On the Power Assignment Problem in Radio Networks

Luzius D. Meisser
Seminar of Distributed Computing WS 2004/05
Eidgenössische Technische Hochschule Zürich
meisserl@student.ethz.ch

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Abstract

This report gives a short overview over the paper *"On the Power Assignment Problem in Radio Networks"* by Clementi et al. [1] ("the paper"). We will shortly discuss their findings as well as the ideas used to prove them.

1 Introduction

1.1 What is *Min* d*D* h-*Range Assignment*?

Given a set S of radio stations in a d-dimensional space, we would like to assign a transmission range r to every station such as they are all connected to each other while minimizing total power consumption.
The sending power P needed for a given transmission range r is defined by

$$P = \gamma * r^{\beta}$$

with $\beta \geq 1$ and $\gamma \geq 1$. This problem is called *Min* d*D* *Range Assignment*. If we additionally introduce a limit for the number of allowed hops, the problem is called *Min* d*D* h-*Range Assignment*. In general, the optimal solution for this problem can only be found by testing all possible assignments. But for some instances of the problem, approximation algorithms are known that are able to find a solution that differs from the optimal by not more than a constant factor.

1.2 Required Knowledge

To understand the underlying paper, it is essential to be familiar with some basic concepts of computer science, namely with the complexity classes P, NP and APX as well as with hardness and completeness. Furthermore, the reader should know the three Ohs of the Oh-Notation: Ω for the lower bound, O for the upper bound and Θ for the optimum.

2 Results

The paper proves that:

1. Min 2D range assignment is NP-complete
2. Min 2D h-range assignment is in APX for well-spread instances
3. Min 2D h-range assignment is in Av-APX
4. Min 3D range assignment is APX-Complete

This section gives a short overview over how these results have been achieved.

2.1 The gadget trick

The main idea used in the proofs is the gadget trick that allows to reduce *min range assignment* to *min vertex cover*. Instances of *min vertex cover* are transformed into instances of *min range assignment* by replacing every edge by a chain of radio stations and a so called gadget, such that solving the obtained instance of *min range assignment* implicitly solves the original instance of *min vertex cover*.

Figure 1 illustrates the gadget idea. When solving the range assignment problem for these

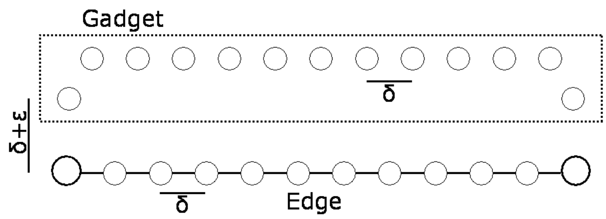

Figure 1: An edge and its gadget

stations, we need to decide on which side to connect the gadget to the edge which corresponds to the decision of picking either the left or the right node to cover the edge in *min vertex cover*. For details, please consult the original paper or the presentation[2] that comes with this report.

2.2 Min 2D range assignment is NP-complete

To prove that *min 2D range assignment* is NP-hard, the paper applies the gadget trick (see 2.1) to reduce it to *min vertex cover restricted to cubic, planar graphs* which is known to be NP-Hard. Since [3] showed that *min 2D range assignment* is in APX, we can conclude that *min 2D range assignment* is NP-complete.

2.3 Min 2D h-range assignment is in APX for well-spread instances

The paper proves this by providing an algorithm that can approximate the optimal range assignment of a well-spread instances of *Min 2D* h-*range assignment* to a constant factor in polynomial time. The algorithm is centralized and the total energy consumption is in $\Theta(\delta(S)^2 * |S|^{1+1/h})$ where S is the set of radio stations, h the number of allowed hops and $\delta(S)$ the minimal distance between two stations in S.

2.4 Min 2D *h*-range assignment is in av-APX

The paper shows that the algorithm used in 2.3 works for *min 2D* h-*range assignment* with high probability if the stations are uniformly distributed on a square. The result provided by the algorithm is in $O(l^2 n^{1/h})$ where l is the side of the square, n the number of stations and h the maximal number of hops. In a second step, the lower bound is proved to be the same and thus the problem to be in av-APX.

2.5 Min 3D range assignment is APX-complete

To prove that *min 3D range assignment* is APX-hard, the paper applies the gadget trick (see 2.1) to reduce it to *min vertex cover restricted to cubic graphs* which is known to be APX-Hard. Since [3] showed that *min 3D range assignment* is in APX, we can conclude that *min 3D range assignment* is APX-complete.

3 Conclusions

3.1 General impression

The paper is theoretical and therefore mainly consists of mathematical proofs. Fortunately, there are some valuable figures that help a lot with understanding the ideas of the proofs. However, it contains some confusing points. There is provably wrong statement in the paper[1] as well as a proof we did not entirely understand[2]. Furthermore, they claim that their results are valid for any $\beta \geq 1$ (see 1.1), but they set $\beta = 2$ for all their proofs. So verifying this claim is left to the reader, which is not trivial. At least to me, this claim is not obvious enough to just believe it without any further proof.

3.2 Restrictions

Obviously, finding an optimal range assignment helps us to connect radio networks. However, there are some problems when applying it in the real world.

3.2.1 Proposed algorithm is centralized

The paper's algorithm requires knowledge of all the stations in the network. It would be interesting to see how well distributed algorithms would perform.

3.2.2 Min range assignment is static

The optimal range assignment is static and does not consider moving stations. What happens if a stations moves? How much movement is allowed without having to recalculate the assignment? These are questions that could be considered in future research.

3.2.3 Power function is the same for all stations

The paper only considers networks in which the power function that calculates the necessary power for a given range is the same for all stations in the network. The results might be different if we allow individual power functions.

[1] Namely the first step in the proof of *Lemma 1* for which it is easy to find a counter-example. But the proof as a whole is still correct if we replace the 3 in $R = \sqrt{|S|/3}$ by a sufficiently large constant, for example 10.

[2] The lower bound proof mentioned in section 2.4

3.2.4 Min range assignment $\neq$ min energy communication

This problem affects the *Min* d*D* h-*Range Assignment* in general. Even if we have found an optimal range assignment, we cannot use it to send messages around in an optimal way. Consider the following example where using the optimal range assignment leads to a non-optimal energy consumption when sending a message from A to B:

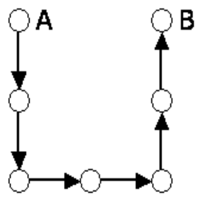

Figure 2: Radio network

If we send the message according to the optimal range assignment, it consumes $6 * 1^2 = 6$ units of energy, but only $2^2 = 4$ units if sent directly from A to B. Thus, if we want to minimize the energy needed for communication in a radio network, solving the *Min* d*D* h-*Range Assignment* might be a first step, but it does not necessarily lead to energy-efficient communication.

References

[1] A.E.F. Clementi, P. Penna and R. Silvestri, *On the power assignment problem in radio networks*, Electronic Colloquium on Computational Complexity (ECCC) vol. 1 no. 054, 2000.

[2] L. Meisser, *On the power assignment problem in radio networks*, Distributed Systems Seminar WS 2004/05, available soon at `http://distcomp.ethz.ch/lectures/ws0405/seminar/index.html`

[3] L. M. Kirousis, E. Kranakis, D. Krizanc and A. Pelc, *Power consumption in packet radio networks*, Proc. of 14th Annual Symposium on Theoretical Aspects of Computer Science (STACS), Lecture Notes in Computer Science, Vol. 1200 (1997) pp. 363-374.