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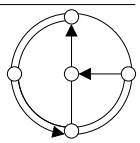

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Principles of Distributed Computing
Exercise 11: Sample Solution

1 How to ping a disrupted flow network

Consider the k disjoint paths, carrying the k units of max flow in the original network. Because the hacker deleted exactly k edges, it must have removed exactly one edge from each of these paths. In addition, these k edges induce an $s-t$ cut: the source is no longer connected to the sink. Thus, there are no other edges in the network that cross this cut.

We can now identify the deleted edges by performing a binary search on each of the k paths (independently). Because each $s-t$ flow path has exactly one edge removed, there is a contiguous prefix set of nodes that are reachable from s, followed by the rest that are not reachable. We can locate the crossover edge by a binary search.

Note that this scheme gives an "implicit" description of the nodes reachable from s. We cannot hope to list them all explicitly in $O(k \log n)$ time, because n can be much bigger than $k \log n$.

2 Paid advertisements on search engines

Construct a 3-partite graph G, with V_1 being the set of n users, V_2 being the set of k demographic groups, and V_3 being the set of m advertisers. Put an edge of capacity ∞ from group u to advertiser v if u is among the desired group for v. Put an edge of capacity 1 from user u to group v if u belongs to the demographic group identified by v.

Next, put a source s, and join it to each user u by an edge of capacity 1. Put a sink t, and join each advertiser v to t, with capacity r_v.

Now the advertising contracts are feasible if and only if there is max flow in the network of value $\sum_v r_v$.