Theorem: \( n - m + f = 2 \)

Proof:

**[Sketch]**

\[
\begin{align*}
  m = 0: & \quad \Rightarrow n - m + f = 1 - 0 + 1 = 2 \checkmark \\
  m > 0: & \quad (\text{assume formula correct for } m-1)
\end{align*}
\]

Tree

- Remove leaf
  \[ \Rightarrow n' = n - 1 \]
  \[ m' = m - 1 \checkmark \]

Not tree

- Remove edge of cycle
  \[ \Rightarrow n' = n - 1 \]
  \[ f' = f - 1 \checkmark \]

Theorem: Simple, connected, planar graph with \( n \) nodes has at most \( 3n - 6 \) edges \((n \geq 3)\)

Proof:

- Each edge bounds at most 2 faces
- Each face bounded by at least 3 edges

\[ 3f \leq 2m \]

\[ n - m + f = 2 \]

\[ 3n - 3m + 3f = 6 \quad \Rightarrow 3n - 6 = 3m - 3f \quad \Rightarrow 3m - 2m = m \]
MST ⊆ RNA

Assume Contradiction: e ∈ MST

e ∉ RNA ⇒ there is a point w in the interior (strictly)

Remove e from MST
⇒ Two trees T_u, T_v
w is e of T_u ∪ T_v
w.l.o.g. w ∈ T_u

We can reconnect T_u with T_v with the edge (w, w)
better MST? &
RNA ∈ CG

by Definition

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<th>GC</th>
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$G \subseteq DT$

by Definition

$e \in G$ if disk $(u,v)$ contains no other node

$DT: e \in DT$ if any disk with $u,v$ on boundary contains no other node

Example:

$DT \subseteq G$
MST connected by definition

DT planar by definition ... however, not quite so easy.

Let planar

Assume not

There is an angle $\geq 90^\circ$

edge $e$ exists $\Rightarrow$ angle $< 90^\circ$ $\iff$
GC contains Minimum Energy Path

Proof:

Let this be MEP

\[ s \rightarrow \ldots \rightarrow t \]

Assume two nodes are not neighbors in GC.
Then, there is a node \( w \) in the circle by \( u, w \).

\[ u \quad (\quad) \quad v \]

If \( uw \) or \( vw \) are not neighbors then you do the same again (recursively)

Otherwise: \( E(u, w) + E(w, v) = uw^k + vw^k \leq uv^k \) (for \( k \geq 2 \))

GC \ominus UDG

Def: UDG

\[ e \in E \text{ of } UDG \iff \text{let } s \bar{1} \].

GC \ominus UDG contains Minimum Energy Path

Proof as above, except the first path is MEP in UDG