Discrete Mobile Centers

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Summary

We have n given points in the plane. The nodes are mobile and they can switch off and on. Usually these nodes have a short communication range and therefore the network topology is affected by node motion. A task for the nodes can be to establish an ad-hoc multi-hop network. A principle is the organization of the mobile nodes into clusters.
The goal is to have a minimal subset of the n nodes which are the centers. Every node is visible to at least one of the centers. Our algorithm should have a O(1)-approximation of the optimal solution with high probability.
The clustering problem in the static version is equivalent to finding the minimum dominating set. The static version of the problem is known to be NP-complete. For the dominating set in an intersection graph several approximation algorithms exist.

Basic algorithm

All points have a unique identifier (1, 2, …, n). This could be a random numbering. The points have a visible range of an axis-aligned square with side length 1.
Each point searches in its visible range the point with the largest index and nominates it to be a center. If the point does not find one with a higher index, then it nominates itself. All points nominated are the centers in our solution. A cluster is formed by a selected center and all the points that nominated it.
The algorithm has an expected approximation factor of $O(\sqrt{n})$

The probability that there are more than $\sqrt{n}\log n \cdot k$ centers is $O\left(\frac{1}{n^{\log n-1}}\right)$

k is the optimal number of centers

Hierarchical algorithm

The hierarchical algorithm proceeds in log log n number of rounds. At each round the basic algorithm is applied to the centers produced by the previous round. For each round the algorithm uses a larger covering ball.
The final output is a constant approximation to the optimal solution.

Kinetic discrete clustering

To check if two points are mutually visible there is over each point a half-size square centered. The algorithm has to handle two kinds of events if two squares start or stop intersecting.

In the paper it is assumed that the points have bounded-degree algebraic motion. Because of this simplification each pair of points can cause only O(1) events. The number of events in the basic algorithm is $O(n^2)$ and in the hierarchical algorithm it is $O(n^2 \log \log n)$.

Comment/Discussion

The hierarchical algorithm is theoretically very interesting, but I am sceptical if it can be used in practice. I think there is a problem because the mobile nodes have to send with different power. Using another distance metric is not so easy either, because GPS devices are mostly not available. Taking the round trip time as distance metric is not practical either. The problem is that the measured times are distributed over a great interval and an exact distance cannot be computed.
A student is working in his master thesis with the hierarchical algorithm and he asked if it is possible to implement this algorithm without using any kind of distance metric. An interesting question, but I think it is difficult to give an answer to this, because in the paper nothing is written about this aspect and for the analysis they use a growing square in each level of the hierarchy. In fact it is one of the central points of the hierarchical algorithm. He mentioned too that it would be nice to have such an algorithm, because the hierarchical algorithm is the fastest known with a constant approximation factor.
For practical use I would recommend the basic algorithm, because it is very simple and gives a good approximation. We do not have to think about problems with the hierarchy and so we do not need a distance metric. In the paper the authors mention that in simulation they observed good performance. A reason can be that if the indexes of the points are uniformly distributed the basic algorithm gives a constant approximation. In practice we can assume that this is given in most situations.
A critical point in the kinetic analysis is the assumption that the points have bounded-degree algebraic motion. Is this a realistic approach? In the paper they do not write anything about the case where this assumption is not fulfilled. The question is, can the algorithm handle more than $O(n^2)$ events? This can happen if a node moves in a circle and this causes more than a constant number of events for any pair of two nodes. More critical is the problem for the hierarchical algorithm, because an event can cause updates for every level of the hierarchy and the data structure for finding points must also be updated.
But the kinetic analysis is interesting anyway, because most other algorithms were only analyzed in a static way. The node motion is an important aspect for mobile devices and the network topology is strongly affected by this. So the simplification is useful to give some theoretical bounds.

Reference

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J. Gao, L. J. Guibas, J. Hershberger, L. Zhang, A. Zhu, SCG 2001