Topology Control

Chapter 3

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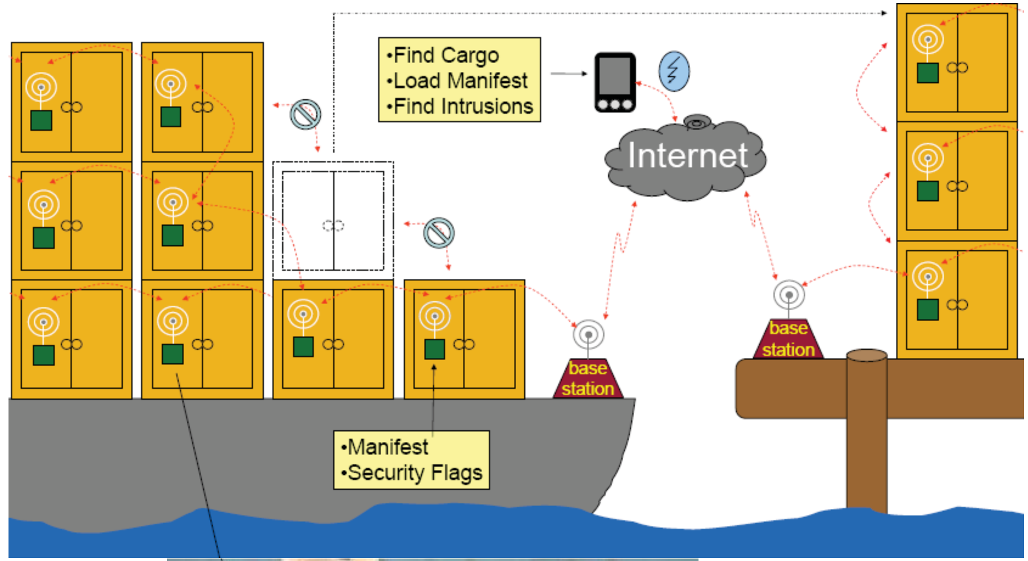
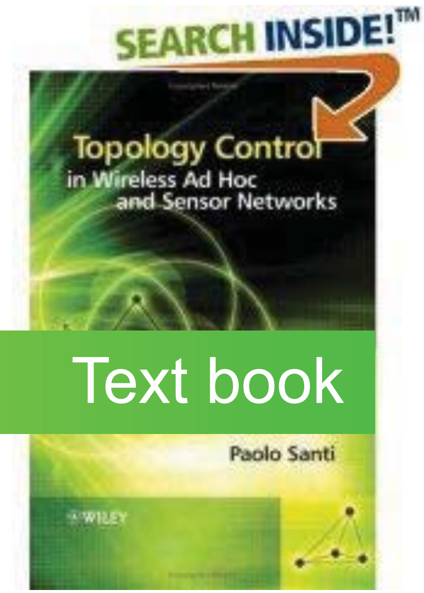
Inventory Tracking (Cargo Tracking)

- Current tracking systems require line-of-sight to satellite.

- Count and locate containers
- Search containers for specific item
- Monitor accelerometer for sudden motion
- Monitor light sensor for unauthorized entry into container



Rating



- Area maturity

First steps — Text book

- Practical importance

No apps — Mission critical

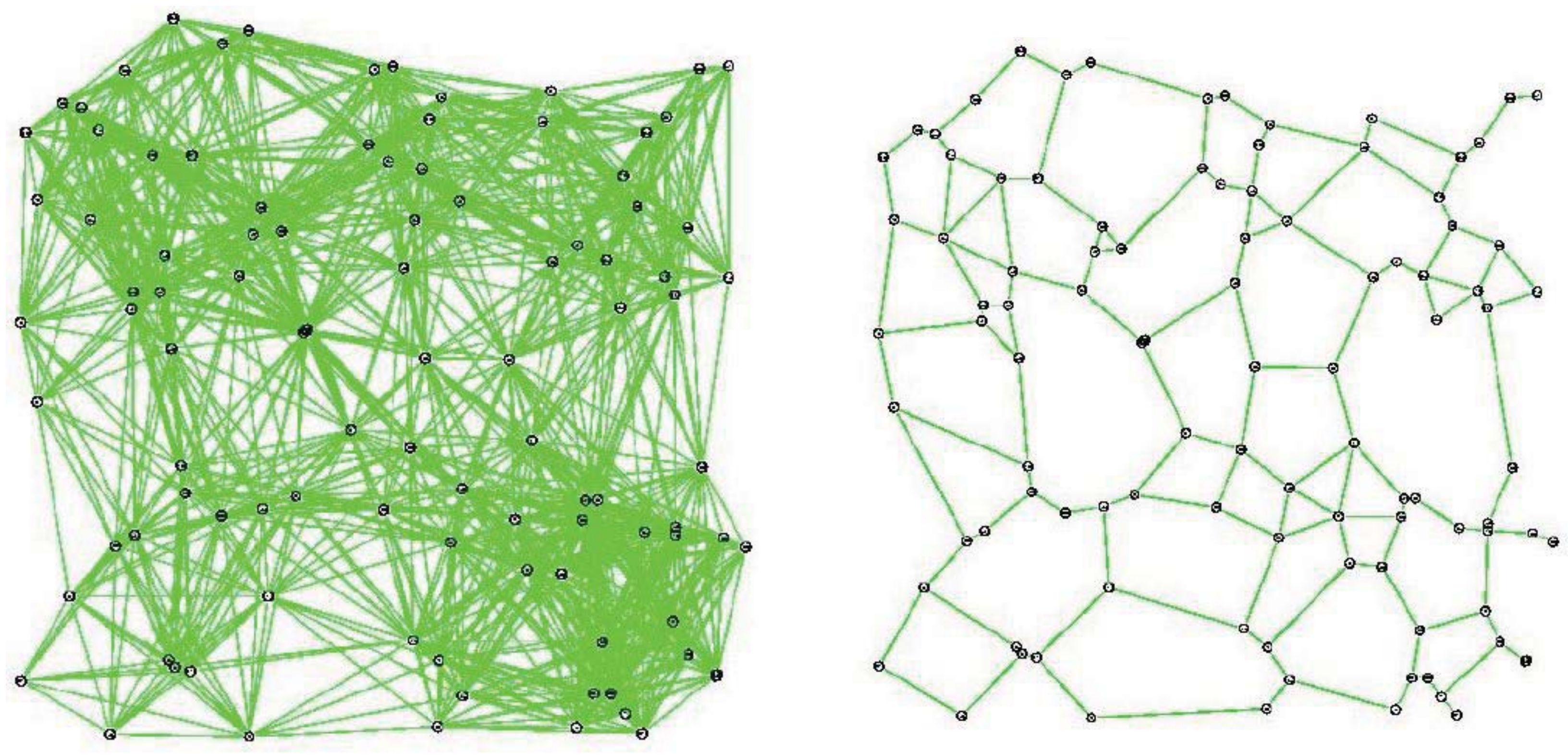
- Theory appeal

Booooooooring — Exciting

Overview – Topology Control

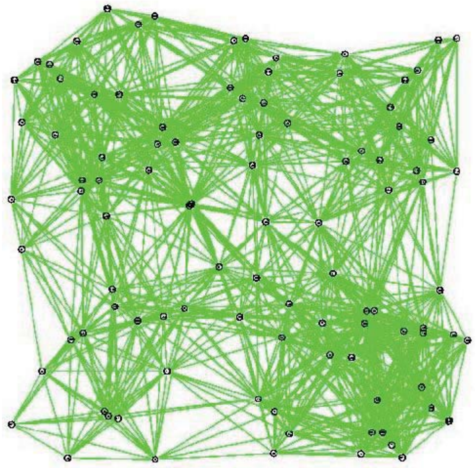
- Proximity Graphs: Gabriel Graph et al.
- Practical Topology Control: XTC
- Interference

Topology Control

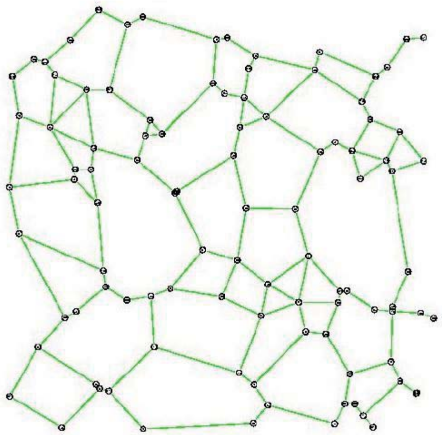


- Drop long-range neighbors: Reduces interference and energy!
- But still stay connected (or even spanner)

Topology Control as a Trade-Off



Topology Control



Network Connectivity
Spanner Property

$d_{TC}(u,v) \leq c \cdot d(u,v)$

Conserve Energy
Reduce Interference
Sparse Graph, Low Degree
Planarity
Symmetric Links
Less Dynamics

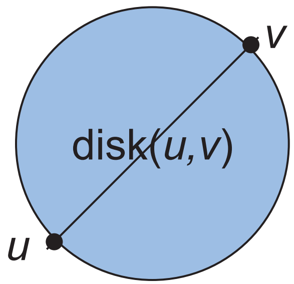
Spanners

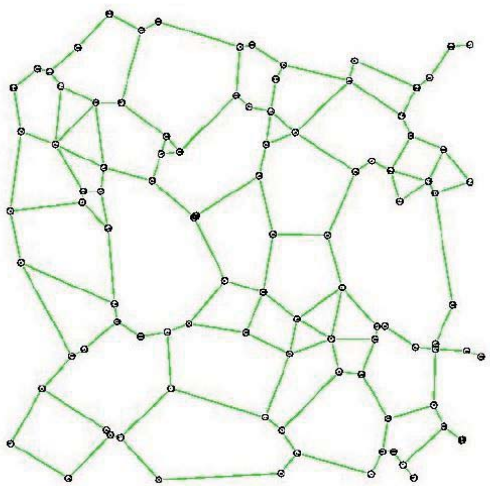
- Let the distance of a path from node u to node v, denoted as d(u,v), be the sum of the Euclidean distances of the links of the shortest path.
  - Writing $d(u,v)^p$ is short for taking each link distance to the power of *p*, again summing up over all links.
- Basic idea: S is spanner of graph G if S is a subgraph of G that has certain properties for all pairs of nodes, e.g.
  - Geometric spanner: $d_S(u,v) \leq c \cdot d_G(u,v)$
  - Power spanner: $d_S(u,v)^\alpha \leq c \cdot d_G(u,v)^\alpha$, for path loss exponent α
  - Weak spanner: path of S from u to v within disk of diameter $c \cdot d_G(u,v)$
  - Hop spanner: $d_S(u,v)^0 \leq c \cdot d_G(u,v)^0$
  - Additive hop spanner: $d_S(u,v)^0 \leq d_G(u,v)^0 + c$
  - (α, β) spanner: $d_S(u,v)^0 \leq \alpha \cdot d_G(u,v)^0 + \beta$
  - The stretch can be defined as maximum ratio d_S/d_G

Gabriel Graph

- Let disk(*u*,*v*) be a disk with diameter (*u*,*v*) that is determined by the two points *u*,*v*.
- The Gabriel Graph GG(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is an edge between two nodes *u*,*v* iff the disk(*u*,*v*) including boundary contains no other points.

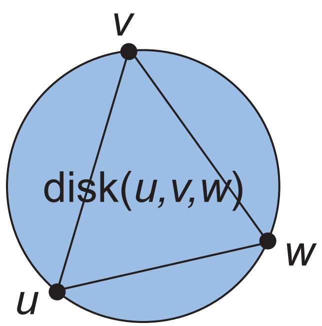
- As we will see the Gabriel Graph has interesting properties.



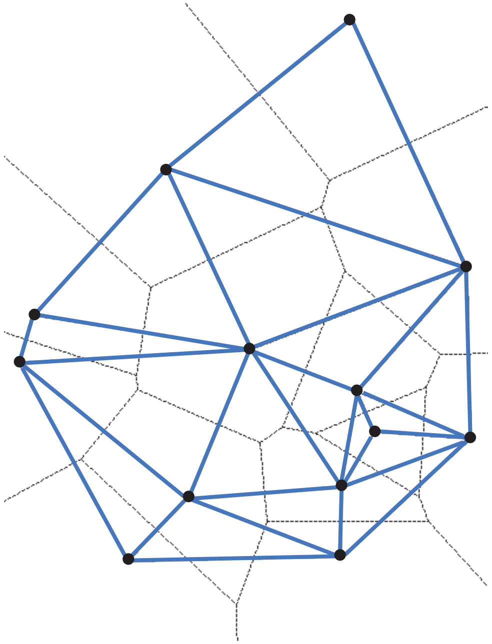


Delaunay Triangulation

- Let disk(u,v,w) be a disk defined by the three points u,v,w.
- The Delaunay Triangulation (Graph) DT(V) is defined as an undirected graph (with E being a set of undirected edges). There is a triangle of edges between three nodes u,v,w iff the disk(u,v,w) contains no other points.

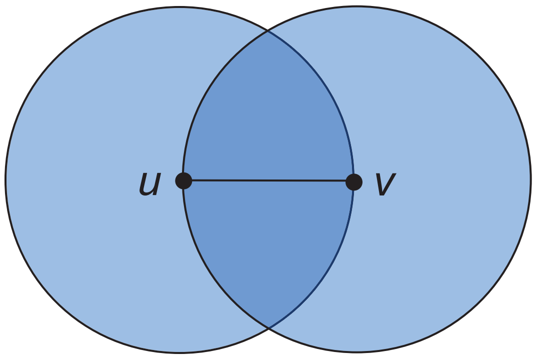


- The Delaunay Triangulation is the dual of the Voronoi diagram, and widely used in various CS areas
  - the DT is planar
  - the DT is a geometric spanner

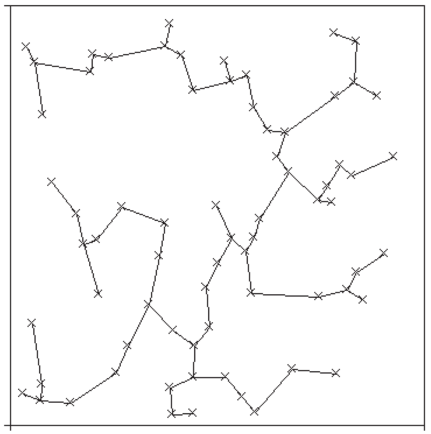


Other Proximity Graphs

- Relative Neighborhood Graph RNG(V)
  - An edge e = (u,v) is in the RNG(V) iff there is no node w in the “lune” of (u,v), i.e., no noe with with (u,w) < (u,v) and (v,w) < (u,v).



- Minimum Spanning Tree MST(V)
  - A subset of *E* of *G* of minimum weight which forms a tree on *V*.



Properties of Proximity Graphs

- Theorem 1:
  MST $\subseteq$ RNG $\subseteq$ GG $\subseteq$ DT

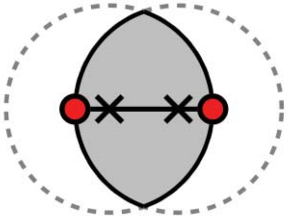
- Corollary:
  Since the MST is connected and the DT is planar, all the graphs in Theorem 1 are connected and planar.

- Theorem 2:
  The Gabriel Graph is a power spanner (for path loss exponent $\alpha \geq 2$). So is GG $\cap$ UDG.

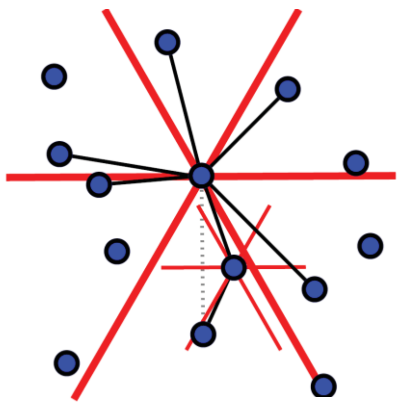
- Remaining issue: either high degree (RNG and up), and/or no spanner (RNG and down). There is an extensive and ongoing search for “Swiss Army Knife” topology control algorithms.

More Proximity Graphs

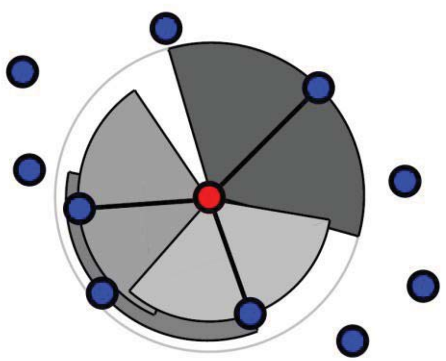
- β-Skeleton
  - Disk diameters are β·d(u,v), going through u resp. v
  - Generalizing GG (β = 1) and RNG (β = 2)



- Yao-Graph
  - Each node partitions directions in k cones and then connects to the closest node in each cone



- Cone-Based Graph
  - Dynamic version of the Yao Graph. Neighbors are visited in order of their distance, and used only if they cover not yet covered angle

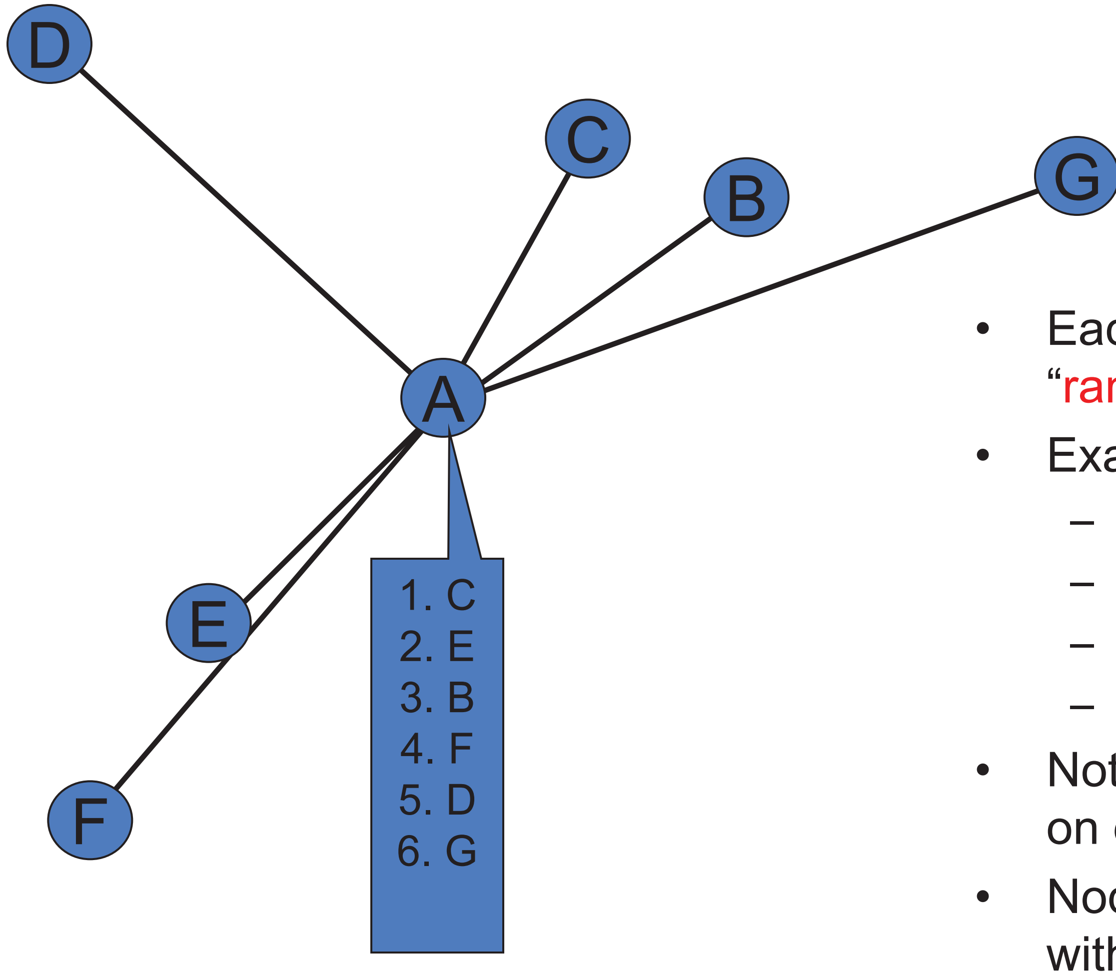


Lightweight Topology Control

- Topology Control commonly assumes that the node positions are known.

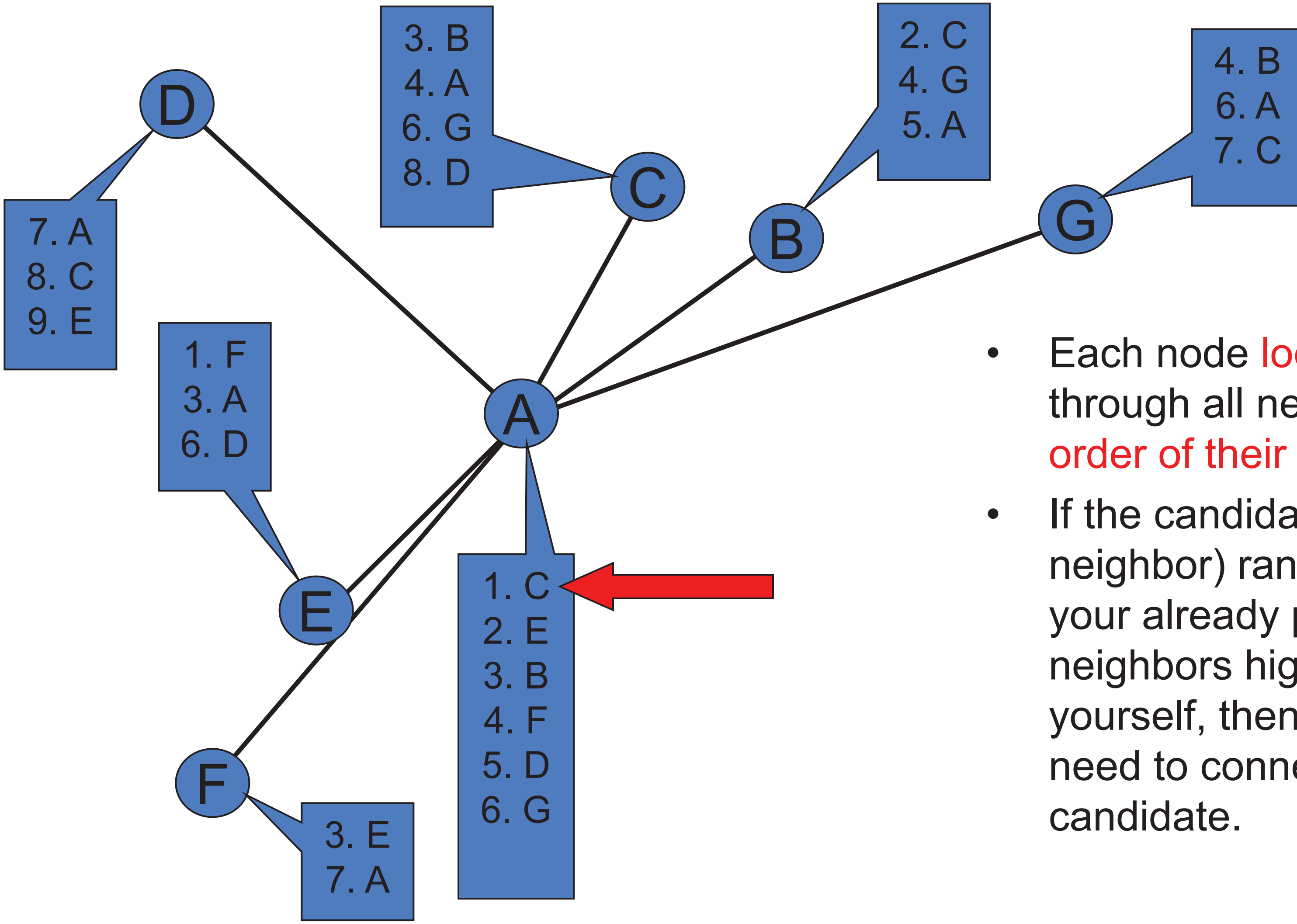
What if we do not have access to position information?

XTC: Lightweight Topology Control without Geometry



- Each node produces "ranking" of neighbors.
- Examples
  - Distance (closest)
  - Energy (lowest)
  - Link quality (best)
  - Must be symmetric!
- Not necessarily depending on explicit positions
- Nodes exchange rankings with neighbors

XTC Algorithm (Part 2)



- Each node locally goes through all neighbors in order of their ranking
- If the candidate (current neighbor) ranks any of your already processed neighbors higher than yourself, then you do not need to connect to the candidate.

XTC Analysis (Part 1)

- Symmetry: A node u wants a node v as a neighbor if and only if v wants u.

- Proof:
  - Assume 1) $u \rightarrow v$ and 2) $u \not\leftarrow v$
  - Assumption 2) $\Rightarrow \exists w$: (i) $w \prec_v u$ and (ii) $w \prec_u v$

In node *u*'s neighbor list, *w* is better than *v*

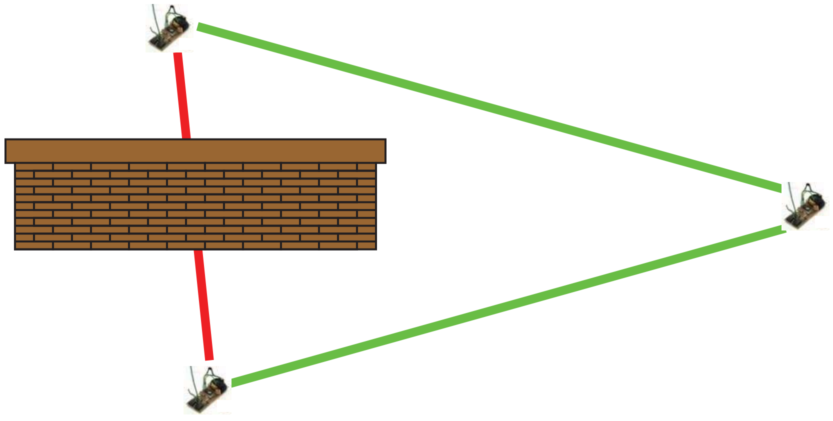
Contradicts Assumption 1)

XTC Analysis (Part 1)

- Symmetry: A node u wants a node v as a neighbor if and only if v wants u.

- Connectivity: If two nodes are connected originally, they will stay so (easy to show if rankings are based on symmetric link-weights).

- If the ranking is energy or link quality based, then XTC will choose a topology that routes around walls and obstacles.

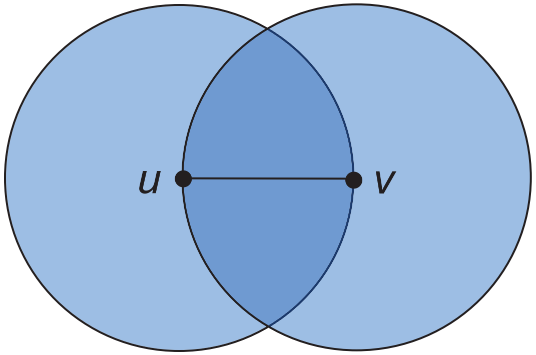


XTC Analysis (Part 2)

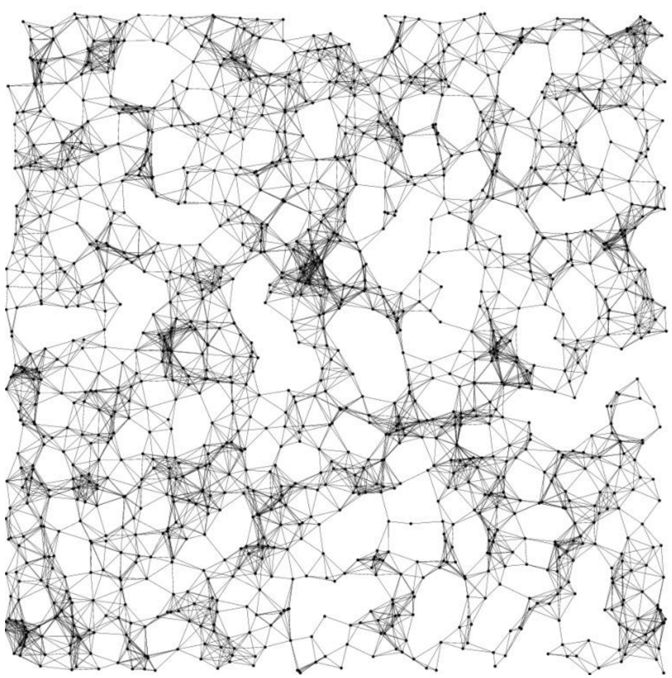
- If the given graph is a Unit Disk Graph (no obstacles, nodes homogeneous, but not necessarily uniformly distributed), then …

- The degree of each node is at most 6.
- The topology is planar.
- The graph is a subgraph of the RNG.

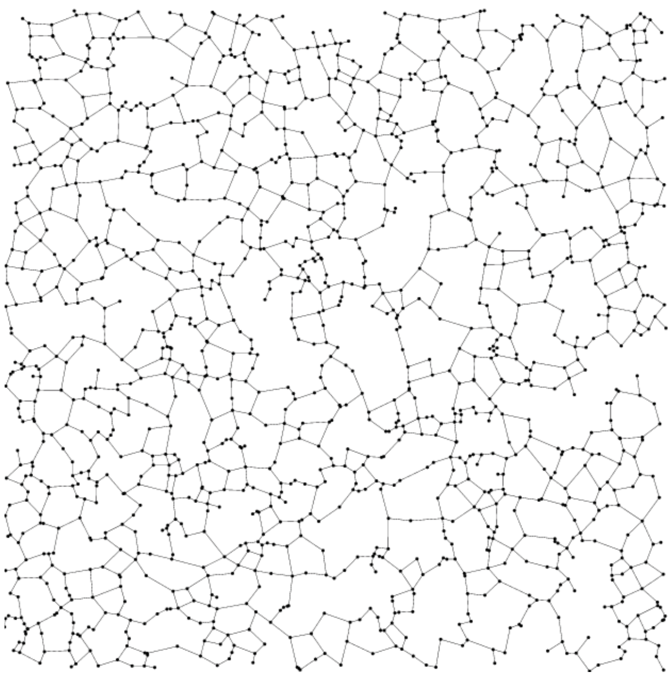
- Relative Neighborhood Graph RNG(V):
  – An edge e = (u,v) is in the RNG(V) iff there is no node w with (u,w) < (u,v) and (v,w) < (u,v).



XTC Average-Case

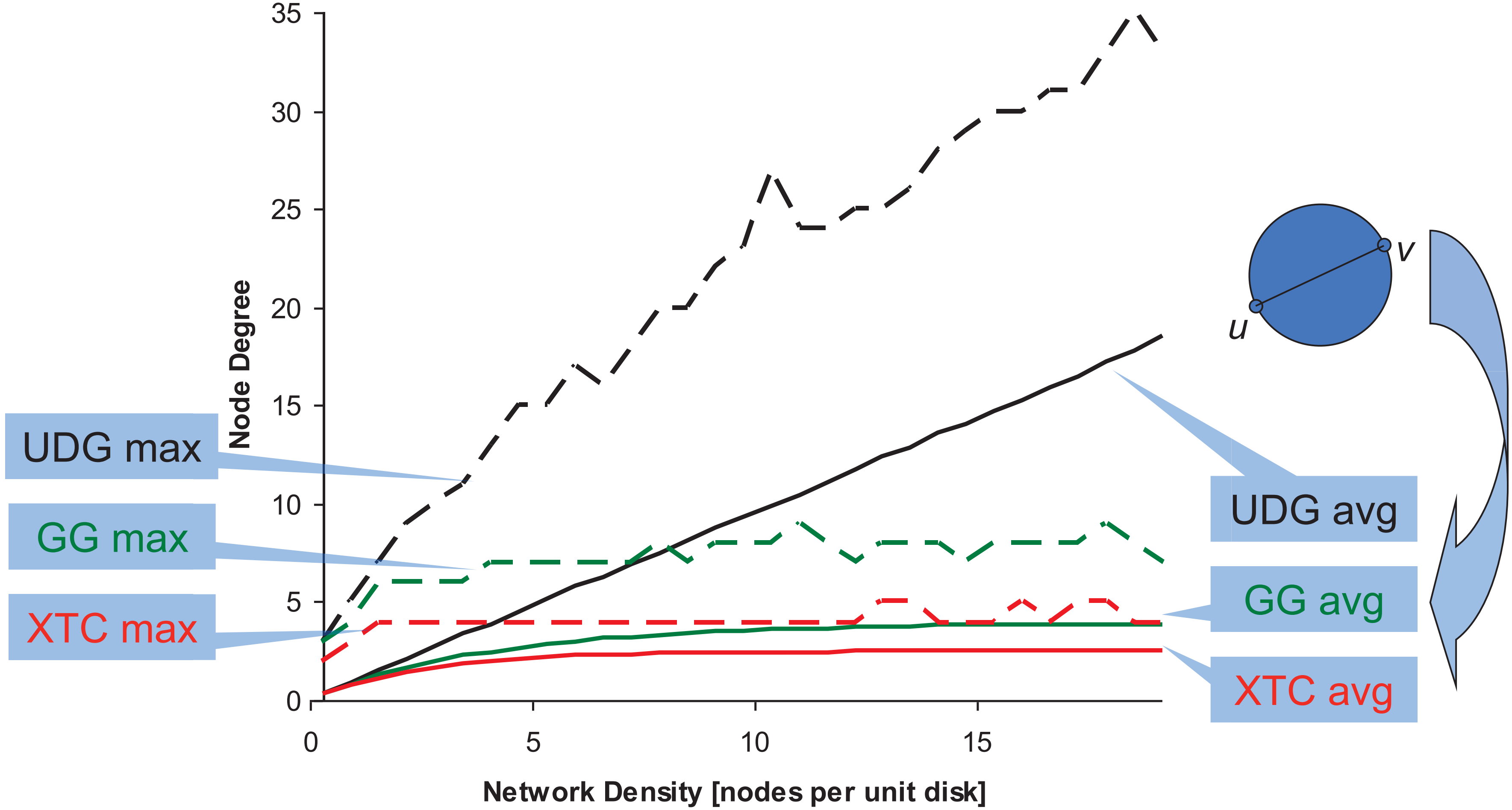


Unit Disk Graph

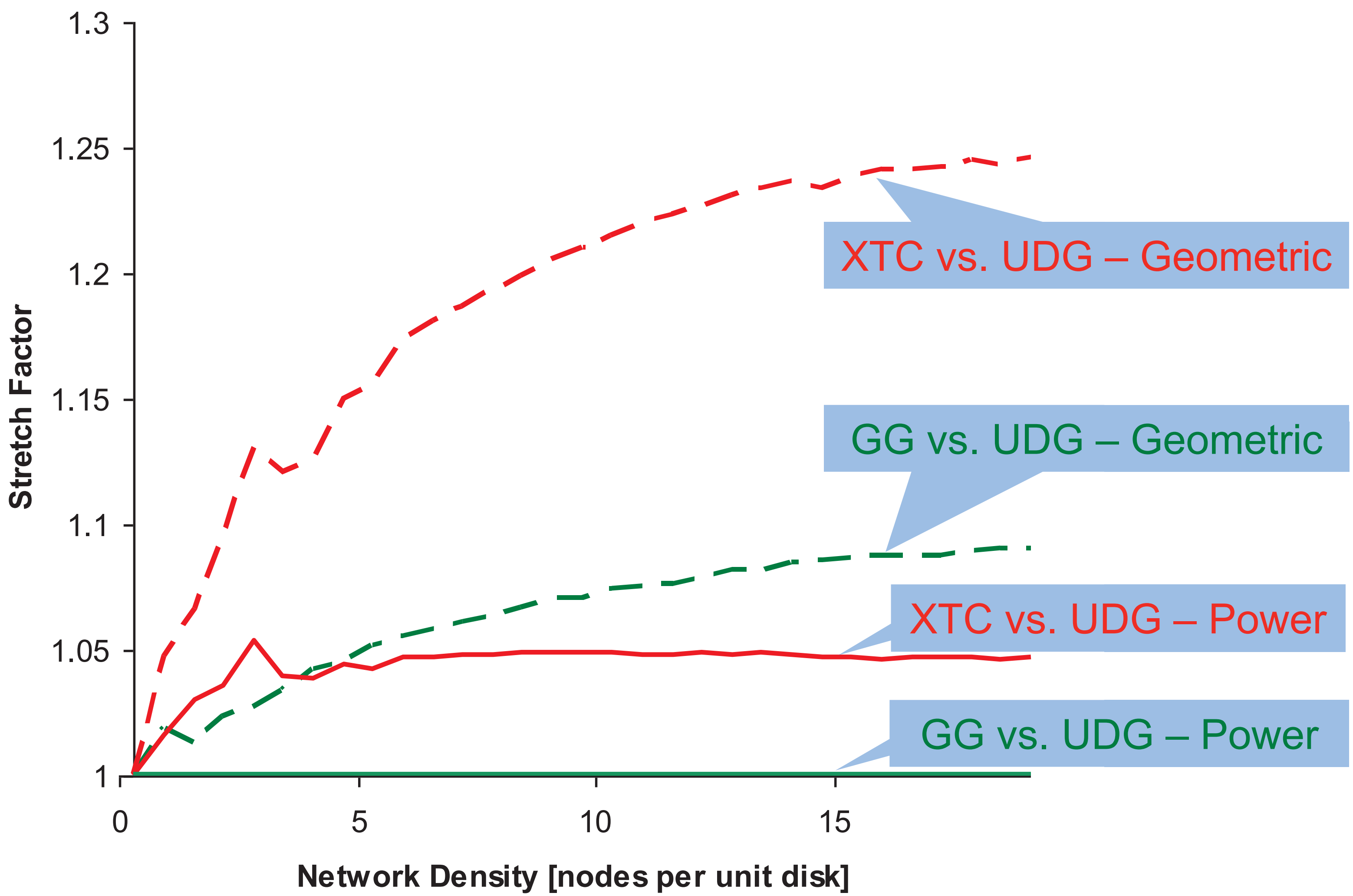


XTC

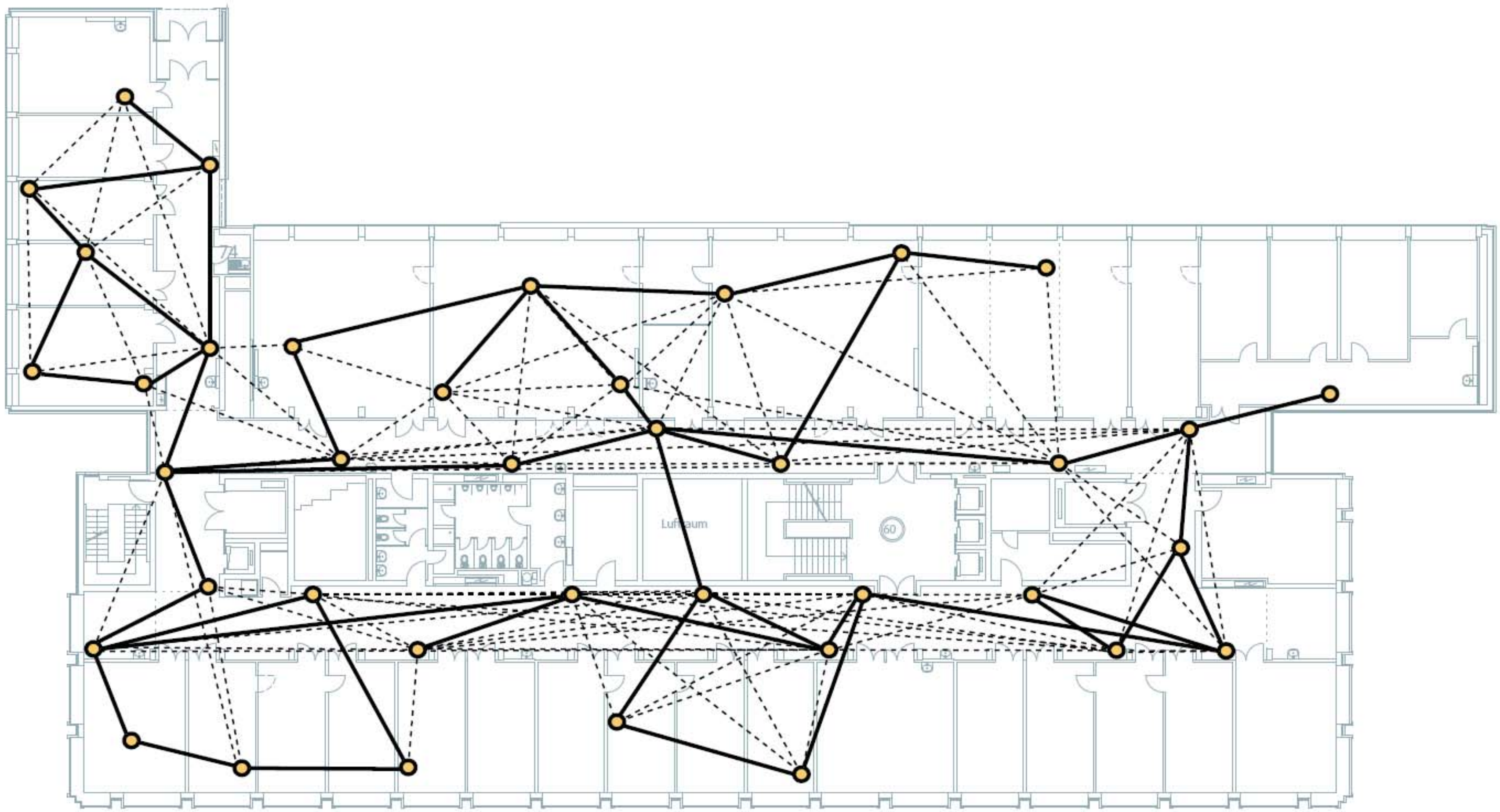
XTC Average-Case (Degrees)



XTC Average-Case (Stretch Factor)

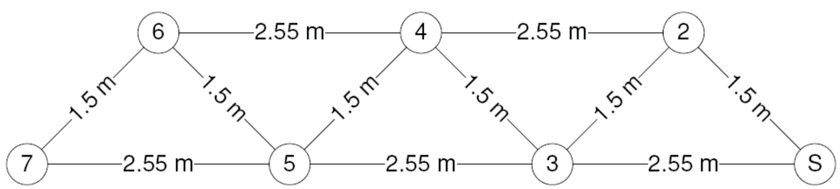


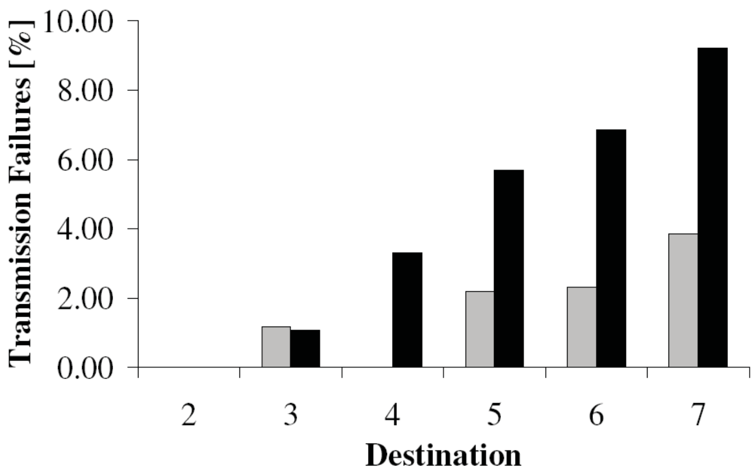
Implementing XTC, e.g. BTnodes v3

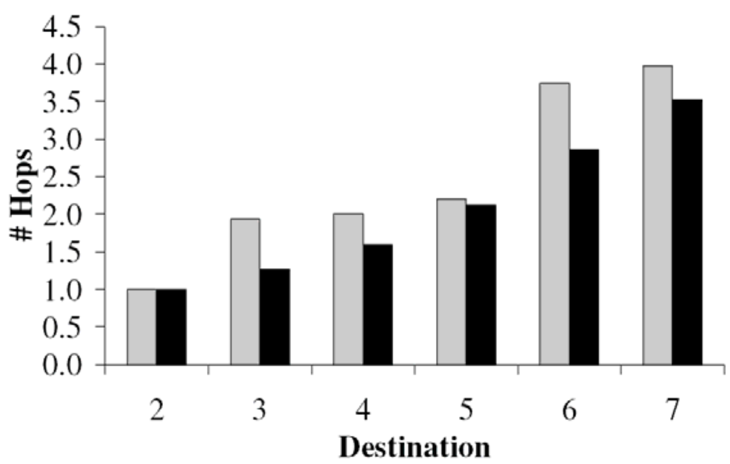


Implementing XTC, e.g. on mica2 motes

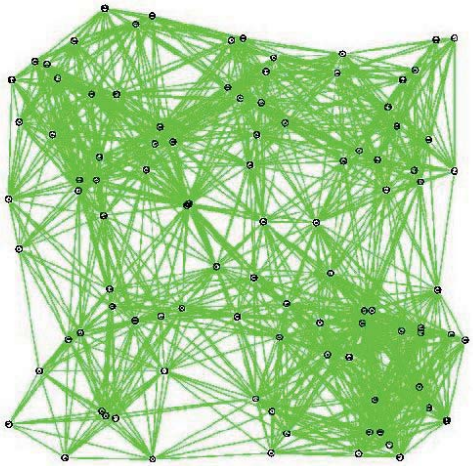
- Idea:
  - XTC chooses the reliable links
  - The quality measure is a moving average of the received packet ratio
  - Source routing: route discovery (flooding) over these reliable links only
  - (black: using all links, grey: with XTC)



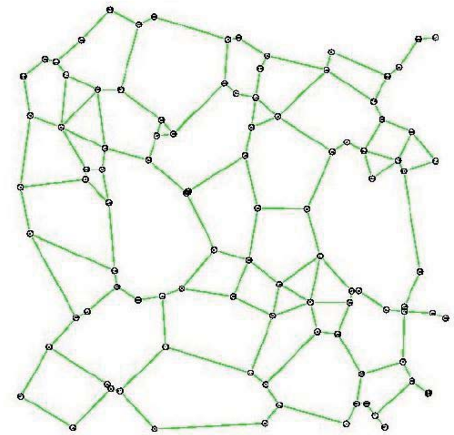




Topology Control as a Trade-Off



Topology Control



Network Connectivity
Spanner Property

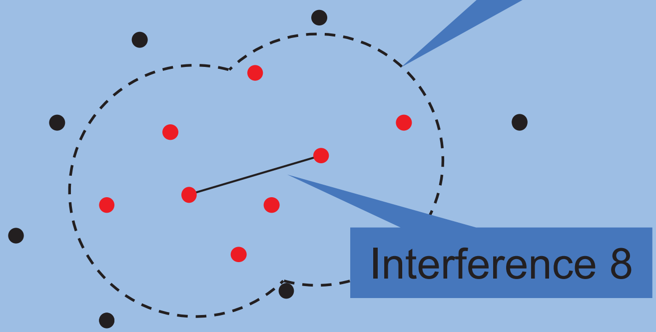
Conserve Energy
Reduce Interference
Sparse Graph, Low Degree
Planarity
Symmetric Links
Less Dynamics

Really?!?

What is Interference?

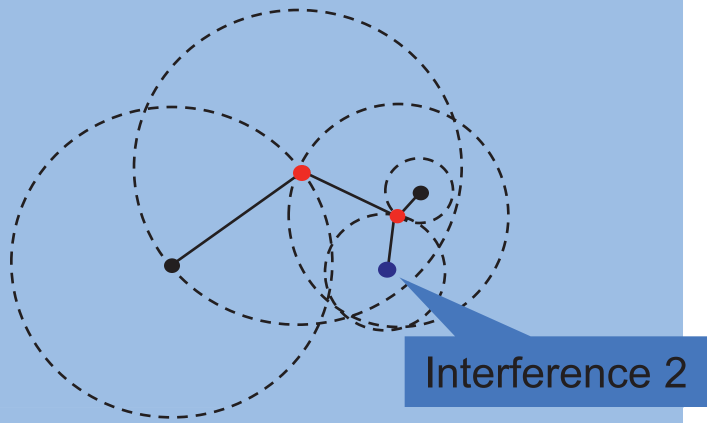
Exact size of interference range does not change the results

Link-based Interference Model



„How many nodes are affected by communication over a given link?“

Node-based Interference Model

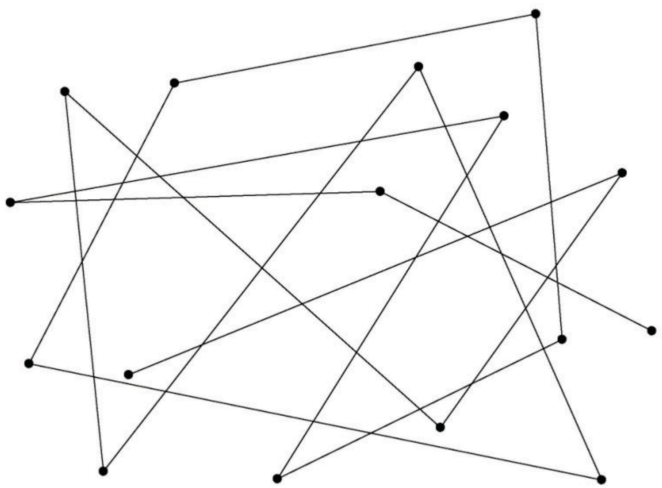


„By how many other nodes can a given network node be disturbed?“

- Problem statement
  - We want to minimize maximum interference
  - At the same time topology must be connected or spanner

Low Node Degree Topology Control?

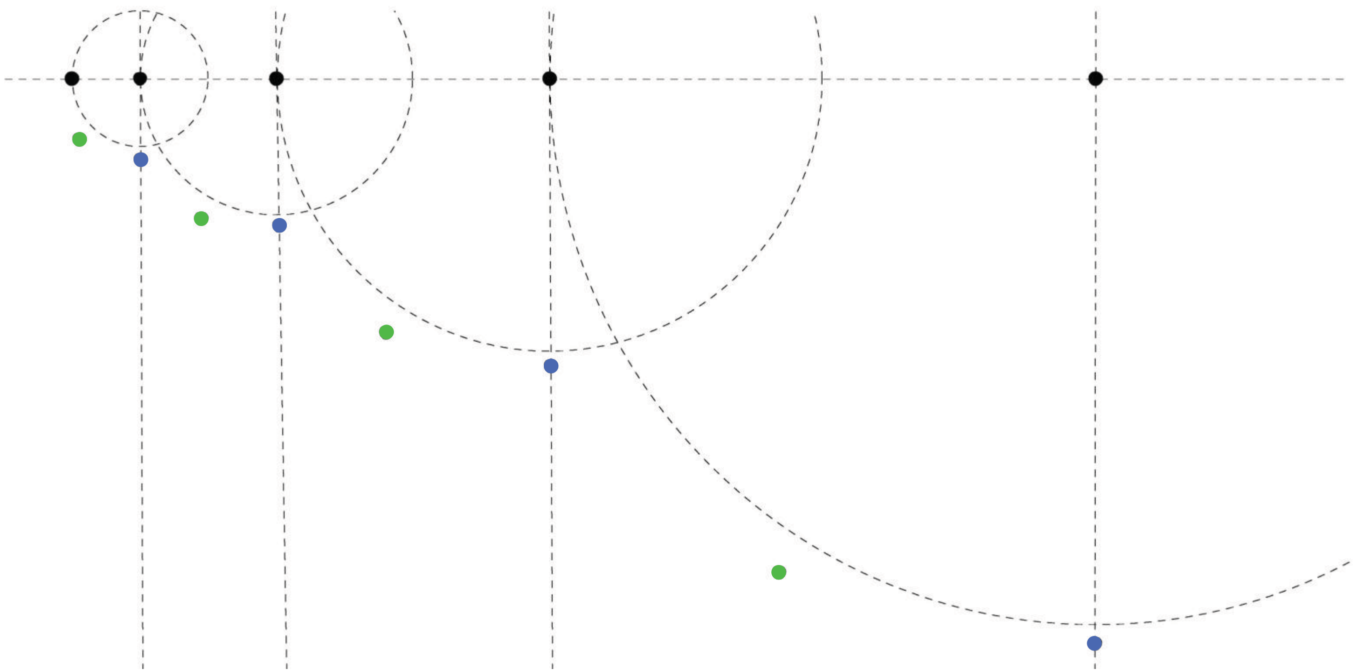
Low node degree does not necessarily imply low interference:



Very low node degree but huge interference

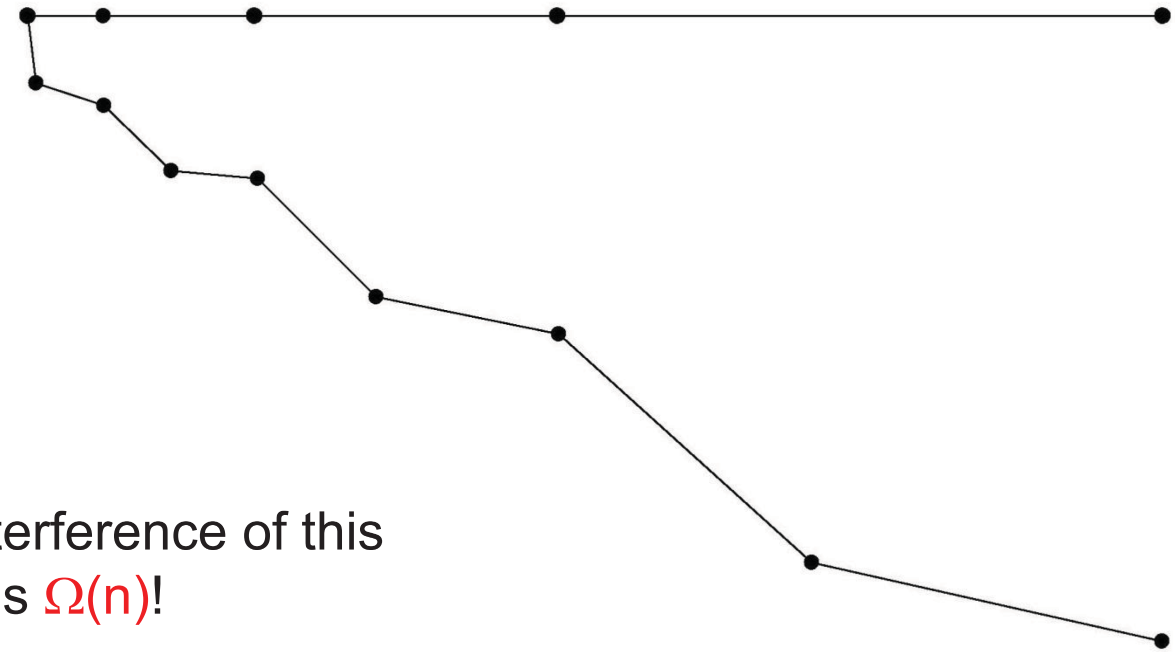
Let's Study the Following Topology!

…from a worst-case perspective



Topology Control Algorithms Produce…

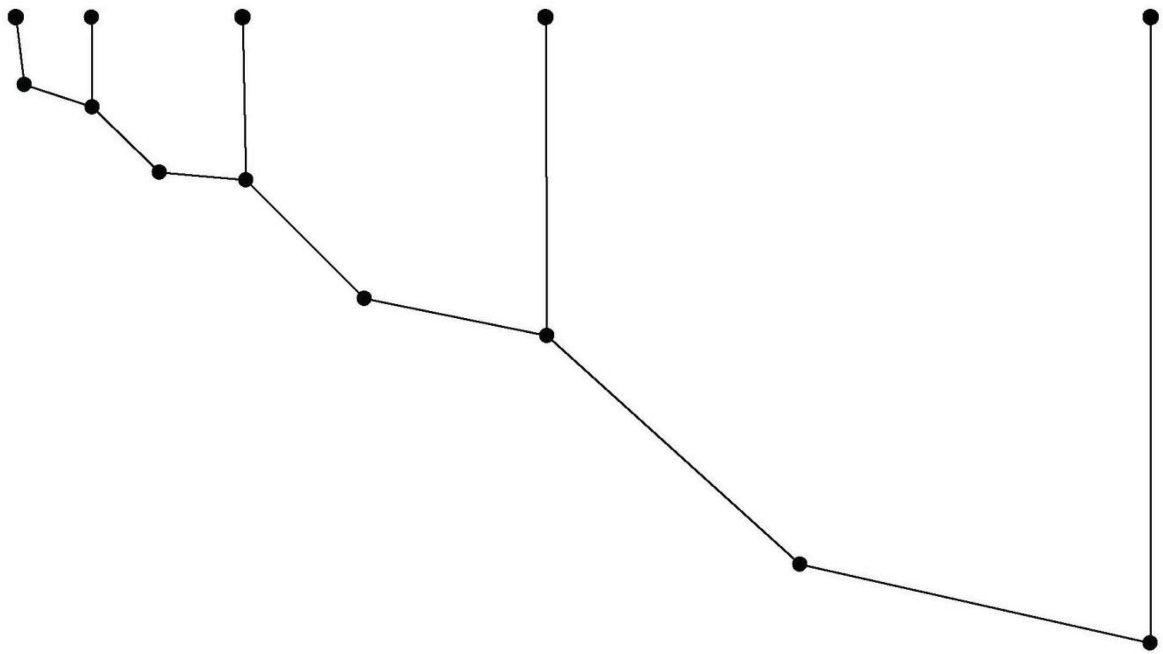
- All known topology control algorithms (with symmetric edges) include the nearest neighbor forest as a subgraph and produce something like this:



- The interference of this graph is $\Omega(n)$!

But Interference…

- Interference does not need to be high…

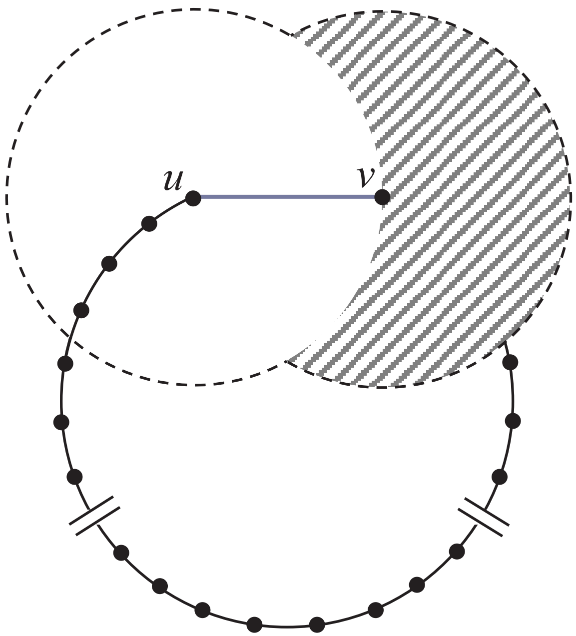


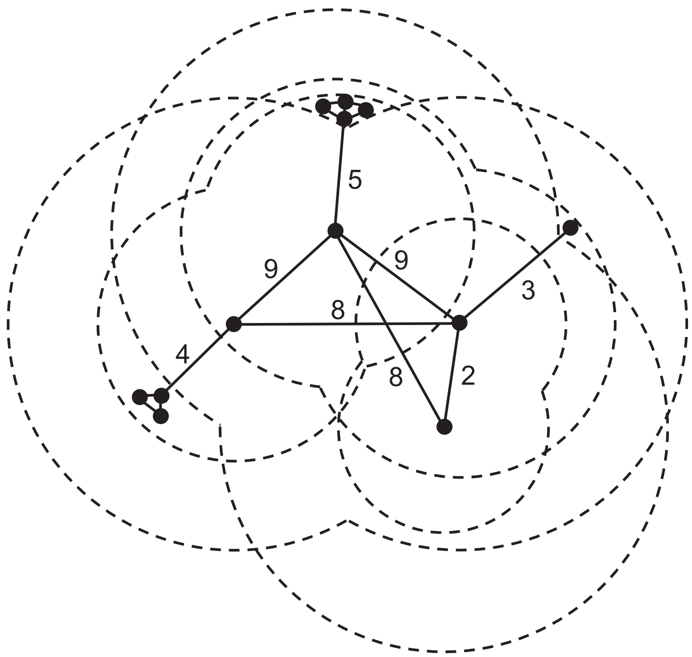
- This topology has interference O(1)!!

Link-based Interference Model

There is no local algorithm that can find a good interference topology

The optimal topology will not be planar





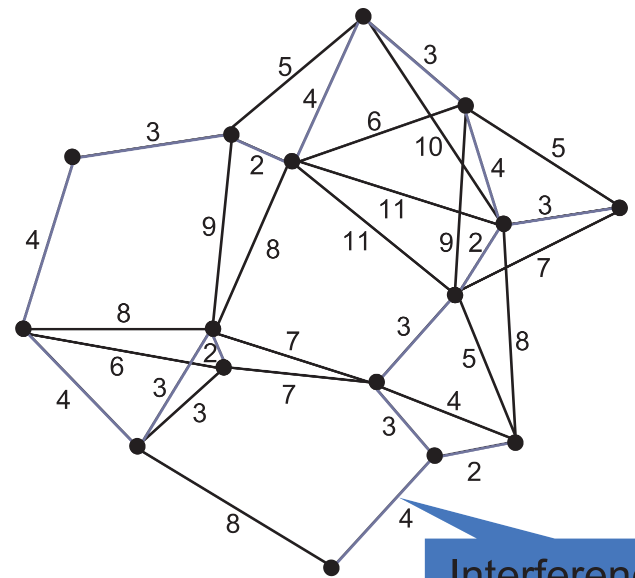
Link-based Interference Model

- LIFE (Low Interference Forest Establisher)
  - Preserves Graph Connectivity

LIFE

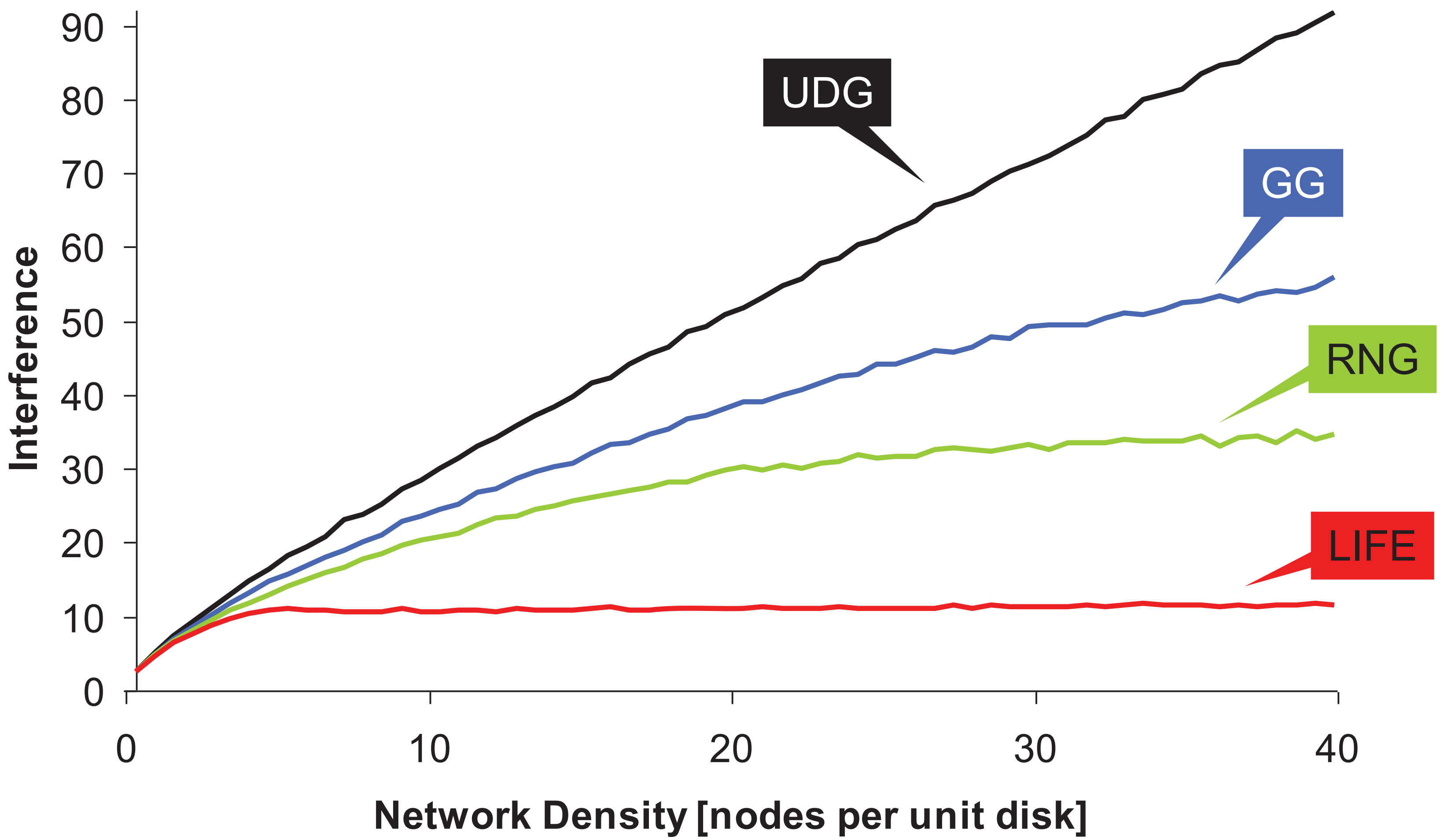
- Attribute interference values as weights to edges
- Compute minimum spanning tree/forest (Kruskal's algorithm)

LIFE constructs a minimum- interference forest



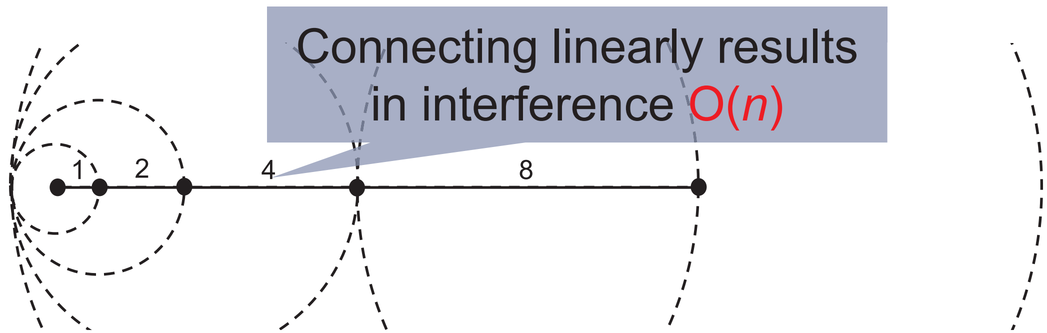
Interference 4

Average-Case Interference: Preserve Connectivity



Node-based Interference Model

- Already 1-dimensional node distributions seem to yield inherently high interference...

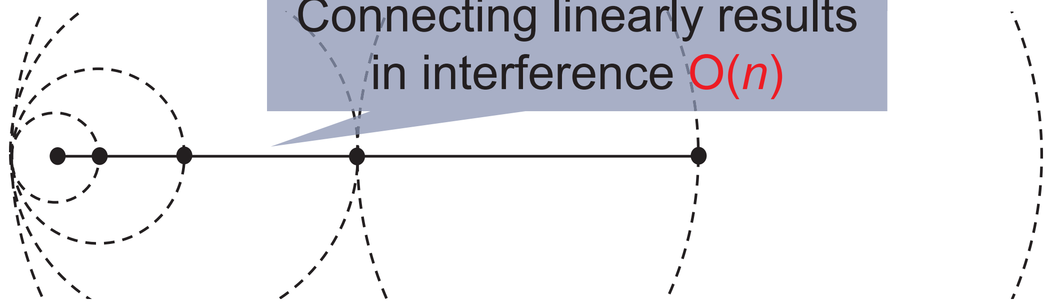


- ...but the exponential node chain can be connected in a better way

Node-based Interference Model

- Already 1-dimensional node distributions seem to yield inherently high interference...

Connecting linearly results in interference O(*n*)



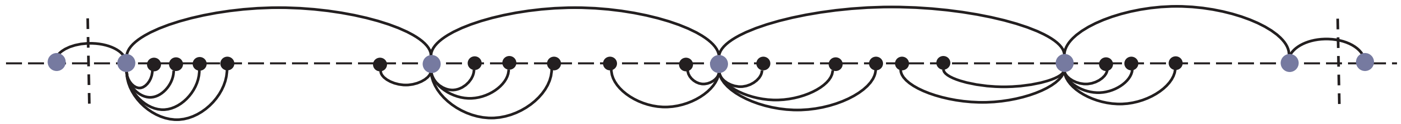
- ...but the exponential node chain can be connected in a better way

$\Rightarrow$ Interference $\in O(\sqrt{n})$

Matches an existing lower bound

Node-based Interference Model

- Arbitrary distributed nodes in one dimension
  - Approximation algorithm with approximation ratio in O($\sqrt[4]{n}$)



- Two-dimensional node distributions
  - Simple randomized algorithm resulting in interference O($\sqrt{n \log n}$)
  - Can be improved to O(√n)

Open problem

- On the theory side there are quite a few open problems. Even the simplest questions of the node-based interference model are open:

- We are given n nodes (points) in the plane, in arbitrary (worst-case) position. You must connect the nodes by a spanning tree. The neighbors of a node are the direct neighbors in the spanning tree. Now draw a circle around each node, centered at the node, with the radius being the minimal radius such that all the nodes' neighbors are included in the circle. The interference of a node u is defined as the number of circles that include the node u. The interference of the graph is the maximum node interference. We are interested to construct the spanning tree in a way that minimizes the interference. Many questions are open: Is this problem in P, or is it NP-complete? Is there a good approximation algorithm? Etc.