Principles of Distributed Computing
Exercise 8

1 Coloring Rings

In Chapter 1, we proved that a ring can be colored with 3 colors in $\log^* n + O(1)$ rounds. Clearly, a ring can only be (legally) colored with 2 colors if the number of nodes is even.

a) Prove that, even if the nodes in a directed ring know that the number of nodes is even, coloring the ring with 2 colors requires $\Omega(n)$ rounds![1]

Since coloring a ring with 2 colors apparently takes a long time, we again resort to the problem of coloring rings using 3 colors.

b) Assume that a *maximal independent set* (MIS) has already been constructed on the ring, i.e., each node knows whether it is in the independent set or not. Give an algorithm to color the ring with 3 colors in this scenario! What is the time complexity of your algorithm? Deduce from this a lower bound for computing a MIS!

2 Ramsey theory

In the classic example for Ramsey theory ($R(3,3)$), it is asked how many people you can invite to a party so that are no three people that mutually know each other, and no three people which are mutual strangers. This can work with five persons, but it will never work with 6, i.e. $R(3,3) = 6$. This could be important for planning a party, since three people that mutually do (not) know each other will form their own subgroup during the party and rarely interact with the other guests. In more mathematical terms this is a coloring problem: Consider the complete Graph K_n with n nodes. Can you assign each edge to one of two colors (for example red and blue), so that there is no K_3 as a subgraph where all edges have the same color? As stated above, you can do this with K_5, but not with K_6: $R(3,3) = R(K_3, K_3) = 6$.

a) However, parties with only 5 people are pretty boring. You change your concept to allow bigger parties: From now on, you want that if three people come together, there is always at least one mutual stranger to the other two. On the other hand, you now raise the number of people that are mutual strangers: There should be no group of p people that mutually do not know each other. How many people (depending on p) can attend your party at most?[2]

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[1] As in the lecture, the message size and local computations are unbounded and all nodes have unique identifiers from 1 to n.

[2] If you have trouble finding a solution, start with $p = 2$ and $p = 3$.