

Discrete Event Systems

Exercise 8

1 Night Watch

In order to improve their financial situation, Stefan and Thomas also work at nights. Their task is to guard a famous Swiss bank which, from a architectonic perspective, looks as follows:

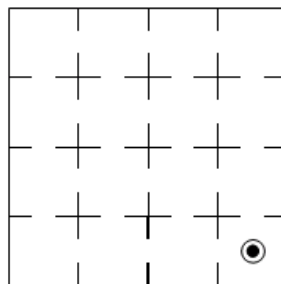


Figure 1: Offices of a Swiss bank.

That is, there are 4x4 rooms, all connected by doors as indicated in the figure.

For security reasons, Stefan and Thomas always stay together. They start in the room on upper left. Every minute, they change to the next room, which is chosen uniformly at random from all possible (adjacent) rooms.

- a) Compute the probability (in the steady state) that Stefan and Thomas are in the room where the thief enters the bank (indicated with \odot)!
- b) Since Stefan and Thomas are extremely fit and also strong, they can easily catch a thief on their own. Thus, they decide that it's smarter to patrol individually. That is, after every minute, each of them chooses the next room *independently*. What is now the probability that at least one of them is in the room where the thief enters?

Note: This exercise is inspired by an exam question of winter term 2004/5.

2 Probability of Arrival

In the script, there is a lemma saying that the probability of arrival can be computed as

$$f_{ij} = p_{ij} + \sum_{k:k \neq j} p_{ik} f_{kj}. \tag{1}$$

Prove this lemma.