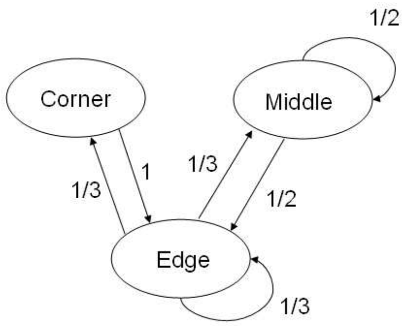
Discrete Event Systems
Exercise 9: Sample Solution

1 Night Watch

a) Observe that the problem is symmetric, e.g., from all four corners, the situation looks the same, and the probability of being in a specific corner room is the same for all corners. The same holds for rooms at the border and for rooms in the middle. Thus, instead of using 16 states, we consider the following simplified Markov chain consisting of 3 states only:



Hence, in the steady state, it holds that

$$P_c = 1/3 \cdot P_e; \quad P_e = 1/3 \cdot P_e + 1/2 \cdot P_m + P_c; \quad 1 = P_c + P_e + P_m$$

Solving this equation system gives: $P_c = 1/6$. The probability of being in a specific corner is therefore $1/6 \cdot 1/4 = 1/24$.

b) Since the two walks are independent, we have

$$1/24 + 1/24 - (1/24)^2 = 0.082.$$

2 Probability of Arrival

The proof is similar to the one about the transition time h_{ij} (see script). We express f_{ij} as a condition probability that depends on the result of the first step in the Markov chain. Recall that the random variable T_{ij} is the *hitting time*, that is, the number of steps from i to j. We get $Pr[T_{ij} < \infty | X_1 = k] = Pr[T_{ij} < \infty]$ for $k \neq j$ and $Pr[T_{ij} < \infty | X_1 = j] = 1$. We can therefore write f_{ij} as

$$\begin{aligned} f_{ij} &= Pr[T_{ij} < \infty] = \sum_{k \in S} Pr[T_{ij} < \infty | X_1 = k] \cdot p_{ik} \\ &= p_{ij} \cdot Pr[T_{ij} < \infty | X_1 = j] + \sum_{k \neq j} Pr[T_{ij} < \infty | X_1 = k] \cdot p_{ik} \\ &= p_{ij} + \sum_{k \neq j} p_{ik} f_{kj}. \end{aligned}$$