, \cdot)$ is public knowledge, while one of its arguments is not (namely, the type t_i).*

  We say that a solution X does not involve agent a_i *if $v_i(X, y_i) = v_i^0$, for a fixed value v_i^0 and for every $y_i \in \Theta_i$. We assume that v_i^0 is public knowledge and that, for every X, it is possible to decide whether X does not involve a_i.*
- Agent payments and utility functions. *For every agent a_i it is possible to define a* payment function $p_i(\cdot)$, *representing some sort of incentive for agent a_i. Then, a function $u_i(X, t_i, P_i)$ expresses the* utility *of a_i of a solution X, given its (true) type t_i and given $p_i(\cdot) = P_i$ (this value represents how much a_i benefits if a solution X is output and a_i receives a payment[2] equal to P_i). This function depends only on the values $v_i(X, t_i)$ and P_i, and represents what agent a_i tries to maximize.*

We use the symbol P^0 to denote the fact that a_i receives no payment. In this case, for every X, we have that $u_i(X, t_i, P^0) = v_i(X, t_i)$.

- Goal. *Find an optimal solution for the true instance, that is, a solution $X^* \in \Phi(\mathcal{I}_T)$ such that*

$$\mu(X^*, \mathcal{I}_T) = \max\{\mu(X, \mathcal{I}_T) \mid X \in \Phi(\mathcal{I}_T)\}. \tag{2}$$

Observe that, because of Constraint (1), it is always possible to obtain a feasible solution. However, our goal is to find an optimal one, which *depends* on the agents' types (i.e., the true instance). In order to solve a MDMax problem we need a suitable combination of a payment scheme and an algorithm which guarantees that (i) no agent has an incentive in misreporting her type and (ii) the algorithm, once provided with the true instance $\mathcal{I}_T$, returns an optimal solution for that. In particular, the usual underlying assumption in mechanism design is that an agent misreports her type only in the case this might improve her utility (see e.g. [15]).

DEFINITION 2 (TRUTHFUL MECHANISM) *A* mechanism *for a* MDMax *problem is a pair $\mathcal{M} = (\mathcal{A}, \mathcal{P})$, where $\mathcal{A}$ is an algorithm computing a solution $\mathcal{A}(\mathcal{I}_R)$ and $\mathcal{P}(\mathcal{I}_R) = \langle p_1(\mathcal{I}_R), \ldots, p_k(\mathcal{I}_R)\rangle$ is the payment scheme. A mechanism $\mathcal{M} = (\mathcal{A}, \mathcal{P})$ for a* MDMax *problem is* truthful *if, for all i,*

$$\forall I_{-i}\ \forall r_i \neq t_i \quad u_i(\mathcal{A}\langle \mathcal{I}_{-i}, t_i\rangle, t_i, p_i\langle \mathcal{I}_{-i}, t_i\rangle) \geq u_i(\mathcal{A}\langle \mathcal{I}_{-i}, r_i\rangle, t_i, p_i\langle \mathcal{I}_{-i}, r_i\rangle).$$

Observe that truthful mechanisms guarantee that, for every a_i, reporting $r_i = t_i$ is the best strategy even when some other agents misreport their type (i.e., $I_{-i} \neq \langle t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_k\rangle$).

Another relevant feature of a mechanism is that of guaranteeing that a truthfully behaving agent a_i incurs in a utility which is not worse than the utility she would obtain if not "participating in the game", that is, if a solution X not involving a_i is computed and a_i receives no payment (see Sect. 3.1):

DEFINITION 3 (VOLUNTARY PARTICIPATION) *A mechanism $\mathcal{M} = (\mathcal{A}, \mathcal{P})$ for a* MDMax *problem satisfies the* voluntary participation condition (VP) *if*

$$\forall a_i \quad \forall I_{-i} \quad u_i(\mathcal{A}\langle \mathcal{I}_{-i}, t_i\rangle, t_i, p_i\langle \mathcal{I}_{-i}, t_i\rangle) \geq v_i^0.$$

Given an instance $\mathcal{I}$, for the sake of simplicity, we denote by $\mathcal{I}_{-i}$ the instance $\langle \mathcal{I}_{-i}, \perp\rangle$, where $\perp \notin \Theta_i$ is a "dummy" value which makes unfeasible every feasible solution involving agent a_i. In the rest of the paper we consider *optimal mechanisms*, that is, mechanisms $\mathcal{M} = (\mathcal{A}, \mathcal{P})$ that use an algorithm $\mathcal{A}$ computing an optimal solution w.r.t. the *reported instance*. A truthful optimal mechanism provides a solution for a MDMax problem: the truthfulness guarantees that the agents, being rational, report their types t_i and then algorithm $\mathcal{A}$ computes a solution $X^* = \mathcal{A}(\mathcal{I}_T)$ satisfying Eq. 2.

3. Truthful mechanisms for consistent problems

In this section we first introduce the class of *consistent problems* (Def. 4) and a family of mechanisms for this class which we call *VCGc mechanisms* (Def. 5). We show that VCGc mechanisms are truthful for consistent problems (Theorem 6) and prove that, under some natural assumptions, VCGc mechanisms are the only truthful mechanisms for consistent problems (Theorem 8).

DEFINITION 4 (CONSISTENT PROBLEM) *A* MDMax *problem is* consistent *if (i) μ is a consistent objective function, i.e., for any instance $\mathcal{I} = (I_P, I)$, with $I = \langle y_1, \ldots, y_k \rangle$, and for any $X \in \Phi(\mathcal{I})$, it holds that $\mu(X, \mathcal{I}) = \bigoplus_i v_i(X, y_i)$, where '$\oplus$' is a suitable operator which enjoys the following properties: associativity, commutativity and monotonicity in its arguments; (ii) the utility function is such that*

$$\forall a_i \quad \forall X \in \overline{\Phi}(I_P) \quad \forall P_i \quad u_i(X, t_i, P_i) = v_i(X, t_i) \oplus P_i.$$

The class of all consistent problems is denoted as consistent*.*

DEFINITION 5 (VCGC MECHANISMS) *A (optimal) mechanism $(\mathcal{A}, \mathcal{P})$ for a consistent problem is a* VCGc mechanism *if, for all i, there exists a function $h_i(\mathcal{I}_{-i})$ such that, denoted $\mu_{-i}(X, \mathcal{I}) = \bigoplus_{j \neq i} v_j(X, y_j)$:*

$$p_i(\mathcal{I}) = \mu_{-i}(\mathcal{A}(\mathcal{I}), \mathcal{I}) \oplus h_i(\mathcal{I}_{-i}). \tag{3}$$

The following theorem generalizes the (proof of the) analogous result in [10] about the truthfulness of VCG mechanisms for utilitarian problems (i.e., the case '$\oplus$'='$+$'). Noticeably, it exploits Constraint (1) (see [12]).

THEOREM 6 *A VCGc mechanism for a consistent problem is truthful.*

We next show that, under some natural assumptions, VCGc mechanisms are the only truthful mechanisms for consistent problems.

DEFINITION 7 (THE CLASS $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{only}}$.) *A consistent problem Π belongs to $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{only}}$ if its operator enjoys the following properties: identity element $i_\oplus$, inverse and strict monotonicity,*[3] *and the type spaces are complete, (i.e., $\forall \mathcal{I}$, $\forall i$, $\{v_i(\cdot, y_i) \mid y_i \in \Theta_i\} = \{f : \Phi(\mathcal{I}) \mapsto \mathbb{R}\}$).*

The proof of the following theorem is a non-trivial adaptation of the proof of a similar result for (a subclass of) utilitarian problems in [9] (see [12]). However, our result is stronger since it shows that *every* consistent problem in $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{only}}$ has essentially a "unique" truthful mechanism: the VCGc mechanism in Def. 5, where the only degree of freedom is on the definition of the function $h_i(\cdot)$.

THEOREM 8 *Let $(\mathcal{A}, \mathcal{P})$ be a truthful mechanism for a problem $\Pi \in \mathsf{C}^{\mathsf{vcgc}}_{\mathsf{only}}$. Then, $(\mathcal{A}, \mathcal{P})$ is a VCGc mechanism for Π.*

3.1 The voluntary participation condition

In practical applications, agents have the freedom/right to put themselves out of the "game" if the final mechanism outcome (i.e., the utility) turns out to be disadvantageous for them. For example, consider the case in which the valuation $v_i(X, t_i)$ represents a cost required to a_i in order to implement the solution X and $p_i(\mathcal{I}_R)$ is the amount of money that a_i receives for that. Agent a_i has the freedom to refuse the payments and to not implement the solution, if the utility deriving from $v_i(X, t_i)$ and $p_i(\mathcal{I}_R)$ is less than 0 (i.e., the utility in case agent a_i does not perform any work nor receives money).

DEFINITION 9 (THE CLASS $\mathsf{C}_{\mathsf{vp}}^{\mathsf{vcgc}}$.) *A consistent problem Π belongs to $\mathsf{C}_{\mathsf{vp}}^{\mathsf{vcgc}}$ if the operator enjoys the following properties: identity element, inverse and strict monotonicity, and*

$$\forall a_i \quad \emptyset \neq \Phi(\mathcal{I}_{-i}) \subseteq \Phi(\mathcal{I}). \tag{4}$$

The following theorem gives a sufficient condition for the existence of VCGc mechanisms which satisfies VP (see Def. 3).

THEOREM 10 *Let Π be a consistent problem in $\mathsf{C}_{\mathsf{vp}}^{\mathsf{vcgc}}$ and $(\mathcal{A}, \mathcal{P})$ be the VCGc mechanism for Π with $h_i(\mathcal{I}_{-i}) = \mu_{-i}(\mathcal{A}(\mathcal{I}_{-i}), \mathcal{I}_{-i})^{-1}$. Then $(\mathcal{A}, \mathcal{P})$ satisfies VP.*

Proof. Consider $X = \mathcal{A}\langle \mathcal{I}_{-i}, t_i \rangle$ and $P_i = p_i\langle \mathcal{I}_{-i}, t_i \rangle$, for any $\mathcal{I}_{-i}$. Since $\mathcal{A}(\mathcal{I}_{-i}) \in \Phi(\mathcal{I}_{-i})$, it holds that $\mu(\mathcal{A}(\mathcal{I}_{-i}), \langle \mathcal{I}_{-i}, t_i \rangle) = \mu_{-i}(\mathcal{A}(\mathcal{I}_{-i}), \mathcal{I}_{-i}) \oplus v_i^0$. Moreover, by Def.s 4 and 5, $u_i(X, t_i, P_i) = v_i(X, t_i) \oplus \mu_{-i}(X, \langle \mathcal{I}_{-i}, t_i \rangle) \oplus h_i(\mathcal{I}_{-i})$, and, by associativity, monotonicity and existence of the inverse:

$$\begin{aligned} u_i(X, t_i, P_i) &= \mu(X, \langle \mathcal{I}_{-i}, t_i \rangle) \oplus \left(\mu_{-i}(\mathcal{A}(\mathcal{I}_{-i}), \mathcal{I}_{-i})^{-1} \oplus (v_i^0)^{-1} \oplus v_i^0\right) \\ &= \mu(X, \langle \mathcal{I}_{-i}, t_i \rangle) \oplus \mu(\mathcal{A}(\mathcal{I}_{-i}), \langle \mathcal{I}_{-i}, t_i \rangle)^{-1} \oplus v_i^0. \end{aligned} \tag{5}$$

>From Condition 4 and from the optimality of $\mathcal{A}$, if follows that $\mu(X, \langle \mathcal{I}_{-i}, t_i \rangle) \geq \mu(\mathcal{A}(\mathcal{I}_{-i}), \langle \mathcal{I}_{-i}, t_i \rangle)$. From the monotonicity of '$\oplus$', we obtain $\mu(X, \langle \mathcal{I}_{-i}, t_i \rangle) \oplus \mu(\mathcal{A}(\mathcal{I}_{-i}), \langle \mathcal{I}_{-i}, t_i \rangle)^{-1} \geq i_{\oplus}$. This, Eq. 5, the monotonicity of '$\oplus$' yield $u_i(X, t_i,$-$P_i) \geq v_i^0$. Hence the theorem follows. □

3.2 Applications to non-utilitarian problems

We now provide two examples of non-utilitarian consistent problems whose operator is '$\oplus$'='$\cdot$' (the MRP problem) and '$\oplus$'='$\min$' (the α-RENT TASK SCHEDULING problem).

The MOST RELIABLE PATH **(**MRP**) problem.** Before introducing the MRP problem, let us consider a general framework in which a truthful mechanism has to be designed on a directed weighted graph $G = (V, E, w)$ that has an edge weight $w_e \in \Theta$ associated with each edge $e \in E$. We are given $s, t \in V$, called the source and the destination, respectively The goal is to find a path from s to t which maximizes the product of the edge weights. Each edge e is owned by a distinct selfish agent a_e [4] which knows the weight $w_e \in \Theta$ (i.e., her type). In the following, we will refer to this problem as a LONGEST MULTIPLICATIVE PATH problem (LMP[Θ]).

The LMP[Θ] problem can be formalized as a consistent problem whenever the valuation functions $v_e(\cdot)$ and the utility functions $u_e(\cdot)$ satisfy

$$v_e(\pi, y_e) = \begin{cases} v_i^0 = 1 & \text{if } e \text{ is not on the path } \pi, \\ y_e & \text{otherwise,} \end{cases} \tag{6}$$

and $u_e(\pi, w_e, P_e) = v_e(\pi, w_e) \cdot P_e$. Since the set of feasible solutions depends ont the topology of the graph only, for 2-connected graphs,[5] Constraint 4 is met. Moreover, for every $\Theta \subseteq \mathbb{R}^+$, the standard product operator is strictly monotone, thus implying that $\mathrm{LMP}[\Theta] \in \mathsf{C}_{\mathsf{vp}}^{\mathsf{vcgc}}$. Hence, by Theorem 10 we obtain the following:

COROLLARY 11 *For every $\Theta \subseteq \mathbb{R}^+$, there exists a truthful mechanism $(\mathcal{A}, p)$ for $\mathrm{LMP}[\Theta]$ which, for 2-connected graphs, also meets VP. In this case, for every $e \in E$, if $X = \mathcal{A}(\mathcal{I}_T)$ and $X_{-e} = \mathcal{A}(\mathcal{I}_{T-e})$:*

$$p_e(\mathcal{I}) = \frac{\mu_{-e}(\mathcal{A}(\mathcal{I}), \mathcal{I})}{\mu_{-e}(\mathcal{A}(\mathcal{I}_{-e}), \mathcal{I}_{-e})} \tag{7}$$

$$u_e(X, t_e, p_e(\mathcal{I}_T)) = \frac{\mu(X, \mathcal{I}_T)}{\mu_{-e}(X_{-e}, \mathcal{I}_{T-e})}. \tag{8}$$

In the following we apply the above result to the MRP problem discussed in Sect. 1. In particular, the message is forwarded from one node to the next one until either (i) the message reaches the destination t or (ii) the link fails. In the latter case, the transmission is lost and a “dummy” message is forwarded throughout the selected path in place of the original one.

In order to satisfy Eq. 6, we use the following rule for the agents' payment. If edge e is not on the chosen path, then the corresponding agent receives a payment equal to $P_e = 1$. Moreover, an agent in the selected path is rewarded after (and only if) her link has *successfully* forwarded the message. Hence, the *true* agent's expected utility is $q_e P_e$. It is easy to see that the MRP problem is the $\mathrm{LMP}[(0,1)]$ problem. Corollary 11 implies the existence of a truthful mechanism $(\mathcal{A}, \mathcal{P})$ which, if at least two disjoint st-paths exists, also meets VP. In this case, Eq.s 7 and 8 yield the following intuitive interpretation of payments and of utilities, respectively:

$$p_e(\mathcal{I}_T) = \frac{Pr[\text{no link in } \pi \text{ fails} | \; e \text{ does not fail}]}{Pr[\text{no link in } \pi_{-e} \text{ fails}]}$$

$$u_e(\pi, q_e, p_e(\mathcal{I}_T)) = \frac{Pr[\text{no link in } \pi \text{ fails}]}{Pr[\text{no link in } \pi_{-e} \text{ fails}]},$$

where π is the best st-path and π_{-e} denotes the best st-path not containing e.

>From Corollary 11 it is possible to obtain analogous results for the ARBITRAGE problem, which is discussed in [12].

The α-RENT TASK SCHEDULING problem. We are given k tasks which need to be allocated to n machines, each of them corresponding to one agent. Let t_j^i denote the minimum amount of time machine i is capable of performing task j and let X_i be the the set of tasks allocated to agent a_i. The goal is to minimize the makespan, that is, the maximum, over all machines, completion time. The type of agent i is given by $t_i = \langle t_1^i, \ldots, t_k^i \rangle$, thus implying $I_T = \langle t_1, \ldots, t_n \rangle$, $I_P = \langle k, n \rangle$ and $\mathcal{I}_T = (I_P, I_T)$. The set of feasible solutions $\Phi(\mathcal{I})$ is the set of all partitions $X = X_1, \ldots, X_n$ of the tasks, where X_i denotes the tasks allocated to agent a_i. For any $\mathcal{I}$, we define

$v_i(X, t_i) = -\sum_{j \in X_i} t^i_j$, that is, the completion time of machine i. Agent a_i is not involved in the solution X if $X_i = \emptyset$. In this case, $v_i(X, \cdot) = 0 = v^0_i$.

We consider the following variant of the TASK SCHEDULING problem defined in [13]. An assignment has to be computed according to the reported types. Each machine i that has been selected (i.e., $X_i \neq \emptyset$) is *rented* for the duration required to perform the tasks assigned to it. The corresponding agent must then receive an amount of money *not larger* than $\alpha - \sum_{j \in X_i} t^i_j = \alpha + v_i(X, t_i)$, where α is a fixed constant equal for all machines. Incentives are provided by defining, for each machine/agent, a *maximum* payment M_i that the machine i will receive if used. In particular, each rented machine is then payed the *minimum* between M_i and $\alpha + v_i(X, t_i)$.

The utility of an agent i is naturally defined as the amount of money derived from the renting of her machine, that is, $\min\{\alpha + v_i(X, t_i), M_i\}$. By letting $P_i := M_i - \alpha$, the previous quantity can be rewritten as

$$\min\{\alpha + v_i(X, t_i), M_i\} = \alpha + \min\{v_i(X, t_i), P_i\}.$$

To formalize the problem as a consistent problem with operator '$\oplus$'='$\min$' it suffices to define $u_i(X, t_i, P_i) = \min(v_i(X, t_i), P_i)$, and to observe that $\mu(X, \mathcal{I}) = \max^n_{i=1} -v_i(X, y_i) = \min^n_{i=1} v_i(X, y_i)$. Hence Theorem 6 implies the following:

COROLLARY 12 *The* α-RENT TASK SCHEDULING *problem is consistent. Hence, it admits a truthful mechanism.*

The fact that the only difference between the α-RENT TASK SCHEDULING problem and the TASK SCHEDULING problem in [13] is on the utility function provides an interesting comparison, since in [13] the authors proved that no exact (or even 2-approximate non-polynomial-time) truthful mechanism exists. Corollary 12 shows that this is due to the fact that the utility functions are quasi-linear.

REMARK 3.1 (ON THE VOLUNTARY PARTICIPATION) *Observe that no mechanism for the* α-RENT TASK SCHEDULING *problem can guarantee the VP condition. Indeed, it suffices to consider instances for which* $\min\{t^i_j\} > \alpha$*, in which case the utilities are always negative. Hence,* α-RENT TASK SCHEDULING $\notin \mathsf{C}^{\mathsf{vcgc}}_{\mathsf{vp}}$.

4. Impossibility results

In this section we investigate extensions of our positive result (Theorem 6) to problems obtained by removing Constraint (1) in the definition of consistent:

DEFINITION 13 (RELAXED CONSISTENT PROBLEM) *A problem is a* relaxed consistent problem *if it satisfies all constraints of Def. 1 except for Constraint (1), as well as the two items in Def. 4. The class of all relaxed consistent problems is denoted as* relaxed consistent*.*

In Sect.s 4.1 and 4.2 we define two subclasses of relaxed consistent and show that problems in these two classes do not admit truthful VCGc mechanisms (Theorem 15) and truthful mechanisms (Theorem 20), respectively. We also prove that the latter class in included in the former (Theorem 20).

4.1 A class with no truthful VCGc mechanisms

Intuitively speaking, we next consider a class of problems for which some non-feasible solution $\hat{X}$ has a measure strictly better than any feasible solution. Moreover, such an unfeasible solution can be output when reporting a false input $\hat{\mathcal{I}}$, that is, $\hat{X} = \mathcal{A}(\hat{I}) \in \Phi(\hat{\mathcal{I}})$. Formally, we have the following:

DEFINITION 14 (THE CLASS $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$.) *A problem* Π *is said to be in the class* $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$ *if it is relaxed consistent and the following holds: (i) the operator '*$\oplus$*' satisfies* strict monotonicity*; (ii) there exist* i, $\widetilde{I} = \langle \widetilde{y}_1, \ldots, \widetilde{y}_k \rangle$ *and* $\widehat{y}_i \in \Theta_i$ *(*$y_i \neq \widehat{y}_i$*) such that, for* $\widetilde{\mathcal{I}} = (I_P, \widetilde{I})$ *and* $\widehat{\mathcal{I}} = \langle \mathcal{I}_{-i}, \widehat{y}_i \rangle$*, it holds that*

$$\mathcal{A}(\widehat{\mathcal{I}}) \notin \Phi(\widetilde{\mathcal{I}}) \text{ and } \mu(\mathcal{A}(\widehat{\mathcal{I}}), \widetilde{\mathcal{I}}) > \mu(\mathcal{A}(\widetilde{\mathcal{I}}), \widetilde{\mathcal{I}}). \tag{9}$$

THEOREM 15 *No problem* $\Pi \in \mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$ *admits a truthful VCGc mechanism.*

Proof. Let $(\mathcal{A}, \mathcal{P})$ be a VCGc truthful mechanism for Π and be $\widetilde{X} = \mathcal{A}(\widetilde{\mathcal{I}})$ and $\widehat{X} = \mathcal{A}(\widehat{\mathcal{I}})$. Then: $u_i(\widetilde{X}, \widetilde{y}_i, p_i(\widetilde{\mathcal{I}}))$ = (by Def.s 4, 5) $v_i(\widetilde{X}, \widetilde{y}_i) \oplus (\mu_{-i}(\widetilde{X}, \widetilde{\mathcal{I}}) \oplus h_i(\widetilde{\mathcal{I}}_{-i}))$ = (by associativity of '$\oplus$' and by Def. 4) $\mu(\widetilde{X}, \widetilde{\mathcal{I}}) \oplus h_i(\widetilde{\mathcal{I}}_{-i})$ < (by Eq. 9 and strict monotonicity of '$\oplus$') $\mu(\widehat{X}, \widetilde{\mathcal{I}}) \oplus h_i(\widetilde{\mathcal{I}}_{-i})$ = (by Def.s 13, 14) $u_i(\widehat{X}, \widetilde{y}_i, p_i(\widehat{\mathcal{I}}))$. This contradicts the truthfulness of $(\mathcal{A}, \mathcal{P})$. □

In the following we provide two examples of problems in the class $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$ which, by Theorem 15, do not admit a truthful VCGc mechanism: KNAPSACK and the 2ND SHORTEST PATH.

The KNAPSACK **problem.** We consider the so called variant 0-1 KNAPSACK of the classical optimization problem, which can be described as follows. We are given a set of n items $\{1, \ldots, n\}$, each one characterized by a *profit* π_i and a *size* σ_i. The goal is to find a set of items such that its total occupancy does not exceed a given capacity B and the total profit is maximized. Hence, the set of feasible solutions is $\Phi(\mathcal{I}) = \{X \in \{0,1\}^n \mid \sum_{i=1}^{n} X_i \sigma_i \leq B\}$ and the total profit of a solution $X \in \Phi(\mathcal{I})$ is given by $\mu(X, \mathcal{I}) = \sum_{i=1}^{n} X_i \pi_i$.

Each item i is associated with an agent a_i that holds a part of the instance and derives from the outcome a utility $u_i(X, t_i, P_i) = P_i + v_i(X, t_i)$, where $v_i(X, y_i) = X_i \pi_i$. Depending on how the private part of the instance is defined we distinguish the following three problem versions, which have have a natural application to the use of a shared communication channel of limited capacity and to a problem of "selling" part of a web page (typically, a marginal strip of fixed width/height) for putting some advertisements (see [6] for a description of the model):

- KNAPSACK[π], where each agent a_i only holds the profit $\pi_i = t_i$ associated with each item i, whereas every size σ_i is public knowledge.
- KNAPSACK[σ], where each agent a_i only holds the size $\sigma_i = t_i$ associated with each item i, whereas every profit π_i is public knowledge.
- KNAPSACK[π, σ] where each agent a_i holds both the profit π_i and the size σ_i associated with each item i, that is, $t_i = \langle \pi_i, \sigma_i \rangle$.

It is worth noticing that only KNAPSACK$[\pi]$ meets Constraint (1), as sizes are public knowledge and $\Phi(\mathcal{I})$ is constant. Then, this proves Theorem16. On the contrary, KNAPSACK$[\sigma]$ and KNAPSACK$[\pi, \sigma]$ satisfy Def. 4 except for Constraint (1). In these case we can state Theorem17.

THEOREM 16 KNAPSACK$[\pi] \in \mathsf{C}^{\mathsf{vcgc}}_{\mathsf{vp}}$. *Hence, it admits a truthful mechanism which also meets VP.*

THEOREM 17 KNAPSACK$[\sigma]$, KNAPSACK$[\pi, \sigma] \in \mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$. *Hence, they do not admit a truthful VCGc mechanism.*

The 2ND SHORTEST PATH **problem.** Let us consider an undirected weighted graph $G = (V, E, w)$ and two nodes $s, t \in V$. The objective is to find a path whose length is minimal among all st-paths that have no minimal length in G. More formally, for any instance $\mathcal{I} = G$, if Φ_{st} is the set of all st-paths in (V, E) and $X_1^*(\mathcal{I}) \subseteq \Phi_{st}$ is the subset of the shortest st-paths, $\Phi(\mathcal{I}) = \Pi_{st}(\mathcal{I}) \setminus X_1^*(\mathcal{I})$. Similarly to the SHORTEST PATH problem mentioned in [13], the valuation function of the agent owing edge e is equal to

$$v_e(\pi, \mathcal{I}_R) = \begin{cases} -r_e & \text{if } e \in \pi, \\ 0 & \text{otherwise.} \end{cases}$$

Utilities are quasi-linear and the objective function is the total weight of the path, that is, $\sum_{e \in \pi} r_e$. By letting $\mu(\pi, \mathcal{I}_R) = \sum_{e \in \pi} -r_e$, and by observing that $\mu(\pi, \mathcal{I}_R) = \sum_{e \in \pi} v_e(\pi, \mathcal{I}_R)$, we can easily prove the following result:

THEOREM 18 *The* 2ND SHORTEST PATH *problem is in* $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$. *Hence, It does not admit a truthful VCGc mechanism.*

In the next section we will strengthen the results of Theorem 17 and of Theorem 18.

4.2 A class with no truthful mechanisms

We next provide a general technique to prove the non-existence of truthful mechanisms for a given problem. We will then apply this result to the KNAPSACK$[\pi, \sigma]$ and to the 2ND SHORTEST PATH problems and show that the reason why VCGc mechanisms fail is not due to its weakness.

DEFINITION 19 (THE CLASS $\mathsf{C}_{\mathsf{none}}$.) *A problem* Π *is said to be in the class* $\mathsf{C}_{\mathsf{none}}$ *if it relaxed consistent and the following holds: (i) the operator '$\oplus$' satisfies strict monotonicity; (ii) there exist* i, ν, $\widetilde{I} = \langle \widetilde{y}_1, \ldots, \widetilde{y}_k \rangle$ *and* $\widehat{y}_i \in \Theta_i$ *(*$y_i \neq \widehat{y}_i$*) such that, for* $\widetilde{\mathcal{I}} = (I_P, \widetilde{I})$ *and* $\widehat{\mathcal{I}} = \langle \widetilde{\mathcal{I}}_{-i}, \widehat{y}_i \rangle$*, it holds that*

$$\mathcal{A}(\widehat{\mathcal{I}}) \notin \Phi(\widetilde{\mathcal{I}}) \;\wedge\; v_i(\mathcal{A}(\widehat{\mathcal{I}}), \widetilde{y}_i) > v_i(\mathcal{A}(\widehat{\mathcal{I}}), \widehat{y}_i) \;\wedge$$
$$v_i(\mathcal{A}(\widehat{\mathcal{I}}), \cdot) \neq \nu \;\wedge\; v_i(\mathcal{A}(\widetilde{\mathcal{I}}), \cdot) = \nu. \quad (10)$$

The class $\mathsf{C}_{\mathsf{none}}$ enjoys the properties stated by the following theorem (see [12]):

THEOREM 20 *The class* $\mathsf{C}_{\mathsf{none}}$ *is included in* $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$*. Moreover, no problem* $\Pi \in \mathsf{C}_{\mathsf{none}}$ *admits a truthful mechanism.*

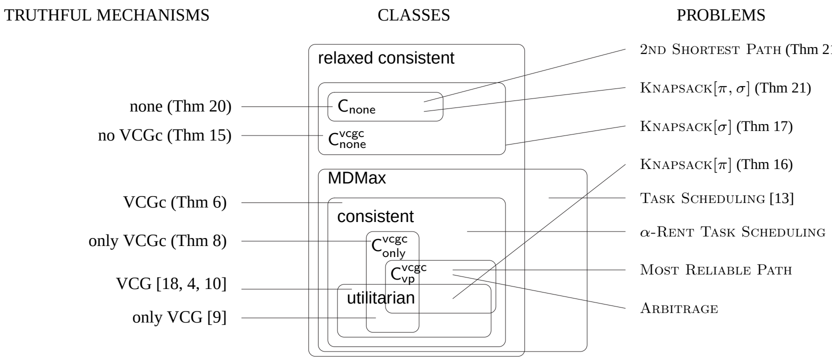
The next result show that, in the case of the 2ND SHORTEST PATH and KNAPSACK$[\pi, \sigma]$ problems, VCGc mechanisms do not fail because inappropriate. Indeed, it can be proved that:

THEOREM 21 *Both the* 2ND SHORTEST PATH *and the* KNAPSACK$[\pi, \sigma]$ *problems are in* $\mathsf{C}_{\mathsf{none}}$*. Hence, none of them admits a truthful mechanism.*

REMARK 4.1 (NECESSITY OF CONSTRAINT (1)) *Observe that if we remove Constraint (1) from the definition of consistent problems, then we obtain the class* relaxed consistent *(Def. 13). Theorem 21 implies that* $\emptyset \neq \mathsf{C}_{\mathsf{none}} \subseteq$ relaxed consistent*. Hence, Constraint (1) is necessary for guaranteeing the existence of truthful mechanisms.*

5. Conclusions and open problems

In the following figure we summarize the results obtained in this work. In particular, we have isolated several classes of problems involving selfish agents which are defined according to some mathematical properties. The inclusions mostly follow



from the definitions, except for the result of Theorem 20. Moreover, the results on the α-RENT TASK SCHEDULING problem and the fact that $\mathsf{C}_{\mathsf{none}} \neq \emptyset$ imply that $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{vp}} \subsetneq$ consistent $\subsetneq$ relaxed consistent. Since the TASK SCHEDULING problem in [13] can be formulated as a MDMax problem, the negative results in [13] also implies that consistent $\subsetneq$ MDMax. It would be interesting to prove analogous separation results among the classes. For instance, if KNAPSACK$[\sigma]$ had a truthful mechanism, then we would obtain $\mathsf{C}_{\mathsf{none}} \subsetneq \mathsf{C}^{\mathsf{vcgc}}_{\mathsf{none}}$. Combinatorial auction is a classic utilitarian problem (see e.g. [14]) which admits VCG mechanisms only. If would be interesting to find a *non-utilitarian* problem in $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{only}}$. Comparing $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{only}}$ and $\mathsf{C}^{\mathsf{vcgc}}_{\mathsf{vp}}$ would be also worthwhile. Investigating classes for which mechanisms that use non-optimal algorithms $\mathcal{A}$ remain truthful is an important issue. Interestingly, Theorem 6 also holds when algorithm $\mathcal{A}$, though non-optimal, is *maximal in its range* (see [14]), thus generalizing one of the results in [14] for utilitarian problems.

Notes

1. We assume that the costs for transmitting are negligible, say equal 0.

2. The term 'payment' does not necessarily mean money as it actually denotes any form of incentive.

3. The inverse of x is denoted by x^{-1} and satisfies $x \oplus x^{-1} = i_{\oplus}$. We say that an operator $\oplus$ satisfies *strict monotonicity* if for every a,a' and b, with $a < a'$, it holds that $a \oplus b < a' \oplus b$.

4. The existence of truthful mechanisms easily extends to a more general setting where each agent owns multiple edges.

5. If the graph is not 2-connected then the problem breaks down to independent subproblems (2-connected components). In this case, it is easy to see that the VP condition cannot be fulfilled.

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