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Principles of Distributed Computing
Exercise 4

1 Deterministic Maximal Independent Set

In the lecture, we discussed a simple maximal independent set (MIS) algorithm in which the decisions of the nodes are based on their identifiers. The time complexity of this algorithm is $O(n)$.

We might hope that if the nodes with the largest degrees, i.e., the largest number of neighbors, decide to enter the MIS, the set of undecided nodes reduces the most. In the following algorithm we try to exploit the knowledge of the node degrees:

Assume that each node knows its degree and also the degrees of all its neighbors. If a node has a larger degree than all its undecided neighbors, it joins the MIS and informs its neighbors. Once a node v learns that (at least) one of its neighbors joined the MIS, v decides not to join the MIS.

Naturally, the algorithm does not make any progress if two or more neighboring nodes share the largest degree. As this is a difficult problem, we will assume in the following that this situation does not occur, i.e., if a node v has the largest degree, then no neighboring node has the same degree as v.[1]

a) Draw a graph that illustrates that this algorithm has a large time complexity for trees! Give a lower bound on the time complexity for trees consisting of n nodes!

b) Construct a graph that shows that the time complexity of this algorithm is even worse for arbitrary graphs than for trees! What is the time complexity?

c*) We will now modify the algorithm: The degree of a node v in any given round is only the number of *undecided* neighbors. Prove (tight) lower and upper bounds for this modified algorithm on arbitrary graphs!

2 Randomized Maximal Independent Set

As we saw in the lecture, randomization helps to solve the MIS problem. In Algorithm 17, a node marks itself with probability $\frac{1}{2d(v)}$, where $d(v)$ denotes the current degree of node v. This means that the larger the degree of a node, the lower the probability that it enters the MIS, which is quite counterintuitive.

We will now consider a different, more intuitive algorithm. Assume that each node knows the maximum degree $\Delta := \max_{v \in V} d(v)$. Each node v marks itself with probability $\frac{d(v)}{2\Delta}$. The rest of the algorithm is analogous to Algorithm 17.

a) Draw a graph that illustrates that the time complexity of this randomized algorithm is large! What is the time complexity for arbitrary graphs consisting of n nodes?

[1]The motivation for this constraint ist that if we prove that the time complexity is large even if there is no conflict in each step, then being able to break ties clearly does not help.