1 The Resilience of a Quorum System

a) Does a quorum system exist, which still works although all nodes of a specific quorum fail? Give an example or prove its nonexistence.

b) Consider the nearly all quorum system, which is made up of \( n \) different quorums, each containing \( n-1 \) servers. What is the resilience of this quorum system?

c) Can you think of a quorum system that contains as many quorums as possible? Note: the quorum system does not have to be minimal.

Basic

2 A Quorum System

Consider a Quorum System with 7 nodes numbered from 001 to 111, where each three nodes fulfilling \( x \oplus y = z \) constitute a quorum. In the following picture this quorum system is represented: All nodes on a line (such as 111, 010, 101) and the nodes on the circle (010, 100, 110) form a quorum.

\[
\begin{array}{c}
111 \\
010 \\
100 \\
001 \\
110 \\
011
\end{array}
\]

a) Of how many different quorums does this system consist and what are its work and its load?

b) Calculate its resilience \( f \). Give an example where this quorum system does not work anymore with \( f + 1 \) faulty nodes.
3 Uniform Quorum Systems

Definitions:

$s$-Uniform: A quorum system $S$ is $s$-uniform if every quorum in $S$ has exactly $s$ elements.

Balanced access strategy: An access strategy $Z$ for a quorum system $S$ is balanced if it satisfies $L_Z(v_i) = L$ for all $v_i \in V$ for some value $L$.

Claim: An $s$-uniform quorum system $S$ reaches an optimal load with a balanced access strategy, if such a strategy exists.

a) Describe in your own words why this claim is true.

b) Prove the optimality of a balanced access strategy on an $s$-uniform quorum system.