Greedy Algorithm is 3-competitive

Let $\sigma$ be a sequence of calls.
Let $D$ be the number of cells in any three mutually adjacent cells.
Then $\text{cost}_{\text{opt}}(\sigma) \geq D$

Let $c_0$ be the cell that was assigned the highest frequency $a_0$.

Note $\text{cost}_{\text{GREEDY}}(\sigma) = a_0$ (by definition).

Let $N^*(c_0)$ be $c_0$ plus 6 neighboring cells.

$$\text{Calls}(N^*(c_0)) \leq D + D + D - 2 \cdot \text{Calls}(c_0)$$

$a_0 \leq 3D$

$$\frac{\text{cost}_{\text{GREEDY}}(\sigma)}{a_0} \leq 3 \cdot D \leq 3 \cdot \frac{\text{cost}_{\text{opt}}(\sigma)}{D} \leq 3$$

Remark: In some cases, cells stand next to each other.
Rand. Call Control is 2.97-competitive

\( \delta \): sequence of calls

\( X(c) \): profit that RANDOM accepted call \( c \)

\[ E[\text{benefit}_\text{random}] = \sum_{c \in \delta} X(c) \]

\( |A(\delta)| = \text{benefit}_{OPT} = |\text{calls OPT accepted}| \)

Amortized benefit:

\[ b(c) = X(c) + \sum_{c' \in N(c)} \frac{X(c')}{d(c')} \quad \text{for each } c \in A(\delta) \]

\[ d(c') = N(c) \cap ACC' \leq 3 \]

\[ E[\text{benefit}_\text{random}] = \sum_{c \in A(\delta)} b(c) \]

\( q \): probability that no call was accepted in \( N(c) \) when \( c \) is presented.

\[ b(c) = X(c) + \sum_{c' \in N(c)} \frac{X(c')}{d(c')} \]

\[ q \cdot p + (1-q) \cdot 0 + \left[ q \cdot \frac{1}{10} \right] + (1-q) \cdot \frac{4}{3} \]

\[ d(c') \leq 3 \]

\[ \Rightarrow b(c) \geq qp + (1-q)/3 \]

\[ q \geq (1-p)^6 \Rightarrow b(c) \geq (1-p)^6 (p - \frac{1}{3}) + \frac{1}{3} \]

\[ \max \]

\[ \frac{db(c)}{dp} = -6 (1-p)^5 (p - \frac{1}{3}) + (1-p)^6 \]

\[ \Rightarrow b(c) = \frac{8112}{2370625} + \frac{1}{3} = 0.33665 \]

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Linearity of expectations:

\[ E[\text{leak}\text{+ranom}] = \sum_{c \in A(\Theta)} b(c) = |A(\Theta)| \times 0.33665 \]

\[ \text{leak opt} \geq |A(\Theta)| \]

\[ S, \text{leak}\text{+ranom} \geq \text{leak opt} [\Theta] \]

\[ S \geq \frac{1}{0.33665} \approx 2.97 \]

\[ \therefore g \geq (1-p)^6 \] was best approximation.

Use different p, depending on how many cells in N(c) are marked.

Ex: N(c) marked = all 6 \( \Rightarrow p_c = 1 \)