Positioning
Chapter 9

Eidgenössische Technische Hochschule Zürich
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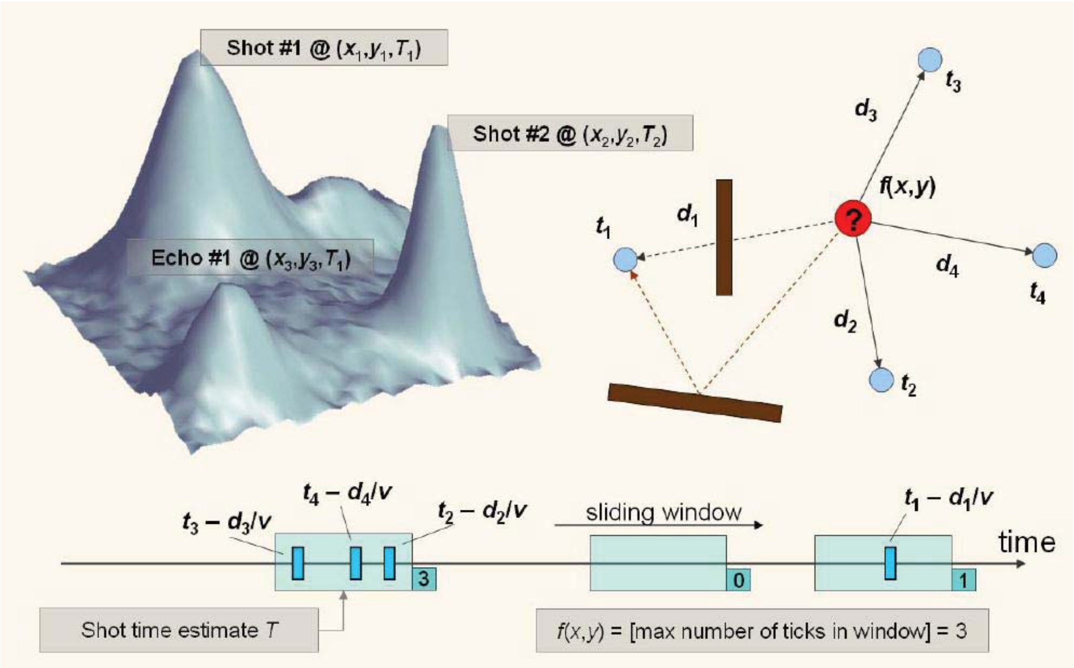
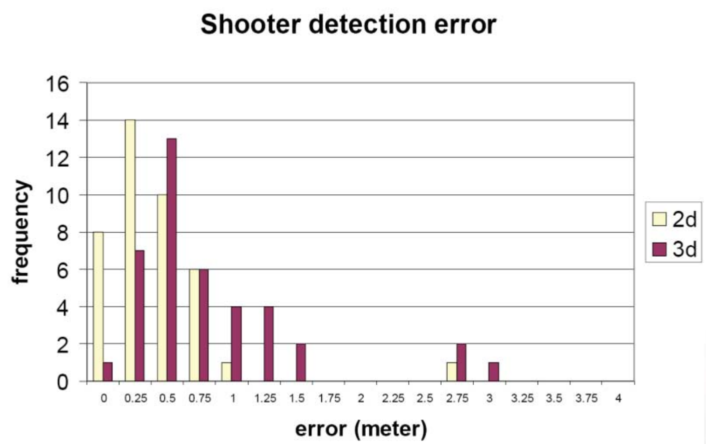
Acoustic Detection (Shooter Detection)





- Sound travels much slower than a radio signal (331 m/s)
- This allows for quite accurate distance estimation (cm)
- Main challenge is to deal with reflections and multiple events

Rating

- Area maturity

First steps — Text book

- Practical importance

No apps — Mission critical

- Theory appeal

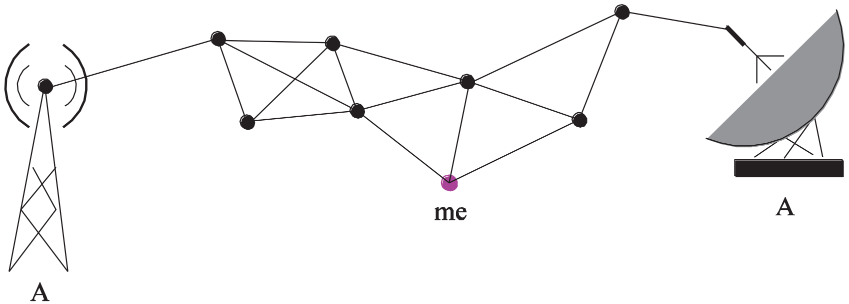
Boooooooring — Exciting

Overview

- Motivation
- GPS, Geodetics, etc.
- Measuring Distances or Angles
- Rigidity Theory
- Anchors & Virtual Coordinates
- Embeddings, Theory and Heuristics
- Boundary Recognition

Motivation

- Why positioning?
  - Sensor nodes without location information are often meaningless
  - Avoid having "costly" positioning hardware
  - Geo-routing



- Why not GPS (or Galileo)?
  - "Heavy, large, and expensive"
  - Battery drain
  - Not indoors
  - Accuracy?

- Solution: equip small fraction with GPS (anchors)

GPS

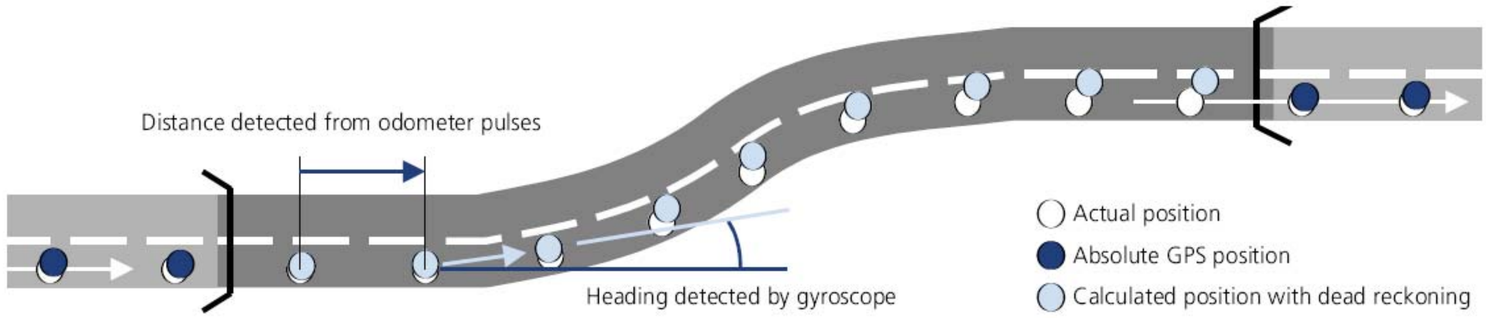
- A lot of recent progress, so attaching a GPS receiver to each sensor node becomes an alternative.

- Example, u-blox
  - footprint size: down to 4x4mm
  - Power supply: 1.8 - 4.8V
  - power consumption: 50 mW
  - power on: < 1 second
  - update rate: 4Hz
  - support for Galileo!

- So GPS is definitely becoming more attractive; however, some of the problems of GPS (indoors, accuracy, etc.) remain.

GPS Extensions

- GPS chips can be extended with other sensors such as gyroscope, direction indications, or tachometer pulses (in cars). With these add-ons, mobile devices get continued coverage indoors.



[u-blox]

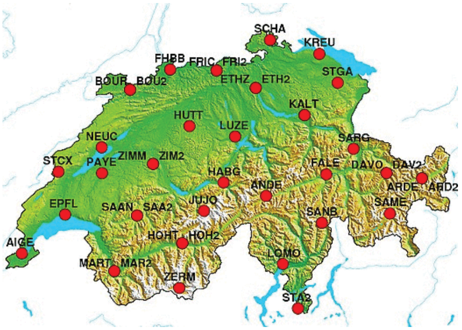
- Affordable technology has a distance error of less than 5% per distance travelled, and a direction error of less than 3 degrees per minute.

Geodetic Networks

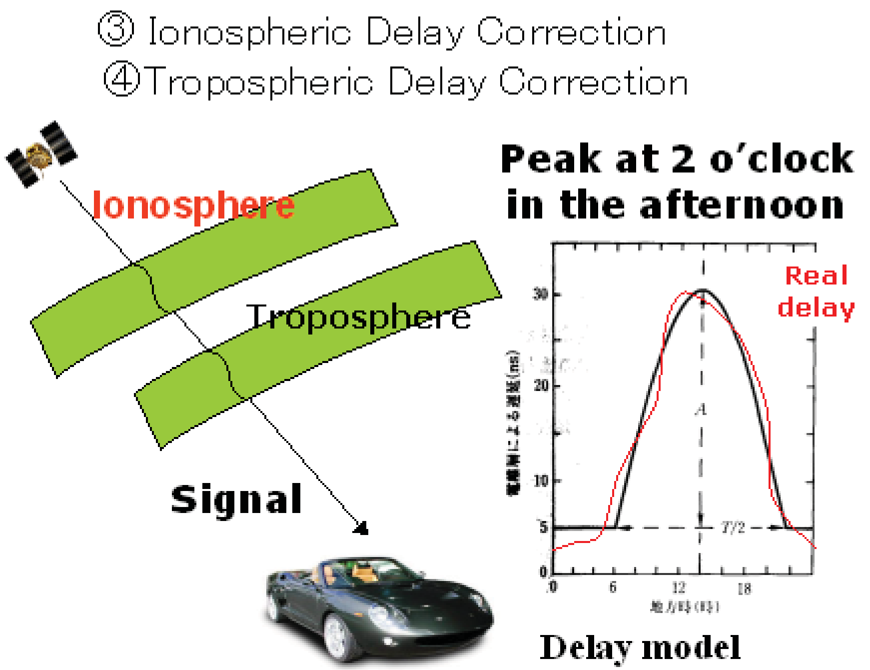
[Geodesy pics by Rainer Mautz]

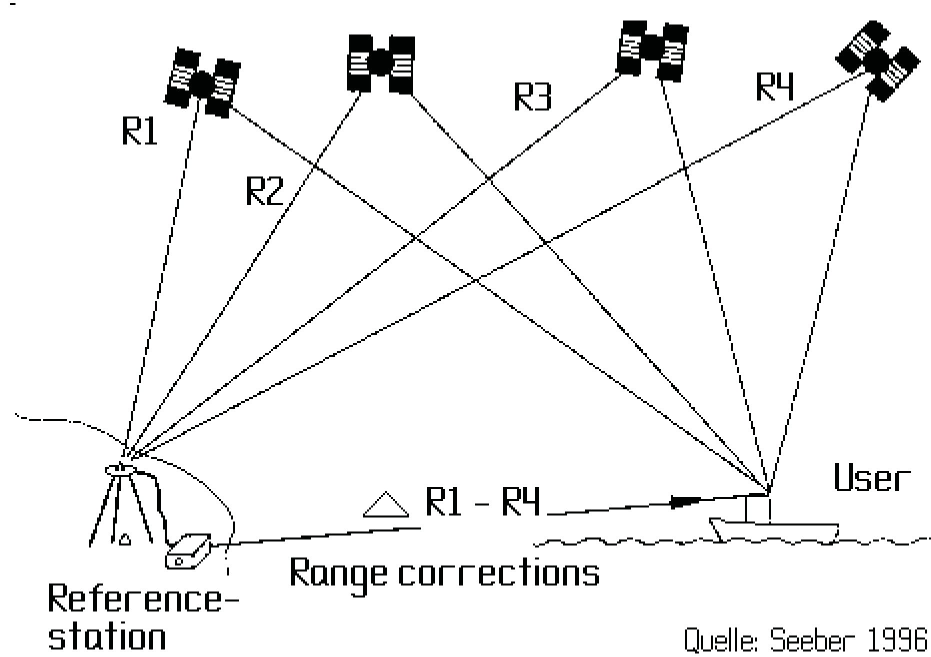
- Optical Sensors
  - Positions through reference points and object points
  - Very accurate (mm)


- GNSS
  - Global Navigation Satellite System
  - Allow phase measurements



GPS meets Geodesy





1. Use of both GPS frequencies L1 & L2
2. Phase measurements
3. DGPS (relative measurements)

→ elimination of ionosphere delays
mm accuracy

More “Distance” Sensors

- Geotechnical Sensors
  - one-dimensional, relative measurements

- Meteorological Sensors
  - correction of other sensors

- Digital Cameras

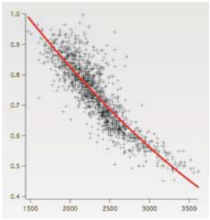
Geodesy vs. Sensor Nodes

availability:	high
timeliness:	real time
reliability:	no failures
hybrid systems:	to be avoided
local installations:	none
accuracy:	mm - cm
coverage:	global

… but:

Measurements with reasonably priced hardware

Distance estimation



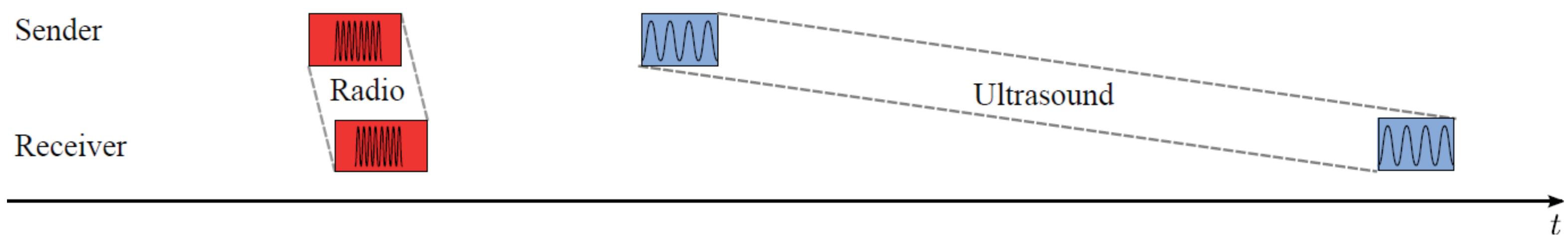
- Received Signal Strength Indicator (RSSI)
  - The further away, the weaker the received signal.
  - Mainly used for RF signals.
- Time of Arrival (ToA) or Time Difference of Arrival (TDoA)
  - Signal propagation time translates to distance.
  - Routing trip time measurements with specific hardware: accuracy 2-3m
  - Better: Mixing RF, acoustic, infrared or ultrasound.

Angle estimation

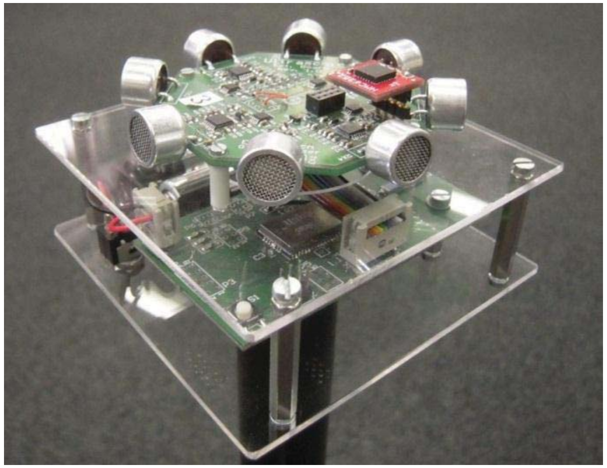
- Angle of Arrival (AoA)
  - Determining the direction of propagation of a radio-frequency wave incident on an antenna array.
- Directional Antenna
- Special hardware, e.g., laser transmitter and receivers.

Example: Measuring distance with ultrasound

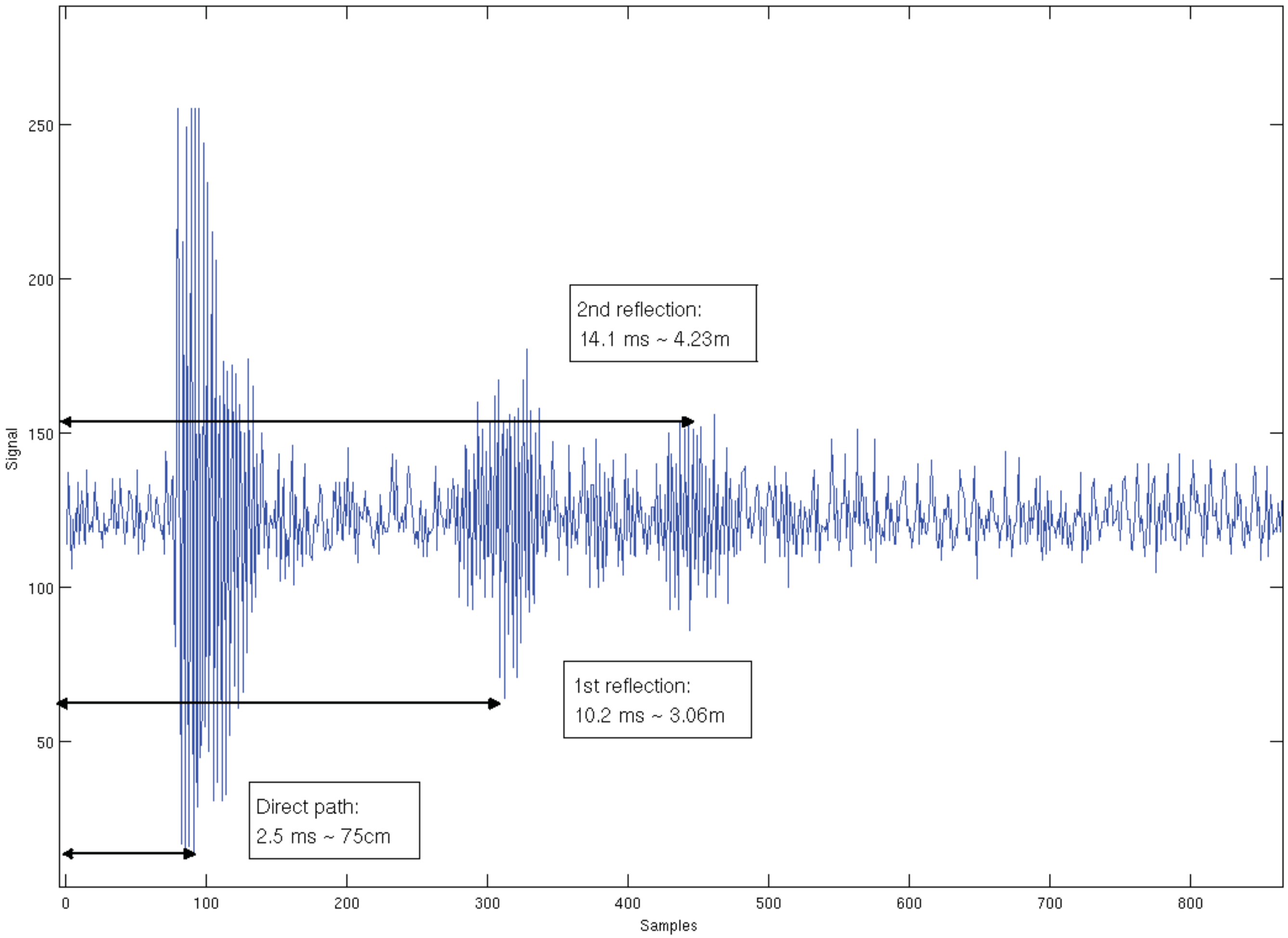
- Particularly interesting if the signal speed differs substantially, e.g. sound propagation is at about 331 m/s (depending on temperature, humidity, etc.), which is of course much less than the speed of light.



- If you have line of sight you may achieve about a 1cm accuracy. But there are problems
  - (Ultra)sound does not travel far
  - For good results you really need line of sight
  - You have to deal with reflections

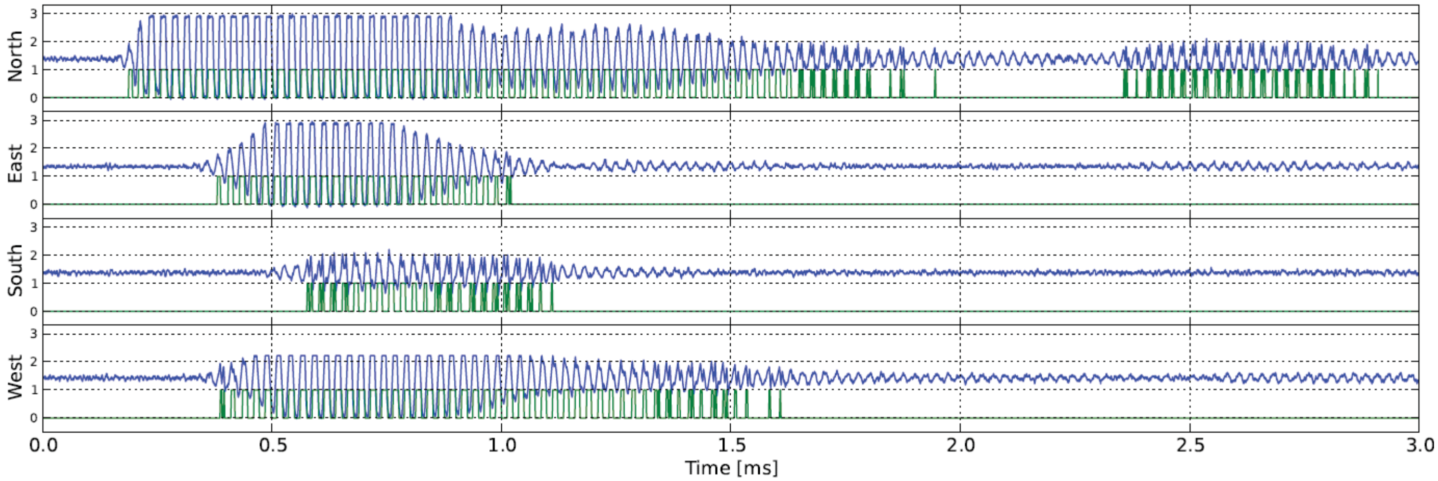


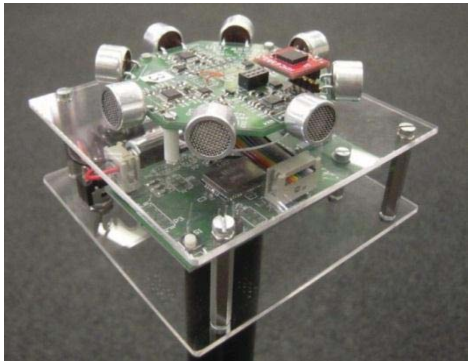
Example: Multipath effects with ultrasound

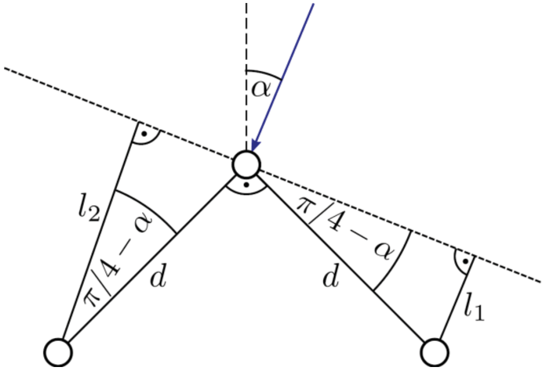


Example: Measuring angles with ultrasound

- By measuring the time of arrival at multiple receivers we can determine the angle of arrival of the ultrasound signal.





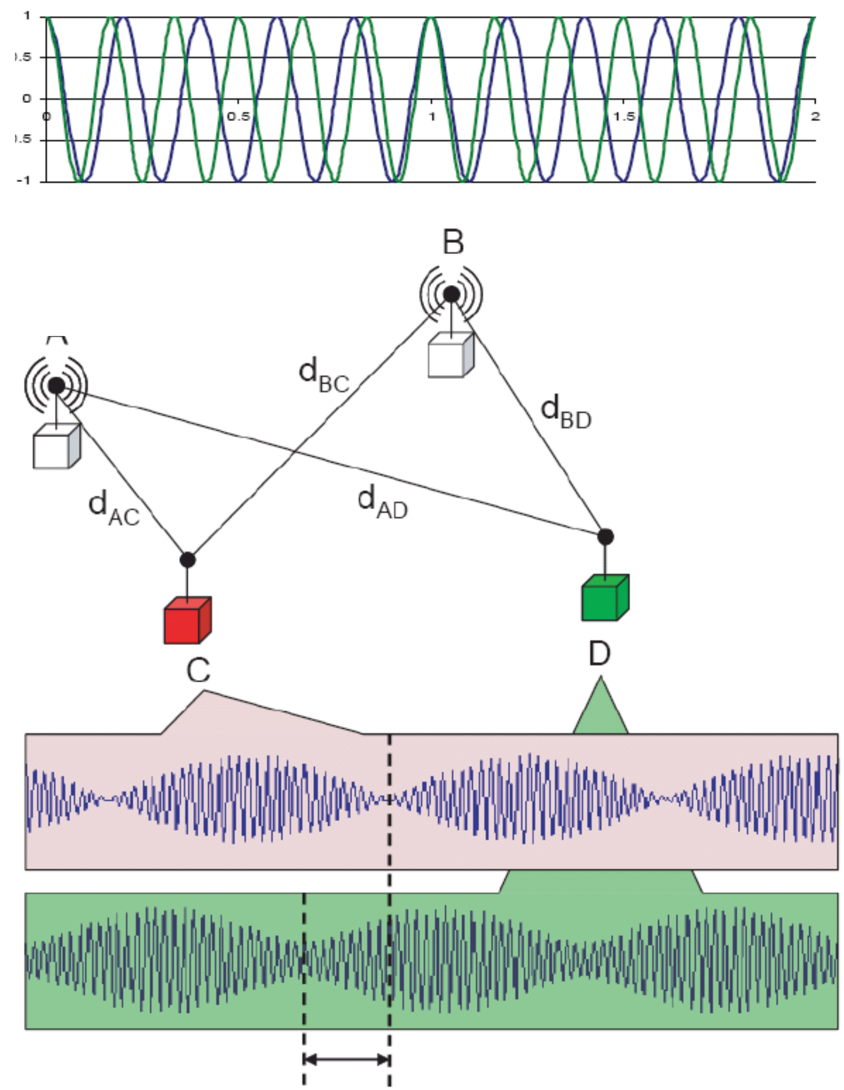


Interferometry

- Interferometry is the technique of superimposing (interfering) two or more waves, to detect differences between them

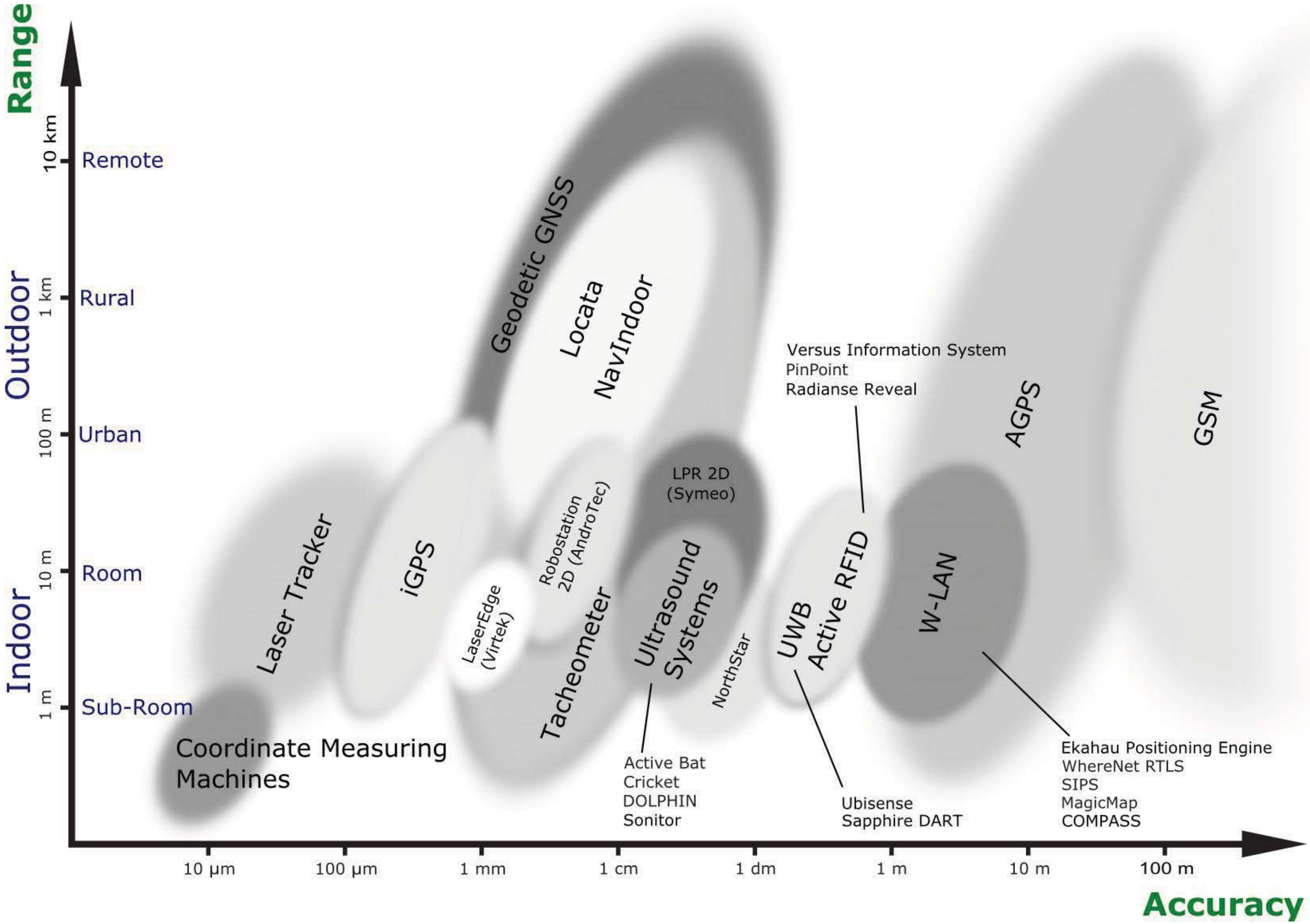
- Signals transmitted with a few hundred Hz difference at senders A and B will give different phase offsets at C and D. Using this, one can compute the total distance of the four points A, B, C, D.

- However, one needs to solve a linear equation system, and one needs very accurate time synchronization (μs order)



$$\text{phase offset} = 2\pi \frac{d_{AD} - d_{BD} + d_{BC} - d_{AC}}{\lambda_{\text{carrier}}} \pmod{2\pi}$$

Positioning Systems: An Overview



Positioning in Networks

- Task: Given distance or angle measurements or mere connectivity information, find the locations of the sensors.

- Anchor-based
  - Some nodes know their locations, either by a GPS or as pre-specified.
- Anchor-free
  - Relative location only. Sometimes called virtual coordinates.
  - Theoretically cleaner model (less parameters, such as anchor density)

- Range-based
  - Use range information (distance or angle estimation)
- Range-free
  - No distance estimation, use connectivity information such as hop count.
  - It was shown that bad measurements don’t help a lot anyway.

Overview

	distance/angle information	connectivity information only
with anchors	Positioning (Solution quality depends on anchor density)	
without anchors	Distance/Angle based Virtual Coordinates	Connectivity based Virtual Coordinates

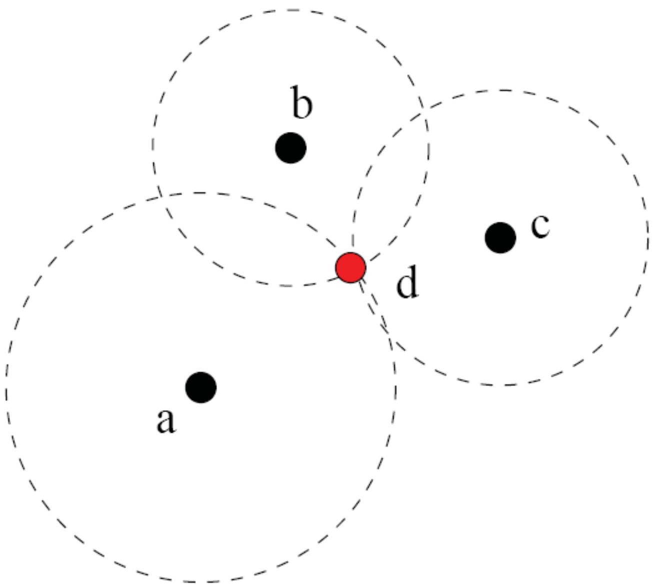
Trilateration and Triangulation

- Use geometry, measure the distances/angles to three anchors.

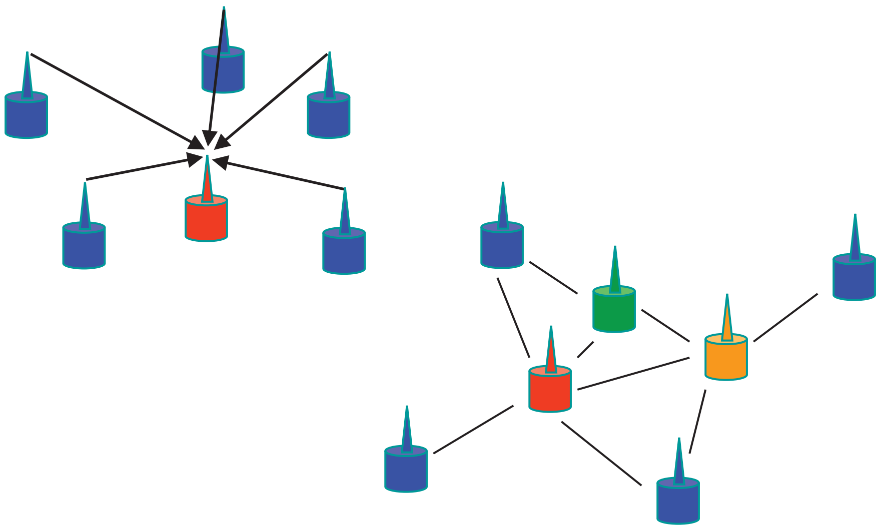
- Trilateration: use distances
  - Global Positioning System (GPS)

- Triangulation: use angles
  - Some cell phone systems

- How to deal with inaccurate measurements?
  - Least squares type of approach
  - What about strictly more than 3 (inaccurate) measurements?

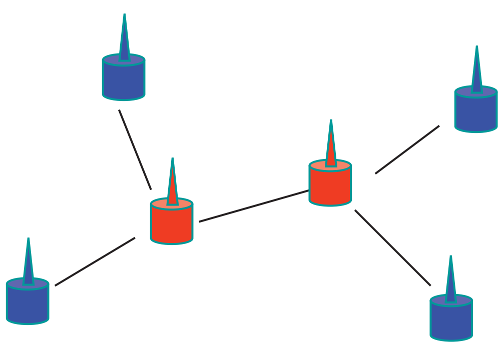


Iterative Multilateration



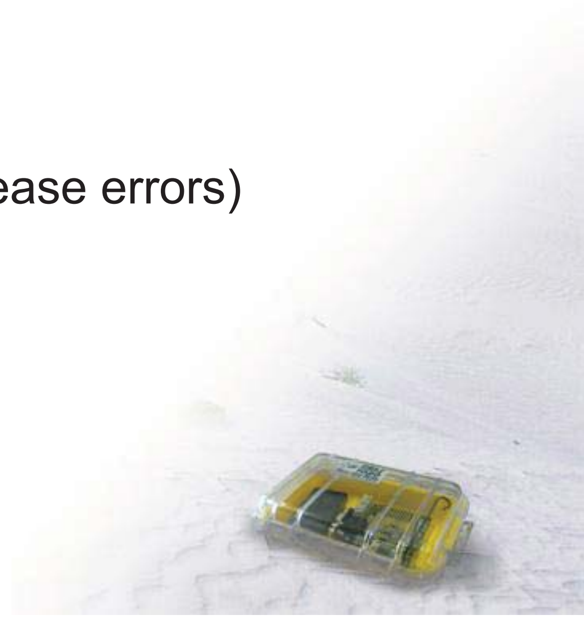
- initial anchor
- becomes anchor in 1^{st} step
- becomes anchor in 2^{nd} step
- becomes anchor in 3^{rd} step

Cooperative Multihop Multilateration:

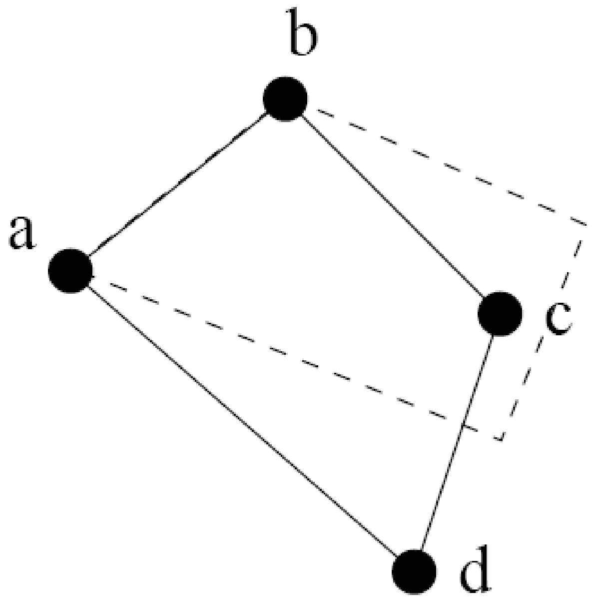


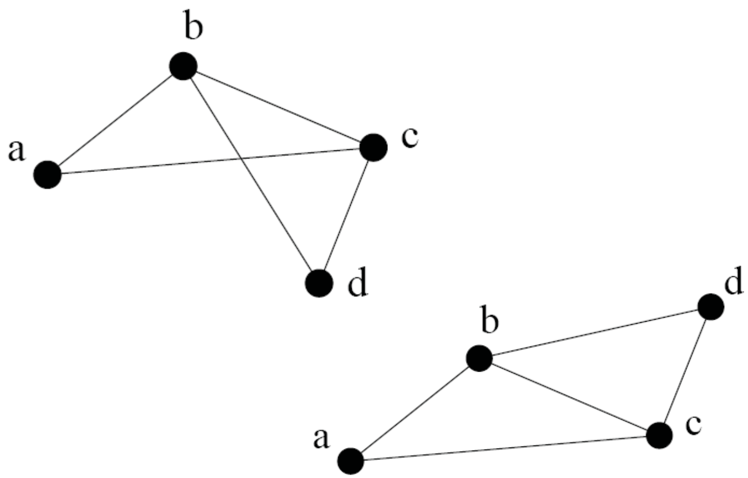
Problems with Distance Measurements

- No information
  - Pair of nodes simply too far apart
  - Physical obstacles: Line-of-sight needed for ultrasound, laser, infrared, …
  - Limitations of ranging hardware: Might not be omni-directional.
- Not enough information
  - Too sparse deployment to determine a “globally rigid” structure
- Inaccurate information
  - Measurement errors will sum up
  - Idea: Reduce “depth” of iterative multilateration
  - Trying to get an over-constrained system (to decrease errors)
  - Problem well known in mobile robotics



Rigidity Theory





[Jie Gao]

- Given a set of rigid bars connected by hinges, rigidity theory studies whether you can move them continuously.
  - Continuous deformations, or flips

Rigidity Theory

- Theory concerned with when we are guaranteed uniqueness of structures in 2D or 3D (or higher D).
  - Applications to structural engineering, molecular structures

- Definition: An *n*-point formation *P* in *d*-space consists of:
  - Coordinates in *d*-space for points $p_1,\dots,p_n$, and
  - A set of edges between some points (indices).
  - In other words, a graph embedded in *d*-dimensional space.

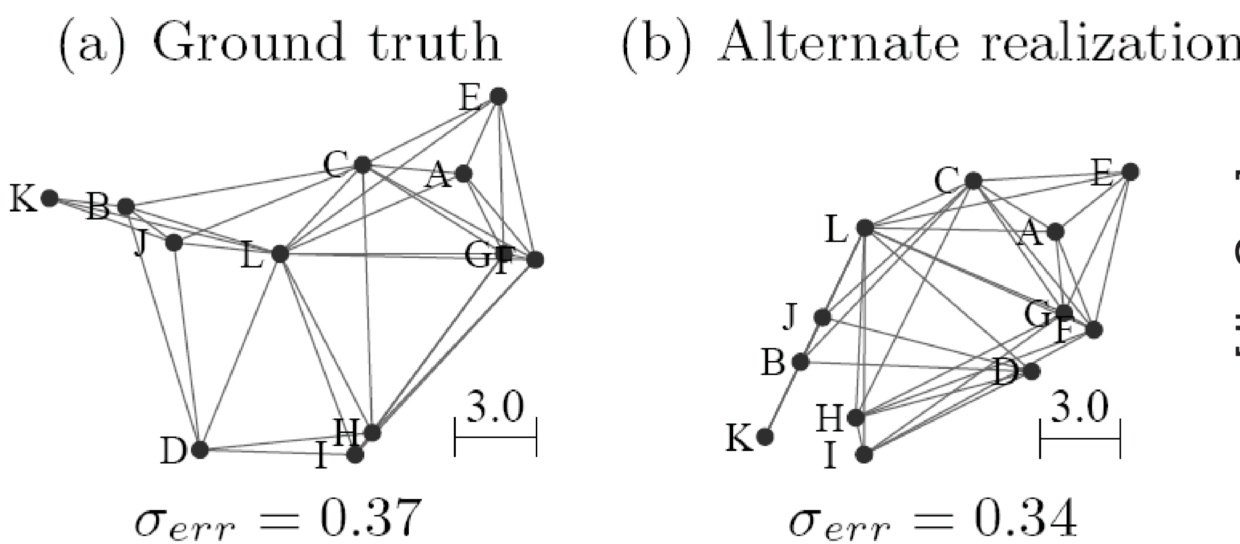
- Definition: An *n*-point formation *P* in *d*-space is globally rigid provided that any other *n*-point formation Q with the same edges and the same distances on those edges is the same as *P*, up to translation, rotation, and reflection.
  - Flips are at least locally rigid. Local rigidity makes sense in e.g. structural engineering
  - Continuous deformations are not rigid at all

Global Rigidity

- There are some simple rules that guarantee global rigidity:
- In 2D, a triangle with distances between all pairs is globally rigid. Starting with a triangle, if we repeatedly add a node and edges (with distances) to at least 3 non-collinear points, it results in a globally rigid point formation.
- In 3D, a tetrahedron (with all 6 distances) is globally rigid. Starting with a tetrahedron, if we repeatedly add a node and edges (with distances) to at least 4 non-coplanar points, it results in a globally rigid point formation.

Noisy distance estimates

- Noise can introduce anomalies
  - We might no longer get exact solutions, but may have some error (difference between given and computed distances). Tolerable, unavoidable.
  - Solutions may no longer be unique, even allowing for small errors: varying the measurements a tiny amount could yield drastically different best solutions.
  - Errors caused by noisy measurements can become compounded through an iterative coordinate assignment procedure.



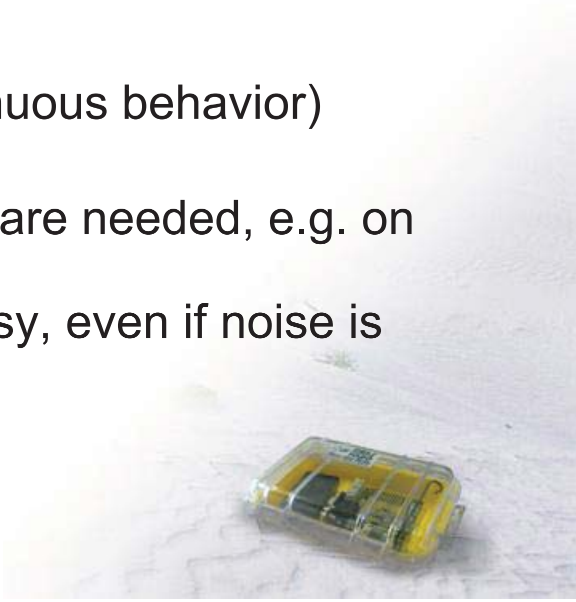
[Jie Gao]

Coping with noisy measurements

- Idea: Robust quadrilaterals
  - Robust with respect to a bound e on error in distance calculations.
  - 4 nodes in 2D, edges (and distances) between all pairs.
  - Nodes spaced so, even with errors, only one realization is possible.

- Method
  - Parameterized by bound e on measurement error.
  - Start with a robust quadrilateral, and then add nodes iteratively.

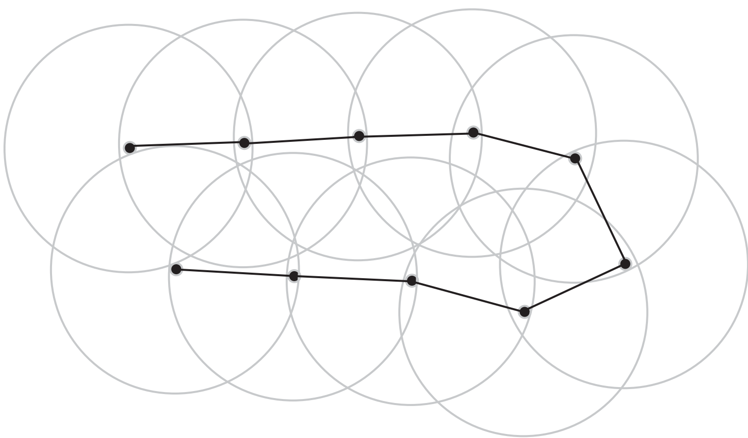
- Discussion
  - Avoids certain kinds of ambiguities (“flip”, discontinuous behavior)
  - But not enough to guarantee global rigidity
  - To guarantee global rigidity, additional constraints are needed, e.g. on angles
  - Intuitively: if angles are too small, then flips are easy, even if noise is bounded



Rigidity alternative: Simple hop-based algorithms

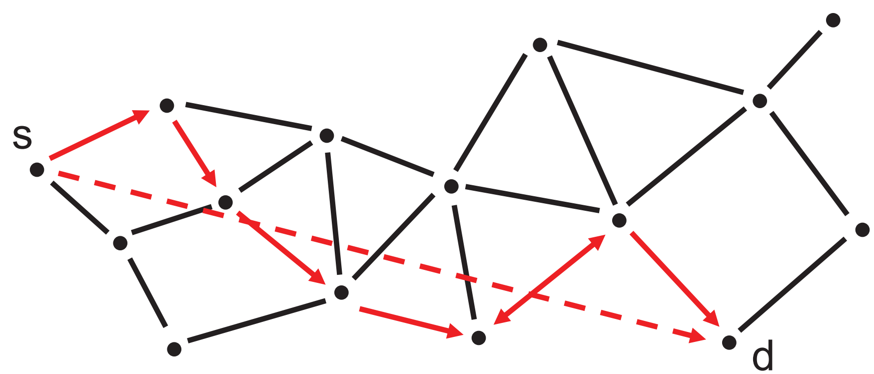
- Algorithm
  - Get hop distance h to anchor(s)
  - Intersect circles around anchors
    - radius = distance to anchor
  - Choose point such that maximum error is minimal
    - Find enclosing circle (ball) of minimal radius
    - Center is calculated location

- In higher dimensions: $1 < d \leq h$
  - Rule of thumb: Sparse graph
    → bad performance



How about no anchors at all...?

- In absence of anchors...
  → ...nodes are clueless about real coordinates.
- For many applications, real coordinates are not necessary
  → Virtual coordinates are sufficient
  → Geometric Routing requires only virtual coordinates
  - Require no routing tables
  - Resource-frugal and scalable



Virtual Coordinates

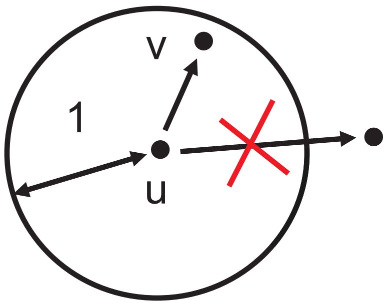
- Idea:
  Close-by nodes have similar coordinates
  Distant nodes have very different coordinates

→ Similar coordinates imply physical proximity!

- Applications
  - Geo-Routing
  - Locality-sensitive queries
  - Obtaining meta information on the network
  - Anycast services („*Which of the service nodes is closest to me?*“)
  - Outside the sensor network domain: e.g., Internet mapping

Model

- Unit Disk Graph (UDG) to model wireless multi-hop network
  - Two nodes can communicate iff Euclidean distance is at most 1



- Sensor nodes may not be capable of
  - Sensing directions to neighbors
  - Measuring distances to neighbors

- Goal: Derive topologically correct coordinate information from connectivity information only.
  - Even the simplest nodes can derive connectivity information

Virtual Coordinates ⟷ UDG Embedding

- Given the connectivity information for each node...

v_1	v_2	v_3	v_4	v_5
v_2	v_1	v_2	v_3	v_1
v_5	v_3	v_4	v_5	v_3
		v_5		v_4

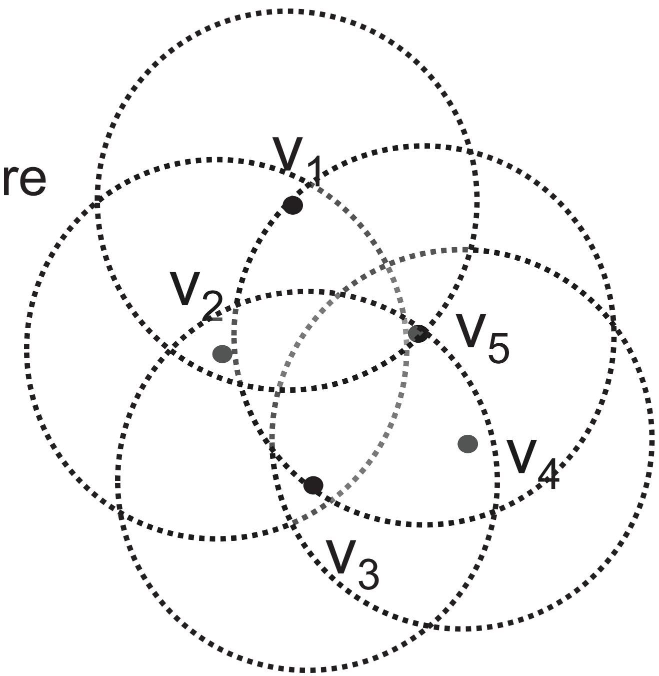
...and knowing the underlying graph is a UDG...

...find a UDG embedding in the plane such that all connectivity requirements are fulfilled! (→ Find a realization of a UDG)

This problem is NP-hard!

(Simple reduction to *UDG-recognition* problem, which is NP-hard)

[Breu, Kirkpatrick, Comp.Geom.Theory 1998]



UDG Approximation – Quality of Embedding

- Finding an exact realization of a UDG is NP-hard.
  → Find an embedding r(G) which approximates a realization.
- Particularly,
  → Map adjacent vertices (edges) to points which are close together.
  → Map non-adjacent vertices („non-edges") to far apart points.
- Define quality of embedding q(r(G)) as:

Ratio between longest edge to shortest non-edge in the embedding.

Let ρ(u,v) be the distance between points u and v in the embedding.

$$q(r(G)) := \frac{\max_{\{u,v\}\in E} \rho(u,v)}{\min_{\{u',v'\}\notin E} \rho(u',v')}$$

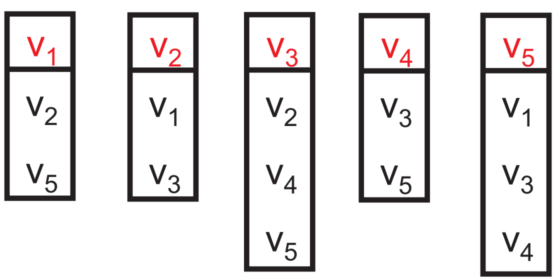
UDG Approximation

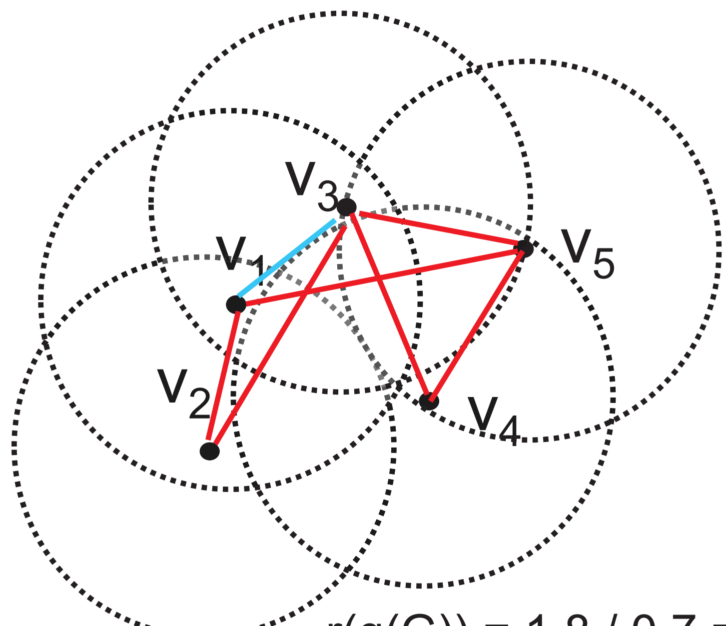
- For each UDG G, there exists an embedding r(G), such that, $q(r(G)) \leq 1$.
  (a realization of G)

$$q(r(G)) := \frac{\max_{\{u,v\}\in E}\rho(u,v)}{\min_{\{u',v'\}\notin E}\rho(u',v')}$$

- Finding such an embedding is NP-hard
- An algorithm ALG achieves approximation ratio α if for all unit disk graphs G, $q(r_{ALG}(G)) \leq \alpha$.

- Example:





r(q(G)) = 1.8 / 0.7 = 2.6

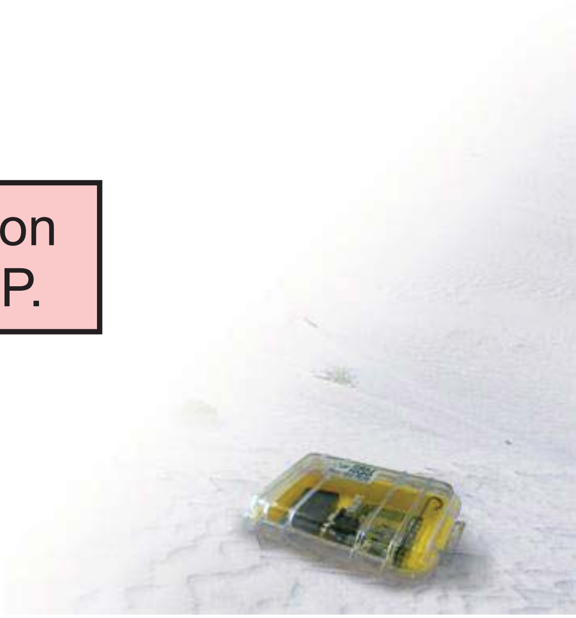
Some Results

- There are a few virtual coordinates algorithms
  Almost all of them evaluated only by simulation on random graphs
- In fact there are very few provable approximation algorithms

> There is an algorithm which achieves an approximation ratio of $O(\log^{2.5} n)$, n being the number of nodes in G. [Pemmaraju and Pirwani, 2007]

Plus there are lower bounds on the approximability.

> There is no algorithm with approximation ratio better than $\sqrt{3}/2 - \epsilon$ unless P = NP.



Approximation Algorithm: Overview

- Four major steps

1. Compute metric on MIS of input graph → Spreading constraints
   (Key conceptual difference to previous approaches!)
2. Volume-respecting, high dimensional embedding
3. Random projection to 2D
4. Final embedding

UDG Graph G with MIS M.

↓

Approximate pairwise distances between nodes such that, MIS nodes are neatly spread out.

↓

Volume respecting embedding of nodes in $\mathbb{R}^n$ with small distortion.

↓

Nodes spread out fairly well in $\mathbb{R}^2$.

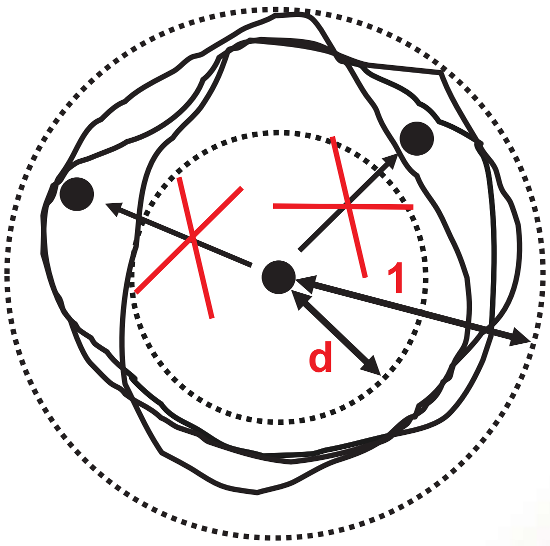
↓

Final *embedding of G in $\mathbb{R}^2$.*

Lower Bound: Quasi Unit Disk Graph

- Definition Quasi Unit Disk Graph:

> Let $V \in \mathbf{R}^2$, and $d \in [0,1]$. The symmetric Euclidean graph $G=(V,E)$, such that for any pair $u,v \in V$
>
> $$\text{dist}(u,v) \leq d \Rightarrow \{u,v\} \in E$$
>
> $$\text{dist}(u,v) > 1 \Rightarrow \{u,v\} \notin E$$
>
> is called *d-quasi unit disk graph*.



Note that between d and 1, the existence of an edge is unspecified.

Reduction

- We want to show that finding an embedding with $q(r(G)) \leq \sqrt{3/2} - \epsilon$, where ε goes to 0 for n → ∞ is NP-hard.

- We prove an equivalent statement:

> Given a unit disk graph G=(V,E), it is NP-hard to find a realization of G as a d-quasi unit disk graph with $d \geq \sqrt{2/3} + \epsilon$, where ε tends to 0 for n→∞.

→ Even when allowing non-edges to be smaller than 1, embedding a unit disk graph remains NP-hard!

→ It follows that finding an approximation ratio better than $\sqrt{3/2} - \epsilon$ is also NP-hard.

Reduction

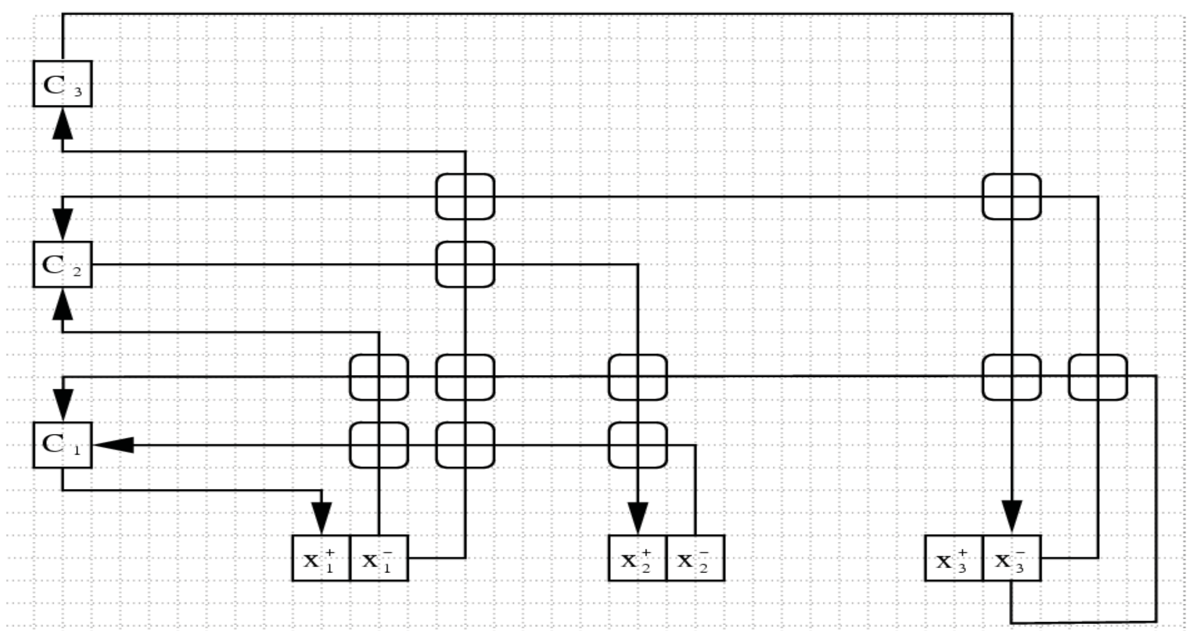
- Reduction from 3-SAT (each variable appears in at most 3 clauses)
- Given a instance C of this 3-SAT, we give a polynomial time construction of $G_C=(V_C, E_C)$ such that the following holds:

  – C is satisfiable $\Rightarrow$ G_C is realizable as a unit disk graph
  – C is not satisfiable $\Rightarrow$ G_C is not realizable as a d-quasi unit disk graph with $d \geq \sqrt{2/3} + \epsilon$

- Unless P=NP, there is no approximation algorithm with approximation ratio better than $\sqrt{3/2} - \epsilon$.

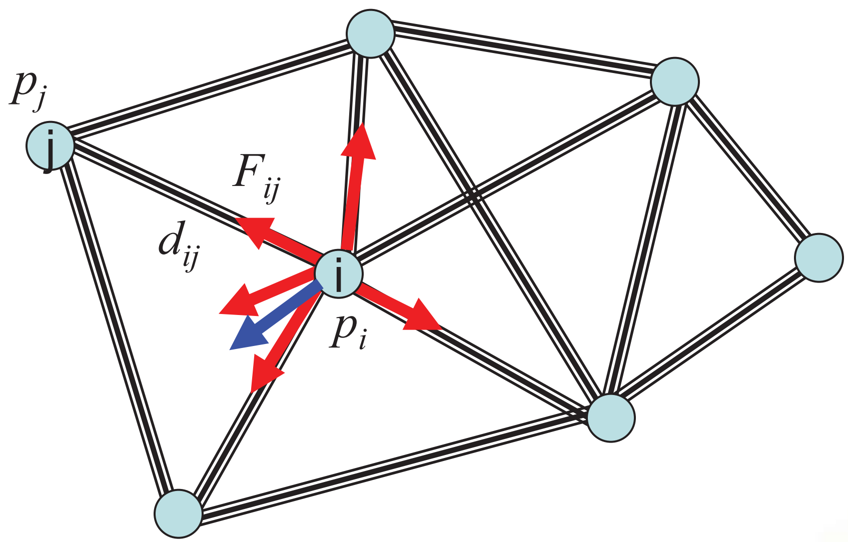
Proof idea

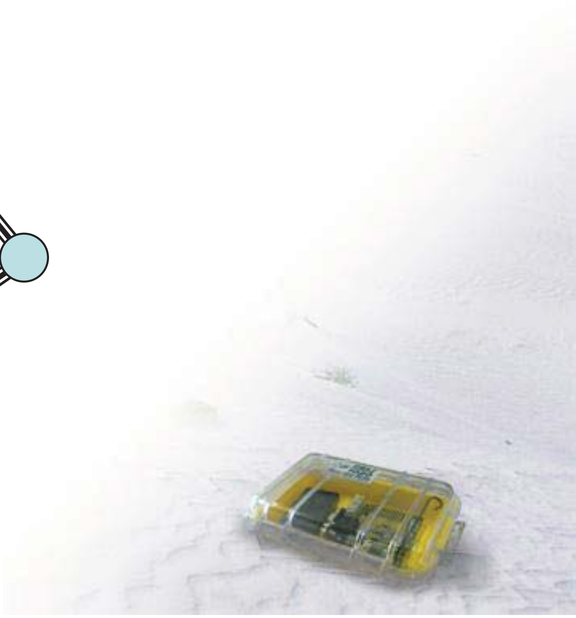
- Construct a grid drawing of the SAT instance.
- Grid drawing is *orientable* iff SAT instance is satisfiable.
- Grid components (clauses, literals, wires, crossings,...) are composed of nodes → Graph G_C.
- G_C is realizable as a d-quasi unit disk graph with $d \geq \sqrt{2/3} + \epsilon$ iff grid drawing is orientable.



Heuristics: Spring embedder

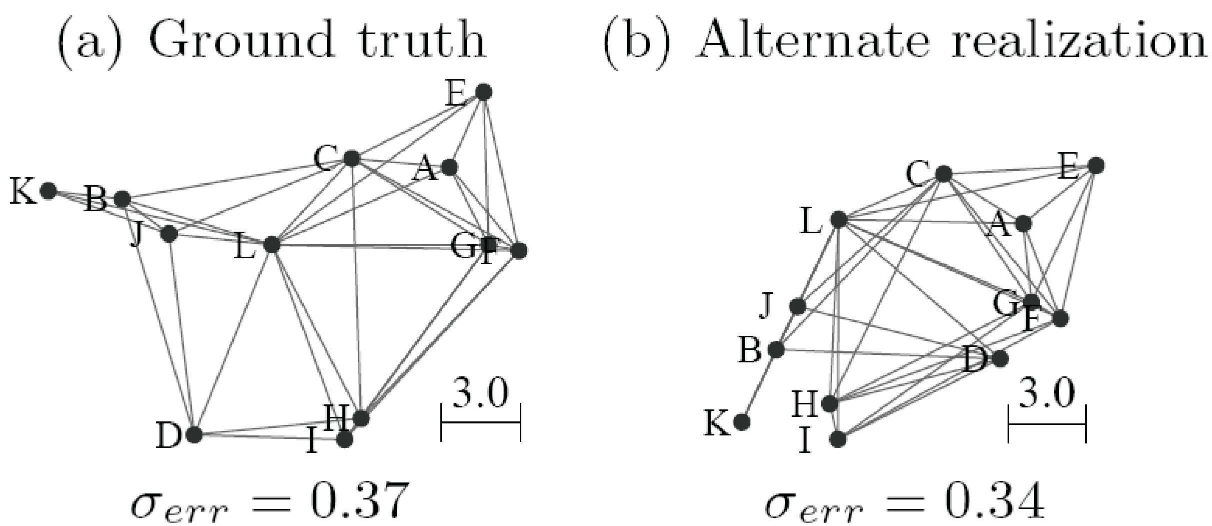
- Nodes are “masses”, edges are “springs”.
- Length of the spring equals the distance measurement.
- Springs put forces to the nodes, nodes move, until stabilization.
- Force: $F_{ij} = d_{ij} - r_{ij}$, along the direction $p_i p_j$.
- Total force on n_i: $F_i = \Sigma\, F_{ij}$.
- Move the node n_i by a small distance (proportional to F_i).





Spring Embedder Discussion

- Problems:
  - may deadlock in local minimum
  - may never converge/stabilize (e.g. just two nodes)
- Solution: Need to start from a reasonably good initial estimation.



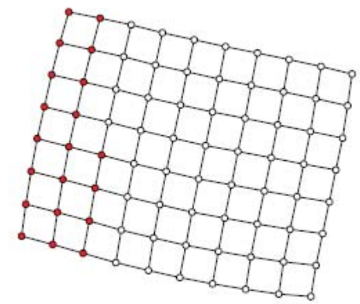
[Jie Gao]

Heuristics: Priyantha et al.

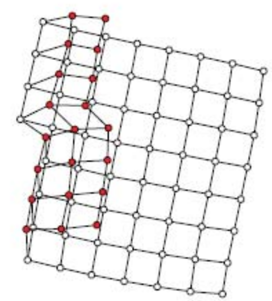
N.B. Priyantha, H. Balakrishnan, E. Demaine, S. Teller:
Anchor-Free Distributed Localization in Sensor Networks, *SenSys*, 2003.

iterative process minimizes the layout energy

$$E(p) = \sum_{\{i,j\}\in E} \left(||p_i - p_j|| - \ell_{ij} \right)^2$$



- fact: layouts can have *foldovers* without violating the distance constraints
- problem: optimization can converge to such a local optimum
- solution: find a good initial layout *fold-free* $\rightarrow$ already close to the global optimum (="real layout")

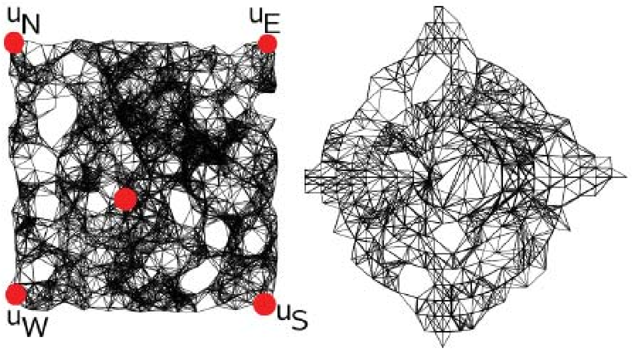


[Fleischer & Pich]

Heuristics: Priyantha et al. (continued)

Phase 1: compute initial layout

- determine periphery nodes u_N, u_S, u_W, u_E
- determine central node u_C
- use polar coordinates



$$\rho_v = d(v, u_C) \quad \theta_v = \arctan\left(\frac{d(v, u_N) - d(v, u_S)}{d(v, u_W) - d(v, u_E)}\right)$$

as positions of node v

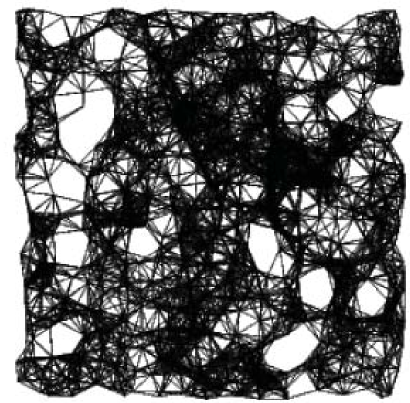
[Fleischer & Pich]

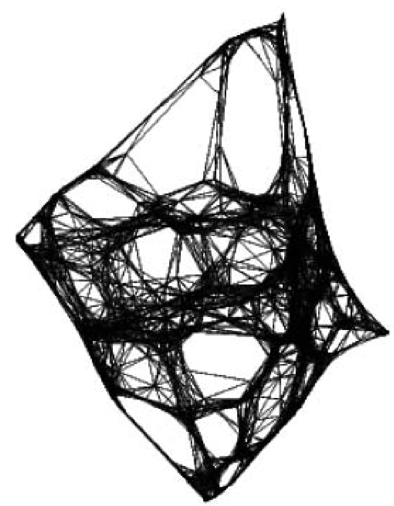
Phase 2: Spring Embedder

Heuristics: Gotsman et al.

C. Gotsman, Y. Koren [5]. **Distributed Graph Layout for Sensor Networks**, *GD*, 2004.

- initial placement: spread sensors
  $\frac{\sum_{\{i,j\}\in E} \exp(-\ell_{ij})||p_i - p_j||^2}{\sum_{i<j} ||p_i - p_j||^2} \rightarrow \min$
- linear algebra:
  minimized by second highest eigenvector v_2 of A where
  $a_{ij} = -\frac{\exp(-\ell_{ij})}{\sum_{j:\{i,j\}\in E} \exp(-\ell_{ij})}$
  $a_{ii} = 1$
- $x, Ax, A^2x, A^3x, \ldots$ converges to v_2
- $x_i \leftarrow \frac{1}{2}\left(x_i + \frac{\sum_{j:\{i,j\}\in E} \exp(-\ell_{ij} x_j)}{\sum_{j:\{i,j\}\in E} \exp(-\ell_{ij})}\right)$
- compute third eigenvector v_3,
  use v_2, v_3 as coordinates

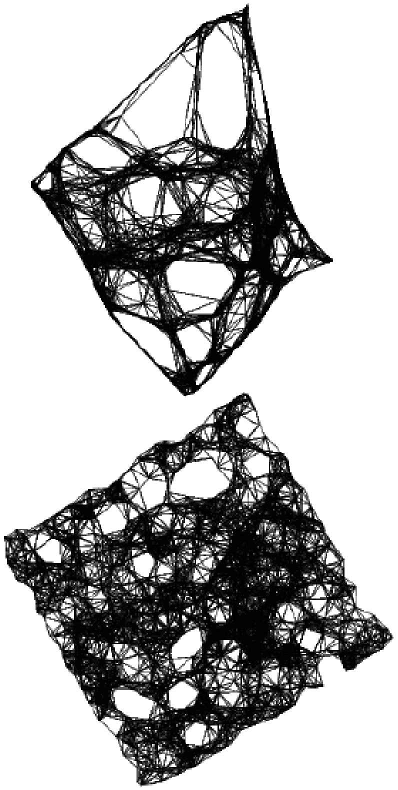




[Fleischer & Pich]

Heuristics: Gotsman et al. (continued)

- distributed optimization (spring model)
- alternative: *majorization*
- compute sequence of layouts $p^{(0)}, p^{(1)}, p^{(2)}, \ldots$ with $E(p^{(0)}) \geq E(p^{(1)}) \geq E(p^{(2)}) \geq \ldots$
  - solve linear equation
    $L^{(t+1)} p^{(t+1)} = L^{(t)} p^{(t)}$
    in distributed manner

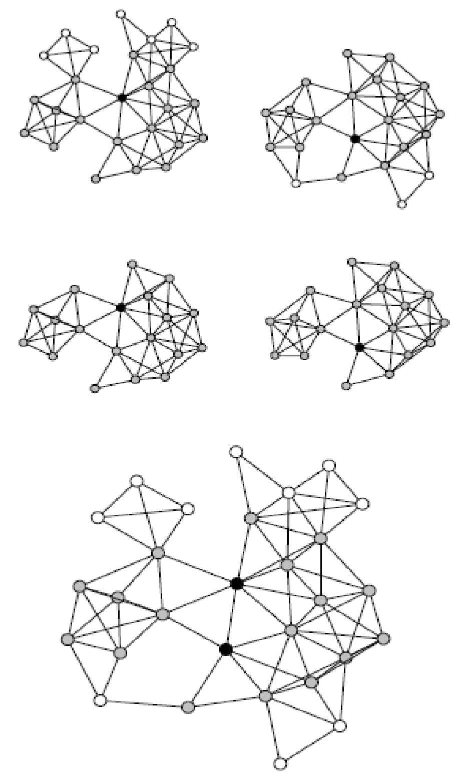


[Fleischer & Pich]

Heuristics: Shang et al.

Y. Shang, W. Ruml [7].
Improved MDS-based Localization, *IEEE Infocom*, 2004.

- compute a local map for each node (local MDS of the 2-hop neighborhood)
- merge local map patches into a global map (use incremental or binary-tree strategy)
- apply distributed optimization to the result

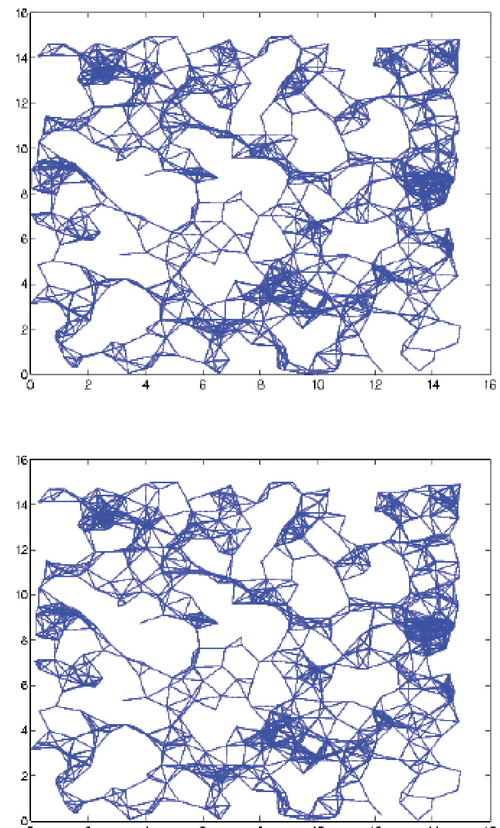


[Fleischer & Pich]

Heuristics: Bruck et al.

J. Bruck, J. Gao, A. Jiang [8]. **Localization and Routing in Sensor Networks by Local Angle Information**, *Mobile Ad Hoc Networking & Computing*, 2005.

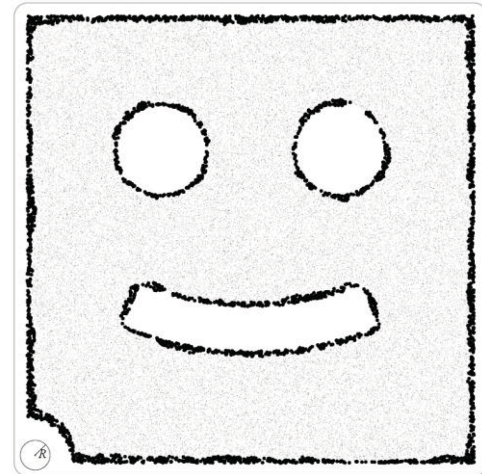
- Choose an edge e as x-axis to obtain absolute angles.
- Form an LP whose variables are the edge lengths $\ell(e)$.
- For all edges $0 \leq \ell(e) \leq 1$.
- For any cycle $e_1, \ldots, e_p$: $\sum_{i=1}^{p} \ell(e_i) \cos \theta_i = 0$ and $\sum_{i=1}^{p} \ell(e_i) \sin \theta_i = 0$.
- Non-adjacent node pair constraints.
- Crossing-edge constraints.

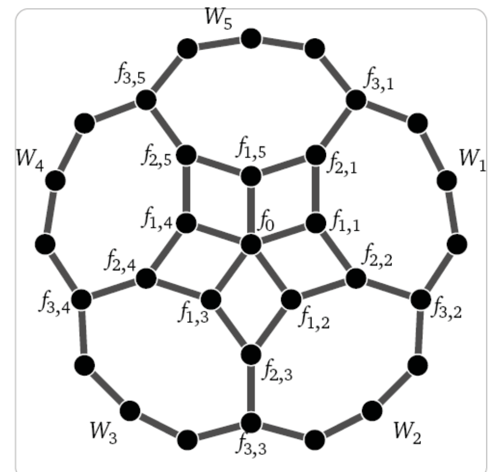


[Fleischer & Pich]

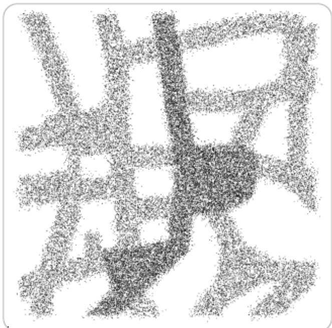
Boundary recognition

- Related problem, given a connectivity graph, what is the boundary?
- So far heuristics only, one heuristic uses the idea of independent nodes; specifically, an interior (non-boundary) node sees enough (e.g. 5) independent neighbors which in turn have a ring around them; these are called “flowers”. Flowers can be grown and connected to compute the boundary.
- However, this is only a heuristic, and does not always work…

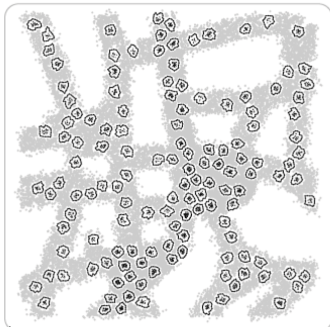




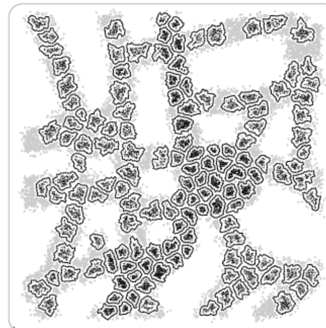
Boundary recognition



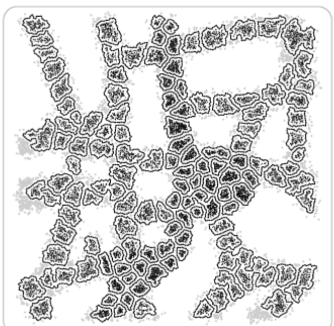
(a) Network



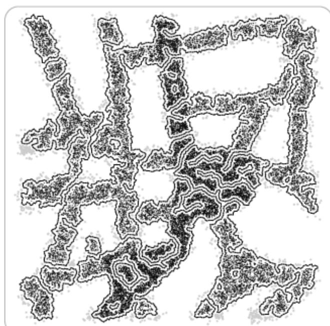
(b) Identified flowers



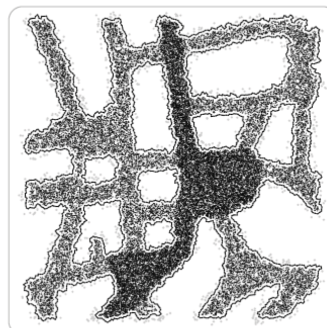
(c) Growing FGDs



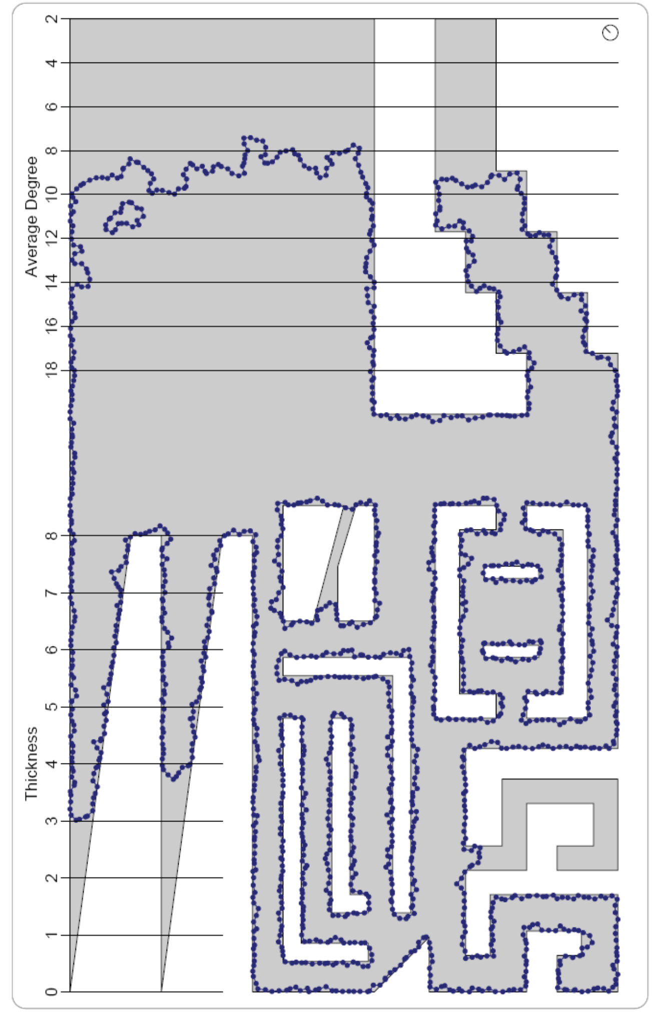
(d) FGDs beginning to merge



(e) Mergings FGDs



(f) Final state



Open problem

- One tough open problem of this chapter obviously is the UDG embedding problem: Given the adjacency matrix of a unit disk graph, find positions for all nodes in the Euclidean plane such that the ratio between the maximum distance between any two adjacent nodes and the minimum distance between any two non-adjacent nodes is as small as possible.

- There is a large gap between the best known lower bound, which is a constant, and the polylogarithmic upper bound. It is a challenging task to either come up with a better approximation algorithm or prove a stronger (non-constant) lower bound. Once we understand this better, we can try networks with anchors, or with (approximate) distance/angle information.

- Generally, beyond GPS this area is in its infancy.