Results for radio broadcast

A Survey

Vijay D'silva
vdsilva@inf.ethz.ch

Department of Computer Science

Agenda

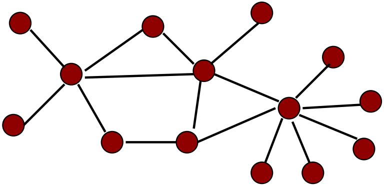
Theorem (Broadcast Potpourri)

In a multi-hop radio network, broadcast using a deterministic algorithm:

- *requires processor ids*
- *is* $\Omega(n)$ *for a family of networks*
- *is* $\Omega(D\log(n))$ *for another family*

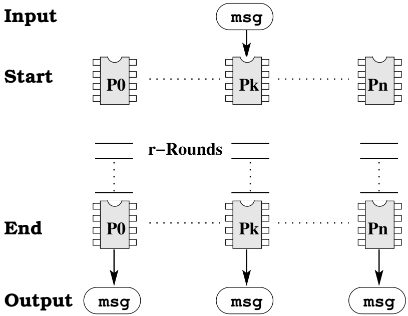
There exists a randomized algorithm achieving broadcast with constant probability in $O(D\log N + \log^2 N)$ *rounds.*

Multi-hop radio network



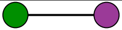
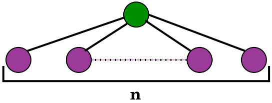
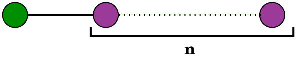
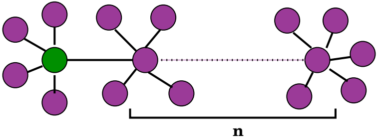
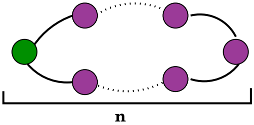
- Network: undirected graph $G = (V, E)$
- Node: $v \in V$, Turing machine
- Nodes transmit or receive
- $\{v, v'\} \in E$ can communicate
- Message received if *exactly* one neighbour transmits
- Otherwise hear noise
- Synchronous rounds

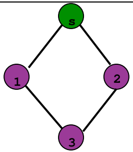
Broadcast



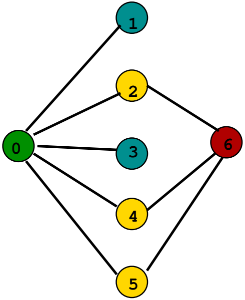
- Finite set of nodes
- Start: source has message
- Protocol: run locally by nodes
- End: all nodes have message
- Nodes don't know topology
- Problem: no collision detection

Assume no node ids. Lower bound for a deterministic algo?

Network	*rounds* $\geq$
	1
	1
	n
	n
	n

Network	*rounds* $\geq$
	Impossible

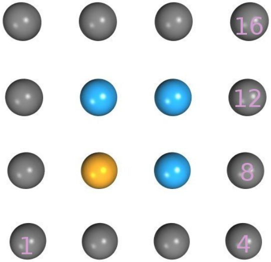
A family of networks



- Family C_n: Source, sink, n layer nodes
- Source connected to all layer nodes
- Some layer nodes connected to sink
- Nodes have ids
- Have to ensure a golden node transmits
- Broadcasting in C_n can be reduced to winning the n^{th} hitting game

The Hitting Game

$Gold = \{6, 9, 15\}$
You win!



$\{11\}$, $\{12, 7, 3\}$, $\{7\}$, $\{9, 6, 10\}$, $\{6\}$

- $Lamps = Blue \cup Gold$
- $Blue \cap Gold = \emptyset$
- Aim: Find a gold lamp
- Initially all lamps off
- Move: $M_i \subseteq Lamps$
- If $|M_i \cap Gold| = 1$, player wins
- If $|M_i \cap Blue| = 1$, switch on
- Else "*Try Again!* "

Adversary Procedure

- For each player strategy, define *Gold* to foil it
- Return *Try Again!* as often as possible
- *Oblivious strategy*: does not depend on referee's answers
- Sufficient to consider oblivious strategies

Find Set. Input: set of moves M_i

$Gold := Lamps$
while winning move M_i exists **do**
  move $Gold \cap M_i$ to *Blue*
  while non-singleton blue move M_j exists **do**
    pick $\ell \in Gold \cap M_j$
    move ℓ to *Blue*
  end while
end while
output *Gold*

Adversary Procedure

Find Set(M_i)

$Gold := Lamps$
while winning M_i exists **do**
  move $Gold \cap M_i$ to $Blue$
  while blue M_j exists and $|M_j| > 1$ **do**
    pick $\ell \in Gold \cap M_j$
    move ℓ to $Blue$
  end while
end while
output $Gold$

Lemma 1

If the procedure returns the set $Gold$,

- $|M_i \cap Gold| \neq 1$
- M_i is a blue move iff M_i is a singleton

Lemma 2

If $|\{M_i\}| \leq \frac{n}{2}$ then the output set $Gold \neq \emptyset$.

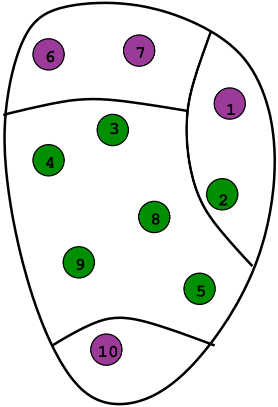
An $\Omega(n)$ lower bound

- Bar-Yehuda, Goldreich, Itai, PODC 1987 and Journal of Computer and System Sciences 1992.
- Successful broadcast reduced to winning hitting game
- Rounds for broadcast and steps for winning strategy differ by constant factor of $\frac{1}{4}$
- n^{th} hitting game cannot be won in less than $\frac{n}{2}$ steps

Theorem (Lower Bound for C_n)

There exists no deterministic broadcast protocol which terminates in less than $\frac{n}{8}$ rounds for any network in C_n.

Towards a better lower bound



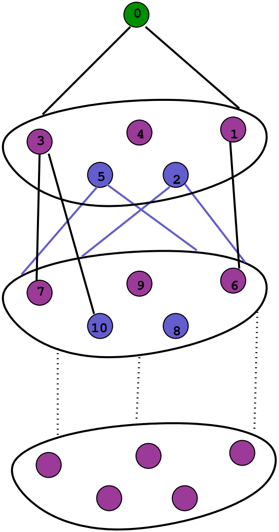
- A set S of k disconnected nodes
- In each round i, compare transmitting and silent nodes
- Retain the majority in S

Lemma: Properties of S

After $1 \leq r \leq \log(\frac{k}{2})$ rounds

- $|S| \geq 2$
- If a pair $v_1, v_2 \in S$, they have both transmitted together or both remained silent in each round.

Another network family



- Construct a graph B_n^D
- Diameter $D \leq \frac{n}{2}$
- Layers with $\frac{n}{D}$ disconnected nodes
- Pick a *collision pair* from the lemma
- Connect to all nodes in next layer
- Other nodes connect to fewer nodes in next layer
- D such layers

An improved lower bound

- Bruschi and Del Pinto, Distributed Computing 1997
- Independently, Chlebus et. al, SODA 2000
- Given protocol and network, find collision pairs
- Message reaches layer i from $i+1$ if exactly one node from collision pairs is used
- $n(1 - i/D)$ candidates for collision pairs

Theorem (Lower Bound for B_n^D)

Given a deterministic broadcast protocol and $n, D \leq \frac{n}{2}$, there exists a network B_n^D which requires $\Omega(D\log(n))$ rounds for broadcast.

Summary: Deterministic Radio Broadcast

- First lower bound: $\Omega(n)$
- Best known lower bound: $\Omega(D \log(n))$
- First distributed algorithm: Diks et al, ESA 1999
- First sub-quadratic algo: $O(n^{11/6})$, Chlebus et al, SODA 2000
- Non constructive upper bound: $O(n \log^2(n))$, Chrobak, Gasieniec and Rytter, FOCS 2000
- Constructive upper bound: Indyk, SODA 2002

A Randomized Protocol

- Bar-Yehuda, Goldreich, Itai, PODC 1987 and Journal of Computer and System Sciences 1992
- Does not require node ids
- Input: upper bound on nodes in network and max degree
- Matches lower bound on some network families

Basic Procedure

Decay(k, msg)

coin := *heads*
steps := 0
while *coin* = *heads* and *steps* $\leq k$ **do**
  send *msg* to neighbours
  flip *coin*
  increment *steps*
end while

Theorem (Decay)

When $d \geq 2$ neighbours of a node v execute `Decay` *starting simultaneously, the probability that v receives a message by time t, $P(t,d)$ satisfies:*

- *As $t \to \infty$, $P(t,d) \geq \frac{2}{3}$*
- *For $t \geq 2\lceil \log(d) \rceil$, $P(t,d) > \frac{1}{2}$*

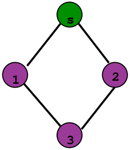
Broadcast Protocol

Broadcast(N, Δ)

$k := 2\lceil\log\Delta\rceil$
$p := \lceil\log(N/\varepsilon)\rceil$
wait till *msg* arrives
for p phases **do**
  wait till $(rnd\ mod\ k) = 0$
  Decay(k, msg)
end for

- N: upper bound on nodes
- Δ: upper bound on max degree
- Runs in p phases
- Execution synchronized in phases to satisfy precondition of theorem.

Broadcast Protocol: Simulation



$N = 6$
$\Delta = 4$
$\epsilon = 0.1$
$k = 2\lceil \log(4) \rceil$
$= 4$
$p = \lceil \log(60) \rceil$
$= 6$

Phase 1	s	v_1	v_2	v_3
0	H, m	Idle	Idle	Idle
1	H, m	Idle, m	Idle, m	Idle
2	H, m	Idle, m	Idle, m	Idle
3	T, m	Idle, m	Idle, m	Idle
Phase 2				
0	H, m	H, m	H, m	Idle
1	T, m	H, m	H, m	Idle
2	T, m	T, m	T, m	Idle
3	T, m	T, m	T, m	Idle
Phase 3				
0	H, m	H, m	H, m	Idle
1	H, m	T, m	H, m	Idle
2	T, m	T, m	H, m	Idle, m
3	T, m	T, m	H, m	Idle, m

Correctness: Message Receipt

Theorem (Correctness: Message Receipt)

If the processors in a network execute `Broadcast`*, starting with a source s, then:*

$$Pr(\textit{all nodes receive } m) \geq 1 - \varepsilon$$

Proof

$$\begin{aligned}
& Pr(\text{some } v \text{ does not receive } m) \\
= \; & Pr(\text{some } v \text{ did not receive } m \text{ but its neighbours did}) \\
\leq \; & \Sigma_{v \neq s} Pr(v \text{ didn't receive } m \text{ but its neighbours did}) \\
\leq \; & n \cdot \left(\frac{1}{2}\right)^{\lceil \log(N/\varepsilon) \rceil} \leq n \cdot \frac{\varepsilon}{N} \\
\leq \; & \varepsilon
\end{aligned}$$

Correctness: Termination - Preliminaries

- $p(\varepsilon) = \Theta(\max(D, \log(N/\varepsilon)))$
- $p(\varepsilon)$: a number of phases considering diameter and conflict delays
- $rcv(v)$: round in which node v receives msg
- $dist(v_1, v_2)$: length of shortest path from v_1 to v_2
- $travel(m, p)$: distance raveled by m in p phases

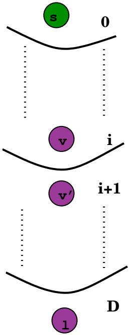
Theorem (Correctness: Time for Broadcast)

If the broadcast protocol runs indefinitely, and v is the last node to receive the message m, then

$$Pr(v \text{ receives } m \text{ in } k \cdot p(\varepsilon) \text{ rounds}) > 1 - \varepsilon$$

Correctness: Termination - Proof

Consider an arbitrary node *v* and the probability that it does not receive a message by a certain round.



Proof

$$
\begin{aligned}
& Pr(\ v \text{ receives after } kp(\varepsilon) \text{ rounds}) \\
= \ & Pr(m \text{ travels less than } dist(v, s)) \\
\leq \ & Pr(m \text{ travels less than } D) \\
= \ & Pr(\Sigma_{i=1}^{p(\varepsilon)} travel(m, 1) < D) \\
& \dots \\
\leq \ & \frac{\varepsilon}{N}
\end{aligned}
$$

$$Pr(rcv(v) \leq k \cdot p(\varepsilon)) > 1 - \frac{\varepsilon}{N} > 1 - \varepsilon$$

Summary: Randomized Radio Broadcast

- Randomization "beats" deterministic impossibility result
- Expected running time: $O(D \log N + \log^2 N)$
- Lower bound: $\Omega(D \log(N/D))$, Kushilevitz and Mansour, SIAM Journal of Computing, 1998
- Alternate proof: Liu and Prabhakaran, COCOON, 2002
- General lower bound on diameter-2 networks: $\Omega(\log^2 n)$, Alon et al. Journal of Computer and System Sciences, 1991
- Broadcast and gossip studied together in recent work

Conclusion

Theorem (Broadcast Potpourri)

In a multi-hop radio network, broadcast using a deterministic algorithm:

- *requires processor ids*
- *is* $\Omega(n)$ *for a family of networks*
- *is* $\Omega(D \log(n))$ *for another family*

There exists a randomized algorithm achieving broadcast with constant probability in $O(D \log N + \log^2 N)$ *rounds.*

Bonus

- Example: Multi-hop radio network
- Reduction of broadcast to hitting games
- Lower bound for randomized algorithms

The sensor my friend is blowing in the wind

- Sensor nodes around 100 cubic millimeter large
- Includes temperature, light sensors, bi-directional wireless communication
- Set of nodes: distributed system
- Many potential applications

Reduction to the hitting game

- Reduce broadcast protocol to restricted protocol in which either source or sink is active
- Reduce restricted protocol to abstract protocol where only middle nodes transmit and either source or sink receives
- Middle nodes know when transmission is successful
- Abstract broadcast achieved when a gold node successfully transmits
- Reduce abstract broadcast to hitting game

Randomized lower bound

- Consider the blue-gold network family
- Yao's minimax principle: reduce proving randomized lower bound to deterministic lower bound on probabilistic input
- Construct probability distribution of inputs
- Lower bound of $\Omega(\log m)$
- Connect D layers with N/D nodes in each
- Some calculations later: $\Omega(D \log(N/D))$