**ETH**

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**Distributed**
**Computing Group**

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# Discrete Event Systems
# Exercise 6

## 1   The Winter Train Problem

We consider two trains T1 and T2 transporting skiers from Sarnen and Lucerne to Engelberg. Because there is only one ground rail track from Stans to Engelberg, at most one train might be between these two villages at any time. There is a switch in Stans, which either connects the track between Sarnen and Engelberg xor the track between Lucerne and Engelberg. After the train conductor has pressed a button m in (Sarnen | Lucerne), its train moves to Engelberg, but might have to wait in Stans until the other train has left the critical section. Once arrived in Engelberg, the train waits for 100s and then returns.

The sensors a1, a2, b1, b2 and c indicate the presence of a train with the value 1, otherwise, the value is 0. The switch in Stans is accessed through a variable G, as indicated in the picture. Finally, the motion of the trains is regulated by assigning 'R', 'L' or 'S' to the train, to move right, left, or stop, respectively.

The situation is shown below. Draw the corresponding State Chart using the notation introduced in the figure!
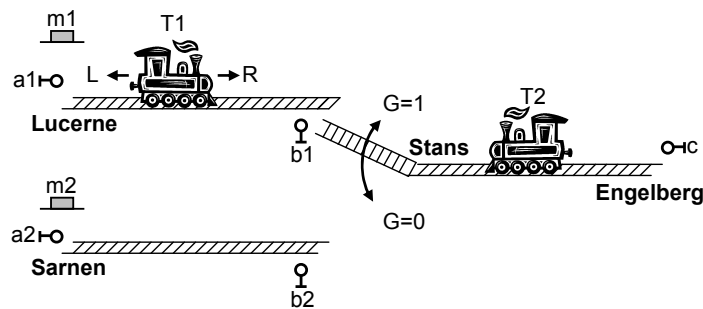


## 2   CNF

Convert the following grammars to Chomsky normal form (CNF).

a)

$$S \rightarrow SAS \mid A$$
$$A \rightarrow 0 \mid 1$$

b)

$$S \rightarrow T1T \mid T$$
$$T \rightarrow T0S \mid T1S \mid U$$
$$U \rightarrow 1U \mid \epsilon$$

# 3  Transducer and Turing Machine

Alice is very happy because she was accepted for an internship at Tintel, one of the world's leading processor manufacturers. Unfortunately, she has only attended the famous DES lecture during her studies at ETH and knows nothing about electronic circuits. Therefore, she wants to solve her first assignment using a transducer - please assist her:

a) Alice first needs to compute the sum $a + b$ of two binary numbers $a$ and $b$. The numbers are represented bitwise with LSB first. I.e. $a$ is given as $a[0]a[1]a[2]a[3]\ldots a[n]$ and $b$ as $b[0]b[1]b[2]\ldots b[n]$. Unfortunately, Alice receives a single string containing both, $a$ and $b$ mixed together in the form $a[0]b[0]a[1]b[1]a[2]b[2]\ldots a[n]b[n]$. Draw the transducer that outputs $a+b$ (in binary) with same bit ordering (LSB first) and only accepts upon a valid input (even number of binary digits).

b) Mister Intal (Alice's supervisor) is only familiar with assembler and the Turing machine. Therefore, Alice is now asked to translate her transducer to a Turing machine. The two numbers $a$ and $b$ are written on the machine tape in the form $a[n]a[n-1]\ldots a[1]a[0] + b[m]b[m-1]\ldots b[1]b[0]$, where $a[n]$ is the MSB of $a$. Note that the two numbers are separated by the '+' symbol, that they have an equal number of bits[1], i.e. $m = n$, and that $\Gamma = \{0|1| + |\square\}$. Initially, the head of the TM points to $a[n]$. At the end, it should point to the MSB of the result.

*Hint* You may extend the alphabet $\Gamma$ to put temporary symbols on the tape.

c) In her last assignment, Alice is asked to implement a *binary to unary converter*. This converter takes a number $a$ in binary (alphabet $\{0|1\}$) and converts it to a unary number $u$ (alphabet $\{1\}$). Initially, the TM head points to the MSB of $a$. At the end, the head should point to the right-most digit of $u$.

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[1]Your machine may crash or produce a wrong result if this condition does not hold.