Chapter 12
POSITIONING

Mobile Computing

Winter 2005 / 2006

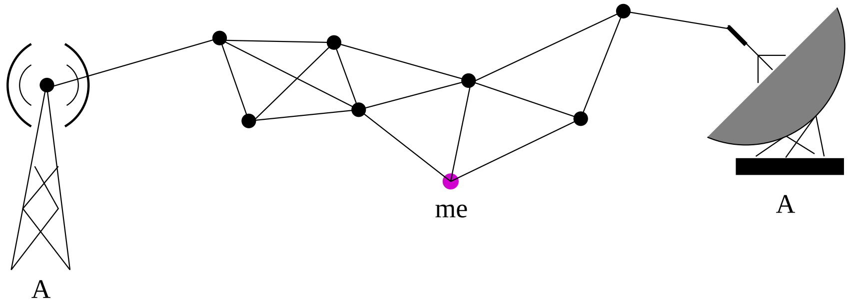
Distributed Computing Group

Overview

- Motivation
- Measurements
- Anchors
- Virtual Coordinates
- Heuristics
- Practice

Motivation

- Why positioning?
  - Sensor nodes without position information is often meaningless
  - Heavy and/or costly positioning hardware
  - Geo-routing



- Why not GPS (or Galileo)?
  - Heavy, large, and expensive (as of yet)
  - Battery drain
  - Not indoors
  - Accuracy?

- Solution: equip small fraction with GPS (anchors)

Measurements

Distance estimation

- Received Signal Strength Indicator (RSSI)
  - The further away, the weaker the received signal.
  - Mainly used for RF signals.
- Time of Arrival (ToA) or Time Difference of Arrival (TDoA)
  - Signal propagation time translates to distance.
  - RF, acoustic, infrared and ultrasound.

Angle estimation

- Angle of Arrival (AoA)
  - Determining the direction of propagation of a radio-frequency wave incident on an antenna array.
- Directional Antenna
- Special hardware, e.g., laser transmitter and receivers.

Positioning (a.k.a. Localization)

- Task: Given distance or angle measurements or mere connectivity information, find the locations of the sensors.

- Anchor-based
  - Some nodes know their locations, either by a GPS or as pre-specified.
- Anchor-free
  - Relative location only. Sometimes called virtual coordinates.
  - Theoretically cleaner model (less parameters, such as anchor density)

- Range-based
  - Use range information (distance estimation).
- Range-free
  - No distance estimation, use connectivity information such as hop count.
  - It was shown that bad measurements don't help a lot anyway.

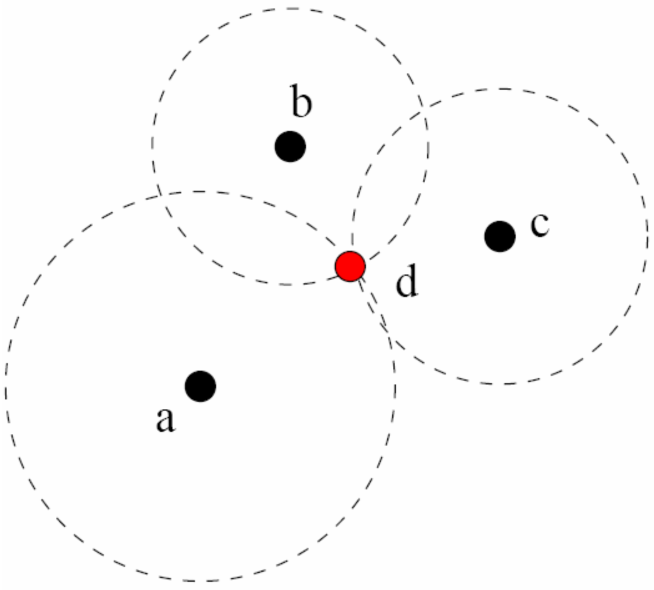
Trilateration and Triangulation

- Use geometry, measure the distances/angles to three anchors.

- Trilateration: use distances
  - Global Positioning System (GPS)

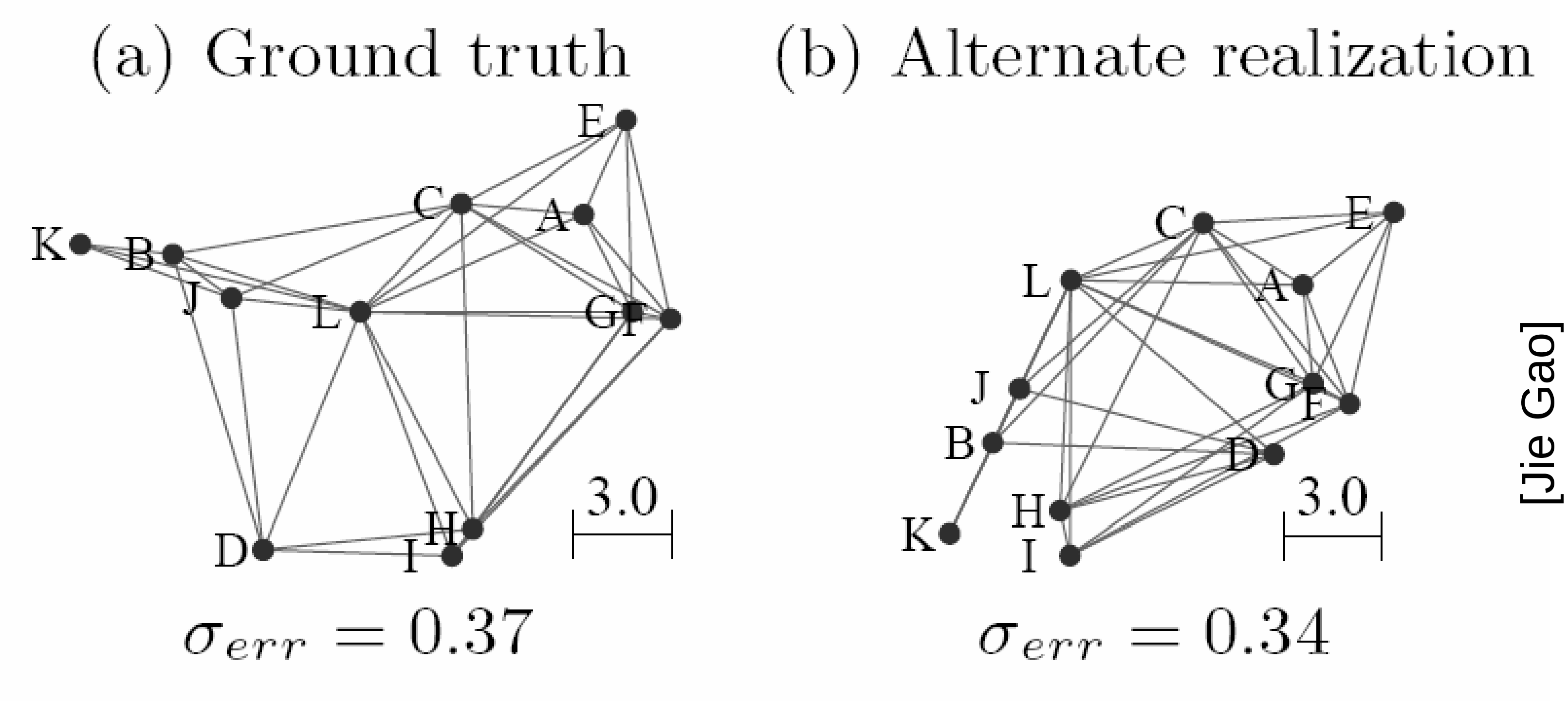
- Triangulation: use angles
  - Some cell phone systems

- How to deal with inaccurate measurements?
  - Least squares type of approach
  - What about strictly more than 3 (inaccurate) measurements?



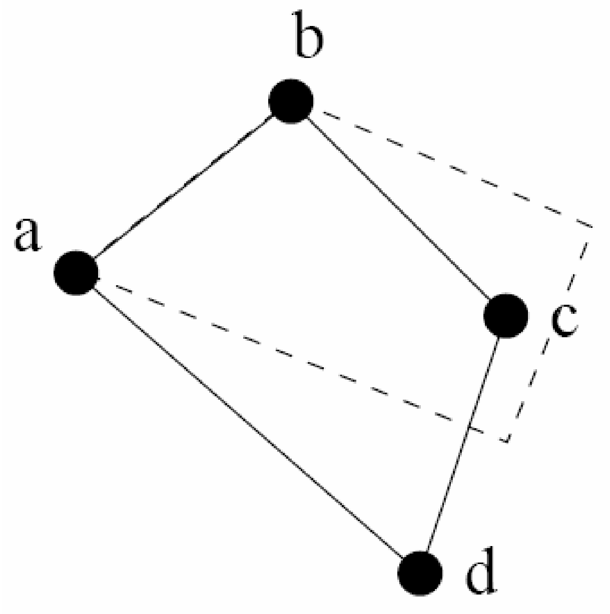
Ambiguity Problems

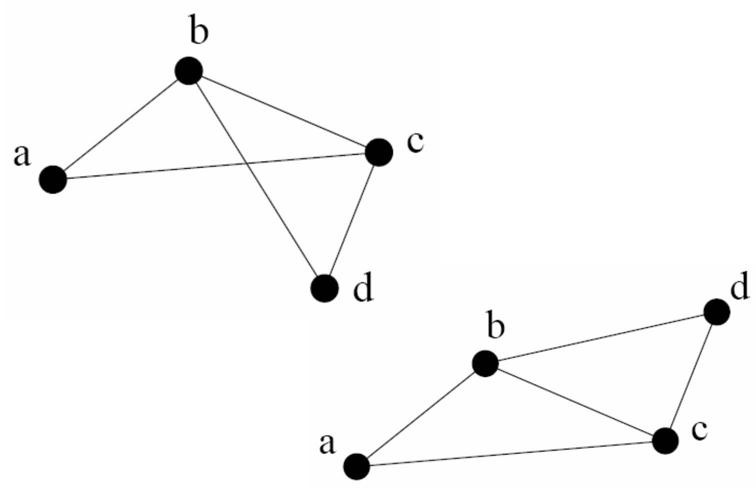
- Same distances, different realization.



[Jie Gao]

Continuous deformation, flips, etc.





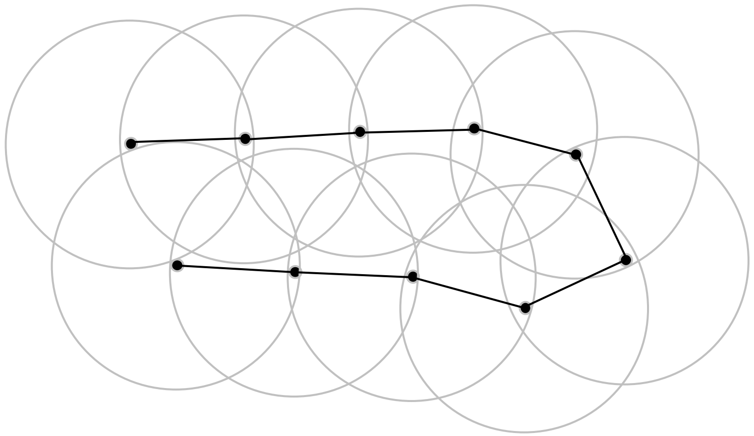
[Jie Gao]

- Rigidity theory: Given a set of rigid bars connected by hinges, rigidity theory studies whether you can move them continuously.

Simple hop-based algorithms

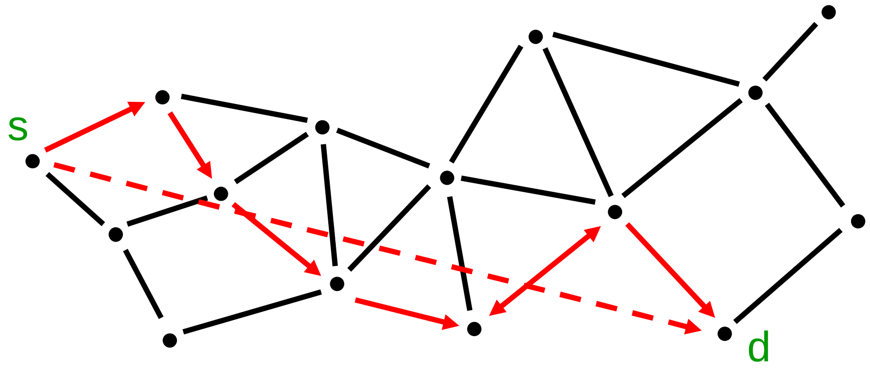
- Algorithm
  - Get graph distance h to anchor(s)
  - Intersect circles around anchors
    - radius = distance to anchor
  - Choose point such that maximum error is minimal
    - Find enclosing circle (ball) of minimal radius
    - Center is calculated location

- In higher dimensions: $1 < d \leq h$
  - Rule of thumb: Sparse graph
    → bad performance



How about no anchors at all...?

- In absence of anchors...
  → ...nodes are clueless about real coordinates.
- For many applications, real coordinates are not necessary
  → Virtual coordinates are sufficient
  → Geometric Routing requires only virtual coordinates
    - Require no routing tables
    - Resource-frugal and scalable



Virtual Coordinates

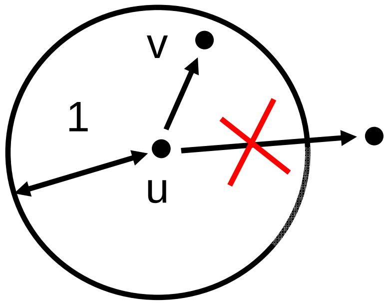
- Idea:
  Close-by nodes have similar coordinates
  Distant nodes have very different coordinates

→ Similar coordinates imply physical proximity!

- Applications
  - Geometric Routing
  - Locality-sensitive queries
  - Obtaining meta information on the network
  - Anycast services („*Which of the service nodes is closest to me?*")
  - Outside the sensor network domain: e.g., Internet mapping

Model

- Unit Disk Graph (UDG) to model wireless multi-hop network
  - Two nodes can communicate iff Euclidean distance is at most 1



- Sensor nodes may not be capable of
  - Sensing directions to neighbors
  - Measuring distances to neighbors

- Goal: Derive topologically correct coordinate information from connectivity information only.
  - Even the simplest nodes can derive connectivity information

Context

	Distance/Angle information	Connectivity information only
With Anchors	Positioning (Solution quality depends on anchor density)	
No Anchors	Distance/Angle based Virtual Coordinates	Connectivity based Virtual Coordinates

next

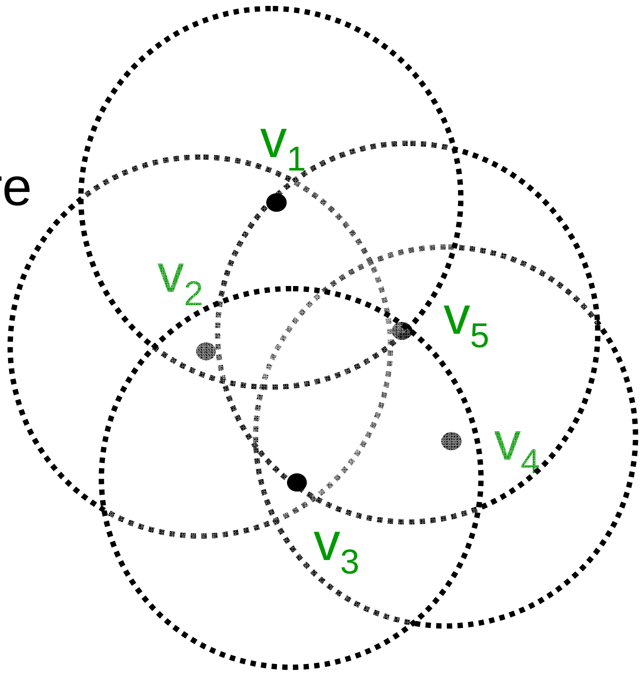
Virtual Coordinates ⟷ UDG Embedding

- Given the connectivity information for each node...

v_1	v_2	v_3	v_4	v_5
v_2	v_1	v_2	v_3	v_1
v_5	v_3	v_4	v_5	v_3
		v_5		v_4

...and knowing the underlying graph is a UDG...

- ...find a UDG embedding in the plane such that all connectivity requirements are fulfilled! (→ Find a realization of a UDG)



This problem is NP-hard!

(Simple reduction to *UDG-recognition* problem, which is NP-hard)

[Breu, Kirkpatrick, Comp.Geom.Theory 1998]

UDG Approximation – Quality of Embedding

- Finding an exact realization of a UDG is NP-hard.
  → Find an embedding r(G) which approximates a realization.

- Particularly,
  → Map adjacent vertices (edges) to points which are close together.
  → Map non-adjacent vertices („non-edges“) to far apart points.

- Define quality of embedding q(r(G)) as:

> Ratio between longest edge to shortest non-edge in the embedding.

Let ρ(u,v) be the distance between points u and v in the embedding.

$$q(r(G)) := \frac{\max_{\{u,v\}\in E}\rho(u,v)}{\min_{\{u',v'\}\notin E}\rho(u',v')}$$

UDG Approximation

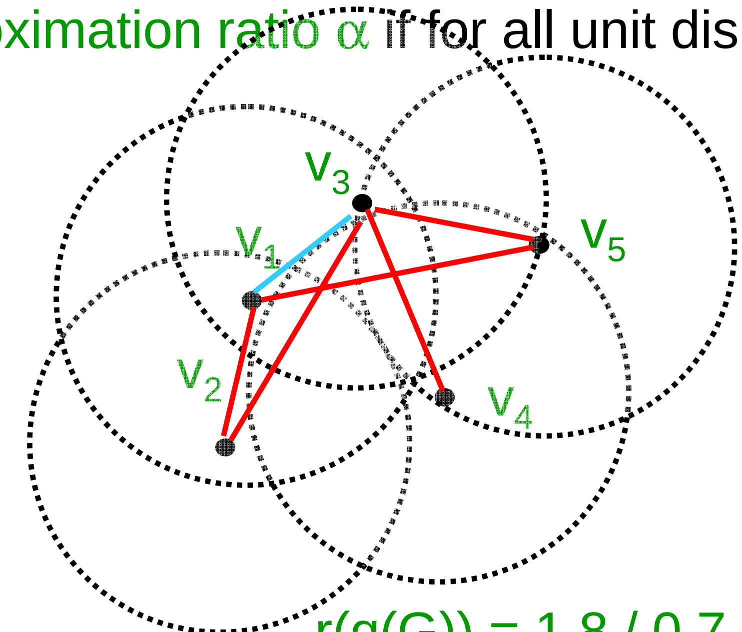
- For each UDG G, there exists an embedding r(G), such that, $q(r(G)) \leq 1$.
  (a realization of G)

$$q(r(G)) := \frac{\max_{\{u,v\} \in E} \rho(u,v)}{\min_{\{u',v'\} \notin E} \rho(u',v')}$$

- Finding such an embedding is NP-hard
- An algorithm ALG achieves approximation ratio α if for all unit disk graphs G, $q(r_{ALG}(G)) \leq \alpha$.

- Example:

v_1	v_2	v_3	v_4	v_5
v_2	v_1	v_2	v_3	v_1
v_5	v_3	v_4	v_5	v_3
		v_5		v_4



r(q(G)) = 1.8 / 0.7 = 2.6

Some Results

- There are a few virtual coordinates algorithms
  All of them evaluated only by simulation on random graphs
- In fact there is only one provable approximation algorithm

There is an algorithm which achieves an approximation ratio of $O(\log^{2.5} n \sqrt{\log\log n})$, n being the number of nodes in G.

- Plus there are lower bounds on the approximability.

There is no algorithm with approximation ratio better than $\sqrt{3/2} - \epsilon$, unless P=NP.

Approximation Algorithm: Overview

- Four major steps

1. Compute metric on MIS of input graph → Spreading constraints
   (Key conceptual difference to previous approaches!)

2. Volume-respecting, high dimensional embedding

3. Random projection to 2D

4. Final embedding

UDG Graph G with MIS M.

↓

Approximate pairwise distances between nodes such that, MIS nodes are neatly spread out.

↓

Volume respecting embedding of nodes in $\mathbb{R}^n$ with small distortion.

↓

Nodes spread out fairly well in $\mathbb{R}^2$.

↓

Final *embedding of G in $\mathbb{R}^2$.*

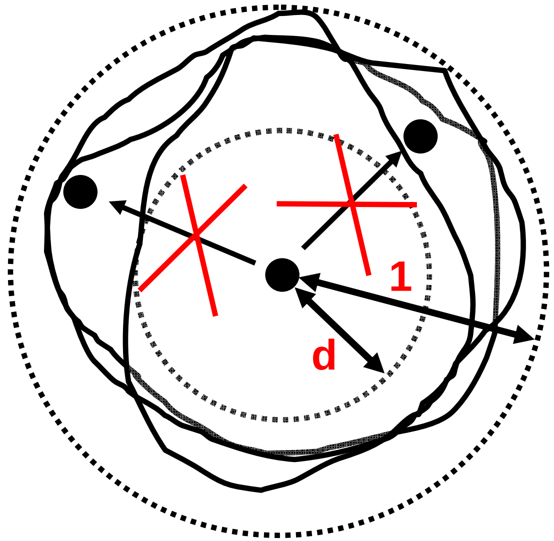
Lower Bound: Quasi Unit Disk Graph

- Definition Quasi Unit Disk Graph:

Let $V \in \mathbf{R}^2$, and $d \in [0,1]$. The symmetric Euclidean graph G=(V,E), such that for any pair $u,v \in V$

- $\text{dist}(u,v) \leq d \Rightarrow \{u,v\} \in E$
- $\text{dist}(u,v) > 1 \Rightarrow \{u,v\} \notin E$

is called *d-quasi unit disk graph*.



- Note that between d and 1, the existence of an edge is unspecified.

Reduction

- We want to show that finding an embedding with $q(r(G)) \leq \sqrt{3/2} - \epsilon$, where ε goes to 0 for n → ∞ is NP-hard.

- We prove an equivalent statement:

> Given a unit disk graph G=(V,E), it is NP-hard to find a realization of G as a d-quasi unit disk graph with $d \geq \sqrt{2/3} + \epsilon$, where ε tends to 0 for n→∞.

→ Even when allowing non-edges to be smaller than 1, embedding a unit disk graph remains NP-hard!

→ It follows that finding an approximation ratio better than $\sqrt{3/2} - \epsilon$ is also NP-hard.

Reduction

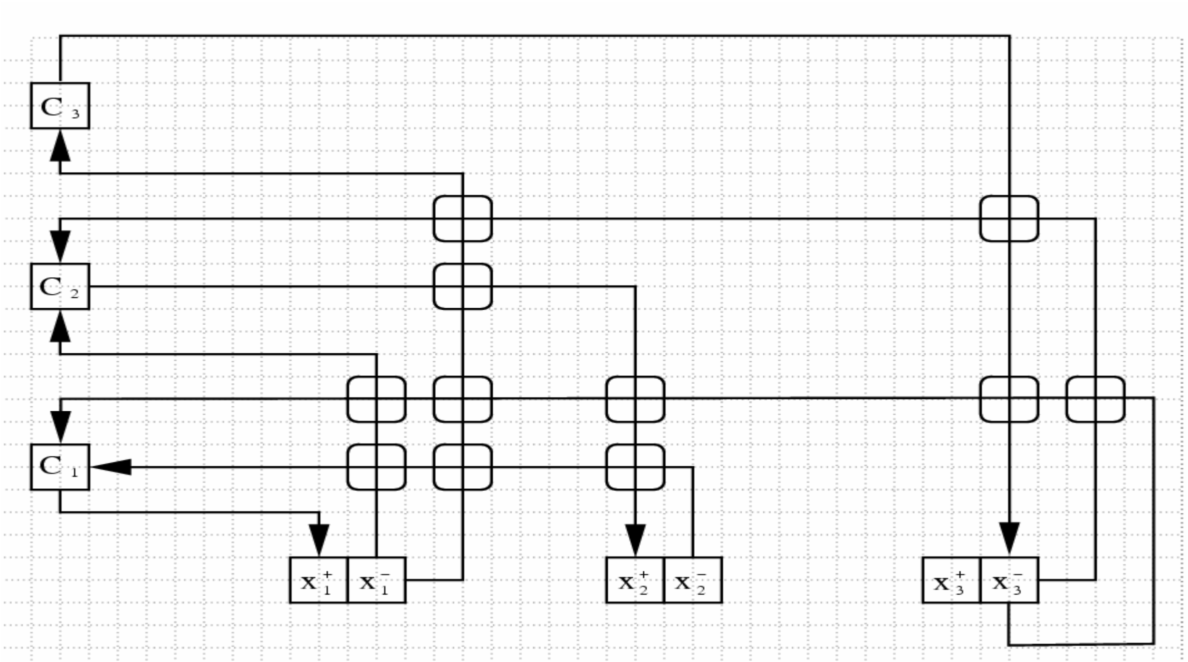
- Reduction from 3-SAT (each variable appears in at most 3 clauses)
- Given a instance C of this 3-SAT, we give a polynomial time construction of $G_C=(V_C, E_C)$ such that the following holds:

> – C is satisfiable $\Rightarrow$ G_C is realizable as a unit disk graph
> – C is not satisfiable $\Rightarrow$ G_C is not realizable as a d-quasi unit disk graph with $d \geq \sqrt{2/3} + \epsilon$

- Unless P=NP, there is no approximation algorithm with approximation ratio better than $\sqrt{3/2} - \epsilon$.

Proof idea

- Construct a grid drawing of the SAT instance.
- Grid drawing is *orientable* iff SAT instance is satisfiable.
- Grid components (clauses, literals, wires, crossings,...) are composed of nodes → Graph G_C.
- G_C is realizable as a d-quasi unit disk graph with $d \geq \sqrt{2/3} + \epsilon$ iff grid drawing is orientable.



Summary

- Virtual coordinates problem is important!
- Natural formulation as unit disk graph embedding.

→ Clear-cut optimization problem.

Upper Bound : $\alpha \in O(\log^{2.5} n \sqrt{\log \log n})$

Lower Bound : $\alpha \geq \sqrt{3/2} - \epsilon$

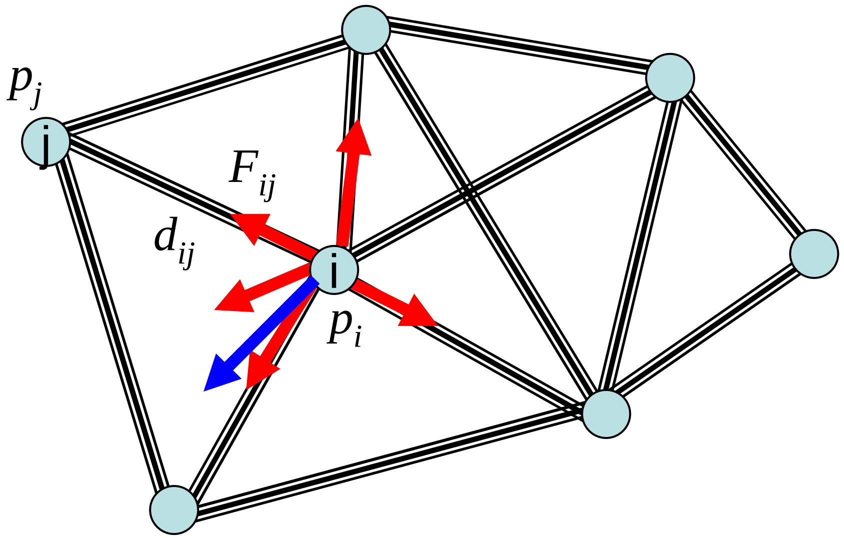
→ Gap between upper and lower bound is huge!

Open Problems:

- Diminish gap between upper and lower bound
- Distributed Algorithm

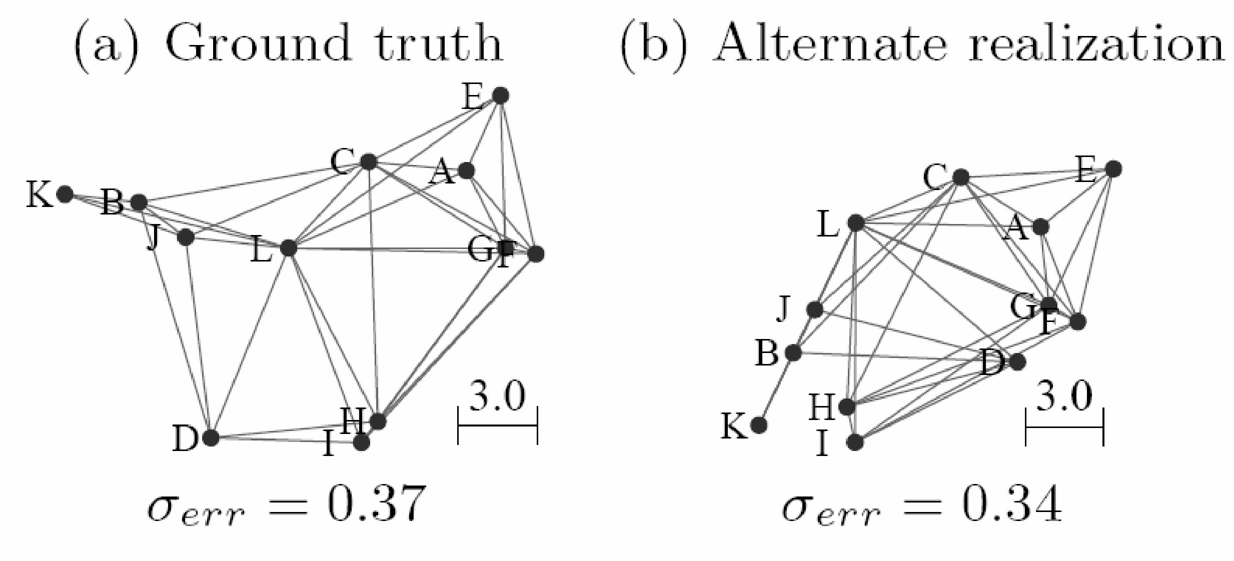
Heuristics: Spring embedder

- Nodes are “masses”, edges are “springs”.
- Length of the spring equals the distance measurement.
- Springs put forces to the nodes, nodes move, until stabilization.
- Force: $F_{ij} = d_{ij} - r_{ij}$, along the direction $p_i p_j$.
- Total force on n_i: $F_i = \Sigma F_{ij}$.
- Move the node n_i by a small distance (proportional to F_i).



Spring Embedder Discussion

- Problems:
  - may deadlock in local minimum
  - may never converge/stabilize (e.g. just two nodes)
- Solution: Need to start from a reasonably good initial estimation.



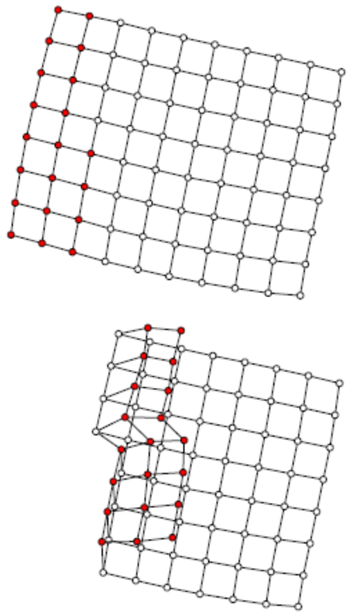
[Jie Gao]

Heuristics: Priyantha et al.

N.B. Priyantha, H. Balakrishnan, E. Demaine, S. Teller:
Anchor-Free Distributed Localization
in Sensor Networks, *SenSys*, 2003.

iterative process minimizes the layout energy

$$E(p) = \sum_{\{i,j\}\in E} \left(||p_i - p_j|| - \ell_{ij} \right)^2$$



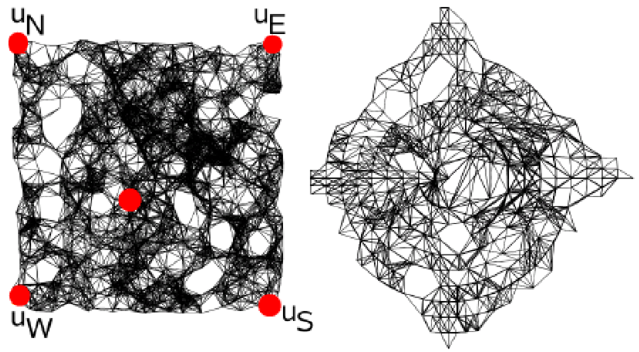
[Fleischer & Pich]

- fact: layouts can have *foldovers* without violating the distance constraints
- problem: optimization can converge to such a local optimum
- solution: find a good initial layout *fold-free* $\rightarrow$ already close to the global optimum (=“real layout”)

Continued

Phase 1: compute initial layout

- determine periphery nodes u_N, u_S, u_W, u_E
- determine central node u_C
- use polar coordinates



$$\rho_v = d(v, u_C) \quad \theta_v = \arctan\left(\frac{d(v, u_N) - d(v, u_S)}{d(v, u_W) - d(v, u_E)}\right)$$

as positions of node v

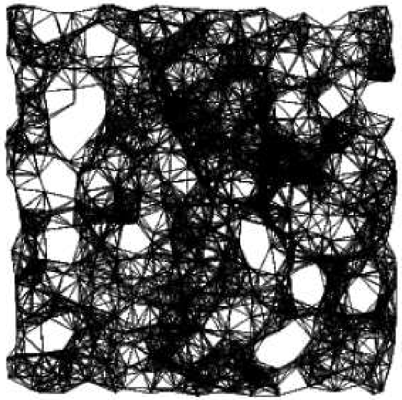
[Fleischer & Pich]

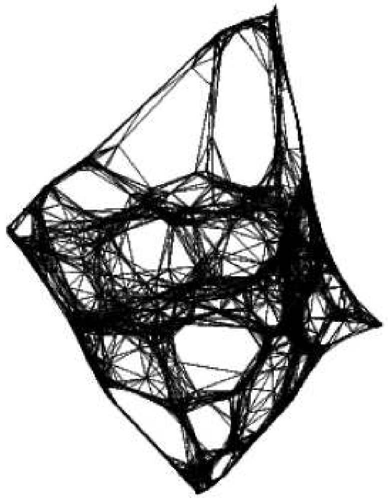
Phase 2: Spring Embedder

Heuristics: Gotsman et al.

C. Gotsman, Y. Koren [5]. **Distributed Graph Layout for Sensor Networks**, *GD*, 2004.

- initial placement: spread sensors
  $$\frac{\sum_{\{i,j\}\in E} \exp(-\ell_{ij})||p_i - p_j||^2}{\sum_{i<j} ||p_i - p_j||^2} \rightarrow \min$$
- linear algebra:
  minimized by second highest eigenvector v_2 of A where
  $$a_{ij} = -\frac{\exp(-\ell_{ij})}{\sum_{j:\{i,j\}\in E} \exp(-\ell_{ij})}$$
  $$a_{ii} = 1$$
- $x, Ax, A^2x, A^3x, \ldots$ converges to v_2
- $x_i \leftarrow \frac{1}{2}\left(x_i + \frac{\sum_{j:\{i,j\}\in E} \exp(-\ell_{ij}x_j)}{\sum_{j:\{i,j\}\in E} \exp(-\ell_{ij})}\right)$
- compute third eigenvector v_3,
  use v_2, v_3 as coordinates

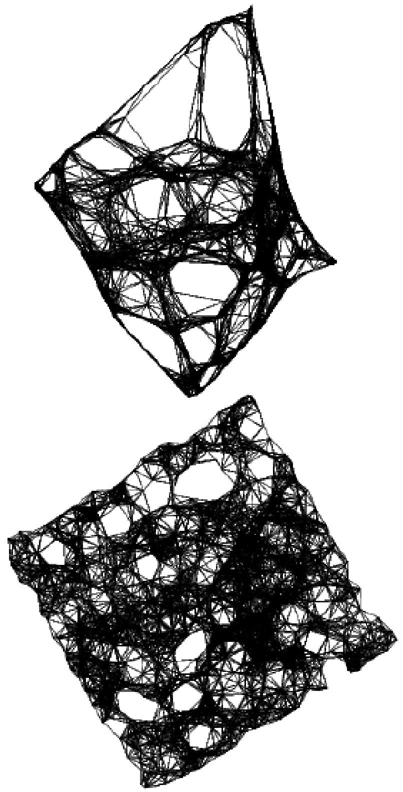




[Fleischer & Pich]

Continued

- distributed optimization (spring model)
- alternative: *majorization*
- compute sequence of layouts $p^{(0)}, p^{(1)}, p^{(2)}, \ldots$ with $E(p^{(0)}) \geq E(p^{(1)}) \geq E(p^{(2)}) \geq \ldots$
  - solve linear equation $L^{(t+1)} p^{(t+1)} = L^{(t)} p^{(t)}$ in distributed manner

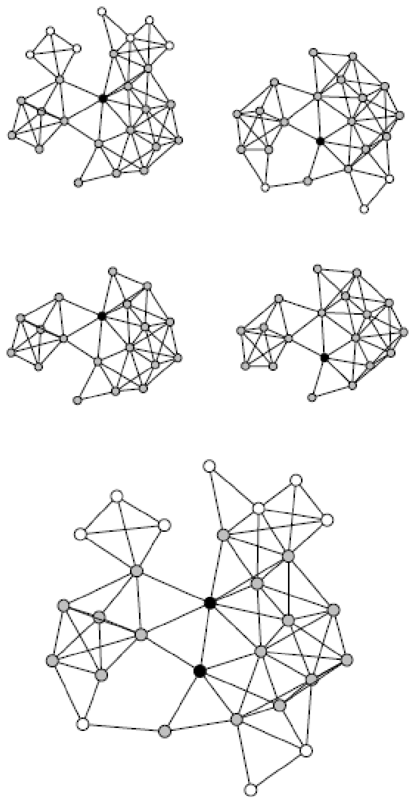


[Fleischer & Pich]

Heuristics: Shang et al.

Y. Shang, W. Ruml [7].
Improved MDS-based Localization, *IEEE Infocom*, 2004.

- compute a local map for each node (local MDS of the 2-hop neighborhood)
- merge local map patches into a global map (use incremental or binary-tree strategy)
- apply distributed optimization to the result

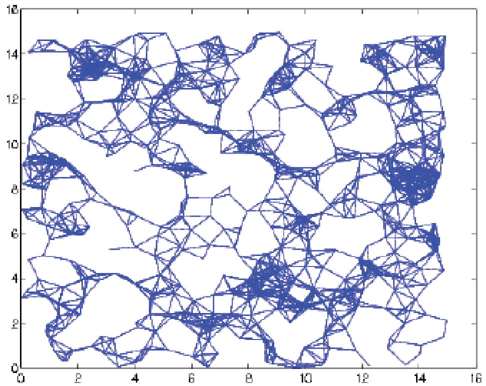


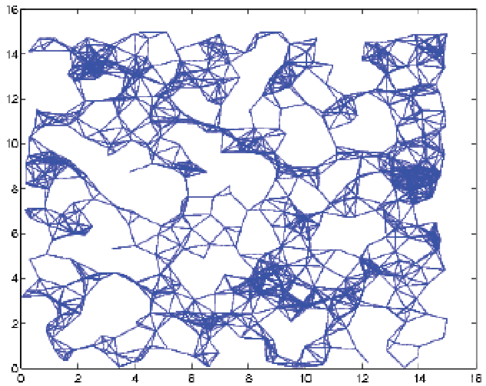
[Fleischer & Pich]

Heuristics: Bruck et al.

J. Bruck, J. Gao, A. Jiang [8]. **Localization and Routing in Sensor Networks by Local Angle Information**, *Mobile Ad Hoc Networking & Computing*, 2005.

- Choose an edge e as x-axis to obtain absolute angles.
- Form an LP whose variables are the edge lengths $\ell(e)$.
- For all edges $0 \leq \ell(e) \leq 1$.
- For any cycle $e_1, \ldots, e_p$: $\sum_{i=1}^{p} \ell(e_i) \cos \theta_i = 0$ and $\sum_{i=1}^{p} \ell(e_i) \sin \theta_i = 0$.
- Non-adjacent node pair constraints.
- Crossing-edge constraints.

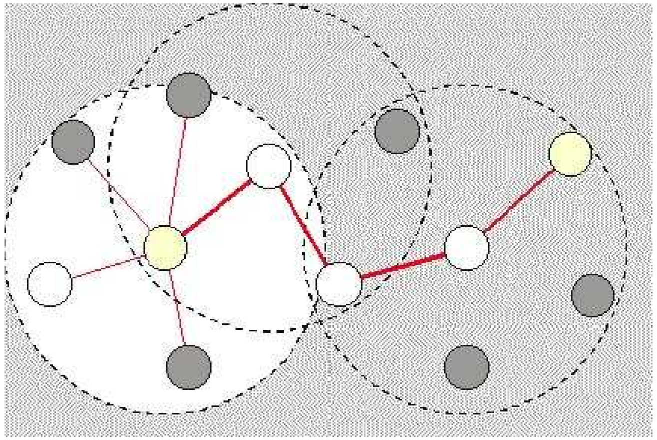




[Fleischer & Pich]

Practical lessons

Theory | Practice



- RSSI in sensor networks: good, but not for “reasonable” localization

- For exact indoor localization
  - Buy special hardware (e.g., UWB)
  - Place huge amount of short range anchors for single-hop localization