Ad Hoc And Sensor Networks
Sample Solution to Exercise 7

Assigned: November 8, 2010
Due: November 15, 2010

1 Slotted Aloha

We define the function $P : \mathbb{R}^2 \to \mathbb{R}$ as

$$P(p, n) := \Pr \text{success} = n \cdot p(1-p)^{n-1}.$$

For a fixed p, $P(p, n)$ is monotone increasing for $n \leq -1/\ln(1-p)$ and monotone decreasing for $n \geq -1/\ln(1-p)$ and therefore $P(p, n)$ is minimized either at $n = A$ or at $n = B$ for $n \in [A, B]$. Therefore, we have to find

$$p_{\text{opt}} := \max_p \left(\min\left\{P(p, A), P(p, B)\right\}\right).$$

For a fixed n, $P(p, n)$ is monotone increasing for $p \leq 1/n$ and monotone decreasing for $p \geq 1/n$ (for $p \in [0, 1]$). Furthermore, $P(1/A, A) \geq P(1/A, B)$ and $P(1/B, B) \geq P(1/B, A)$ for $B \geq A + 1$ and therefore the intersection between $P(p, A)$ and $P(p, B)$ is between the maxima of $P(p, A)$ and $P(p, B)$, respectively. Thus p_{opt} is found where $P(p_{\text{opt}}, A) = P(p_{\text{opt}}, B)$.

$$\begin{aligned}
A * p_{\text{opt}} * (1 - p_{\text{opt}})^{A-1} &= B * p_{\text{opt}} * (1 - p_{\text{opt}})^{B-1} \\
\frac{A}{B} &= (1 - p_{\text{opt}})^{B-1-(A-1)} = (1 - p_{\text{opt}})^{B-A} \\
p_{\text{opt}} &= 1 - \sqrt[B-A]{\frac{A}{B}}.
\end{aligned}$$

For $A = 100$ and $B = 200$, we get

$$p_{\text{opt}} = 0.006908 = \frac{1}{144.8}.$$

2 Broadcast

Student A is right.

An exemplary algorithm:

Source originating the broadcast: Transform the message m as follows: Replace a 1 with 10 and append 11 at the end and at the front of a message, i.e. message m =10110 becomes message $m' =$ 11 10010100 11. Transmit "Hello" in round i if bit i of m' is 1. If a node is not the source it waits until it detects twice a non-free channel for two consecutive rounds. It decodes a non-free channel as 1 and a free channel as 0. It can easily reconstruct the message m by ignoring 11 at the beginning and end and replacing 10 with 1 for the bits received in between the first received 11 and the second 11. As soon as a node decoded the entire message m, it starts to transmit the same m' in the same way as the source.