Discrete Event Systems
Exercise 3

1 Regular Languages and Finite Automaton

Consider the NFA \( A \) in Figure 1 and assume that \( \Sigma = \{0, 1\} \).

(i) Transform the NFA into an equivalent deterministic finite automaton.

(ii) Which regular language is accepted by \( A \)?

2 Non-Regular Languages

(i) Consider the following language \( L_1 \):

\[ L_2 = \{0^a1^b0^c1^d \mid a, b, c, d \geq 0 \text{ and } a = 1, b = 2, \text{ and } c = d \} \]

Is the language \( L_1 \) regular? Prove your answer!

(ii) Consider the following slightly adapted language \( L_2 \):

\[ L_2 = \{0^a1^b0^c1^d \mid a, b, c, d \geq 0 \text{ and if } a = 1 \text{ and } b = 2, \text{ then } c = d \} \]

Is the language \( L_2 \) regular? Be careful when proving your answer!

\(^1\text{All problems in this series have appeared in previous exams.}\)
3 Adapting a Finite Automaton

Consider the DFA in Figure 3, which accepts the language $L$ and let the alphabet be $\Sigma = \{0, 1\}$. Further, let $\Phi(L)$ be defined as $\Phi(L) = \{w \in \Sigma^* \mid \exists x \in \Sigma^*, |x| = |w| \text{ and } wx \in L\}$. That is, $\Phi(L)$ denotes the set of first halves of all strings in $L$.

(i) Give a regular expression that describes the language $L$.

(ii) Construct a DFA which accepts a string $w$ if and only if $w \in \Phi(L)$. 

![Figure 2: DFA B.](image-url)