Principles of Distributed Computing
Exercise 5

1 Pancake Networks

In the lecture, you have encountered several different graphs as underlying network structures (Chapter 5). Here, we will look at another prominent example, the *Pancake graph* P_n.

Define P_n as follows: the vertex set is

$$V(P_n) = \{v_1 v_2 \dots v_n \mid v_i \in [n] \text{ and } v_i \neq v_j \ \forall i \neq j\} \tag{1}$$

where we use $[n] = \{1, 2, \dots, n\}$. In other words, $V(P_n) = S_n$, the group of all permutations on n elements. There exists an edge of dimension i for $2 \leq i \leq n$ when

$$e_i = (u_1 u_2 \dots u_i \dots u_n, v_1 v_2 \dots v_i \dots v_n) \in E(P_n) \iff \\ v_j = u_{i-j+1} \text{ for } 1 \leq j \leq i \text{ and } v_j = u_j \text{ for } i < j \leq n \tag{2}$$

or, we can say that an edge e_i represents a *prefix reversal*

$$v_1 v_2 \dots v_i v_{i+1} \dots v_n \longleftrightarrow v_i \dots v_2 v_1 v_{i+1} \dots v_n. \tag{3}$$

For the following questions, where appropriate, give your answers in terms of $N := |V(P_n)|$ (approximately), the number of vertices, as well as n.

a) Draw (nicely!) P_n for $n = 2, 3, 4$. Try to describe a pattern for drawing P_n for any n.

b) What is the degree of each vertex in P_n?

c) Can you give bounds on the diameter $D(P_n)$ of the pancake network?

d) (optional) Show that P_n is Hamiltonian for $n \geq 3$.

The pancake graph has recently been proposed for P2P networks, owing its usefulness to the above and other properties.