Chapter 7
GEOMETRIC ROUTING

Distributed
Computing
Group

Mobile Computing
Summer 2004

Overview

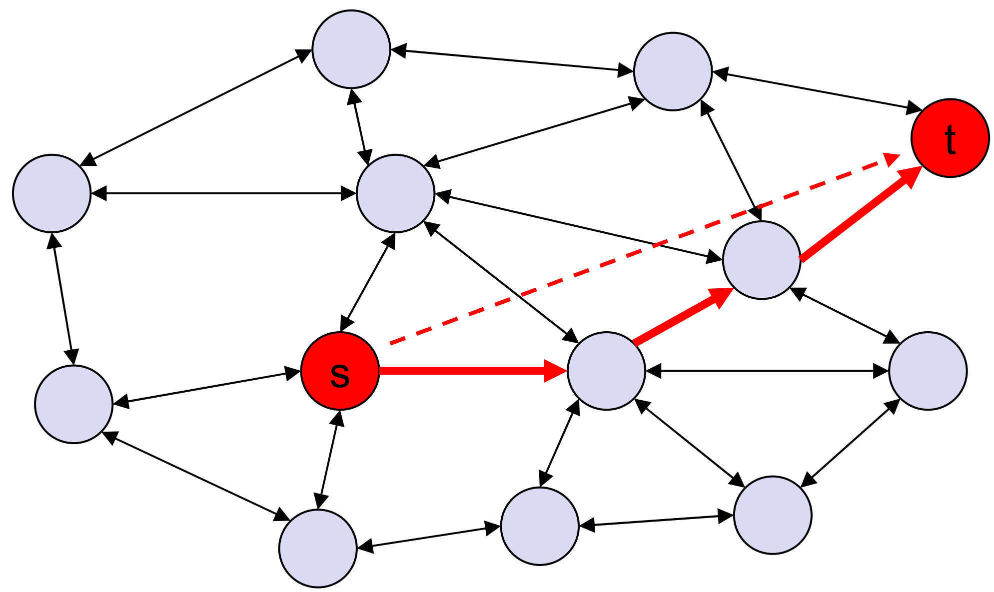
- Geometric routing
- Greedy geometric routing

- Euclidean and planar graphs
- Unit disk graph
- Gabriel graph and other planar graphs

- Face Routing
- Adaptive Face Routing
- Lower bound
- Greedy (Other) Adaptive Face Routing

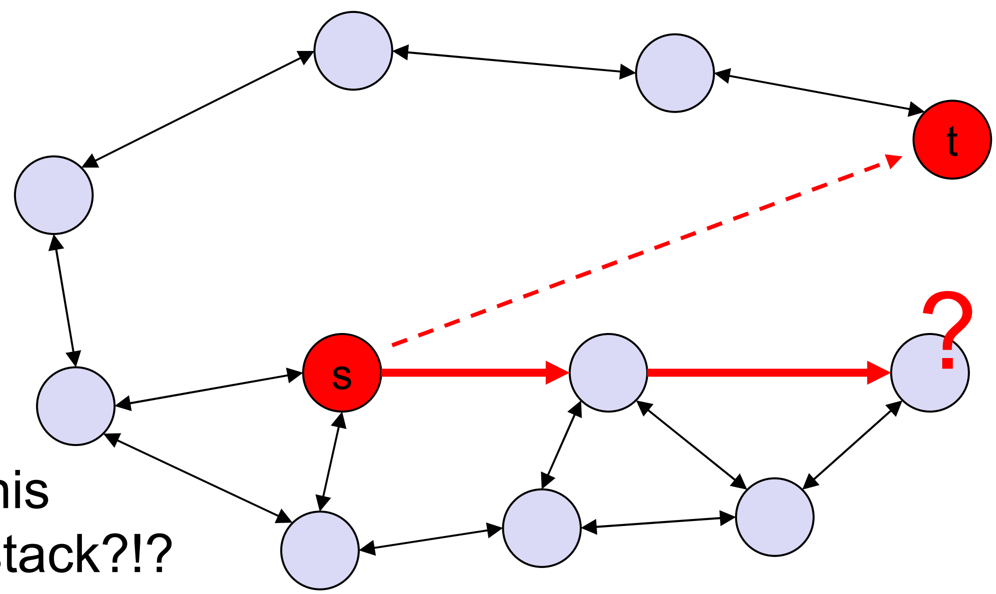
Geometric (Directional, Position-based) routing

- …even with all the tricks there will be flooding every now and then.
- In this chapter we will assume that the nodes are location aware (they have GPS, Galileo, or an ad-hoc way to figure out their coordinates), and that we know where the destination is.
- Then we simply route towards the destination



Geometric routing

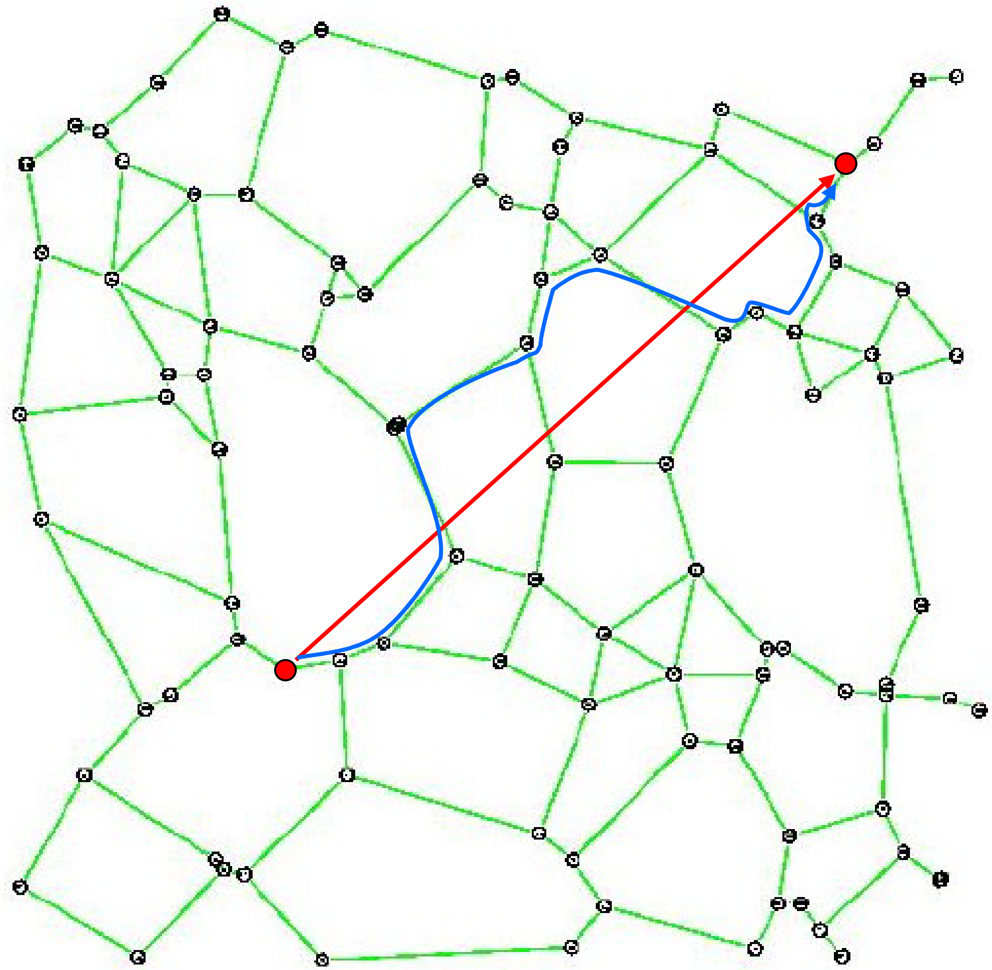
- Problem: What if there is no path in the right direction?
- We need a guaranteed way to reach a destination even in the case when there is no directional path…
- Hack: as in flooding nodes keep track of the messages they have already seen, and then they backtrack* from there



*backtracking? Does this mean that we need a stack?!?

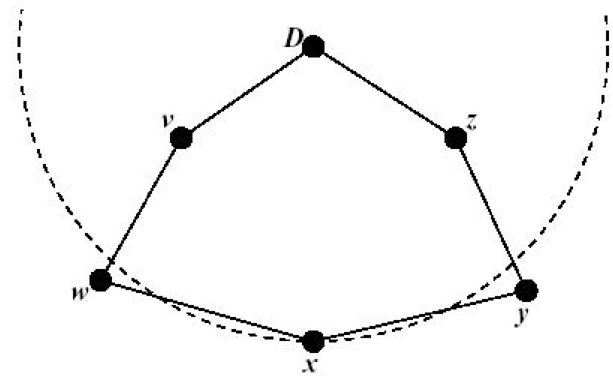
Greedy routing

- Greedy routing looks promising.
- Maybe there is a way to choose the next neighbor and a particular graph where we always reach the destination?

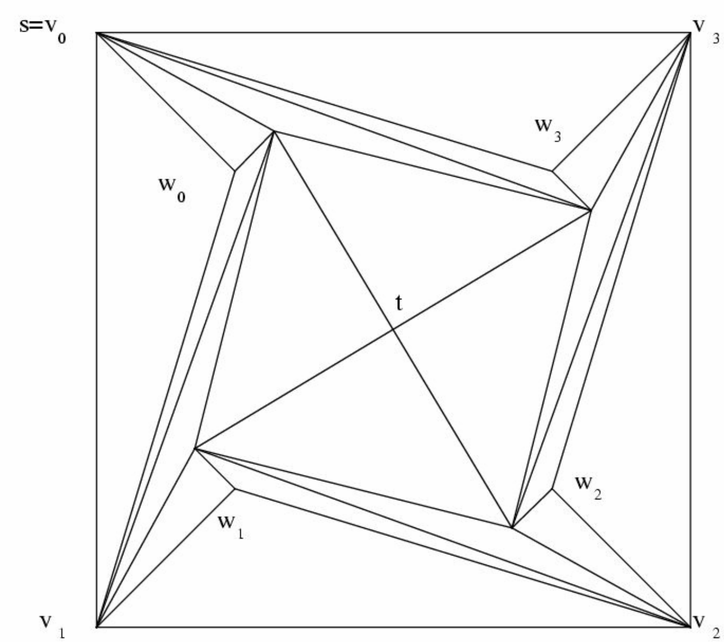


Examples why greedy algorithms fail

- We greedily route to the neighbor which is closest to the destination: But both neighbors of x are not closer to destination D

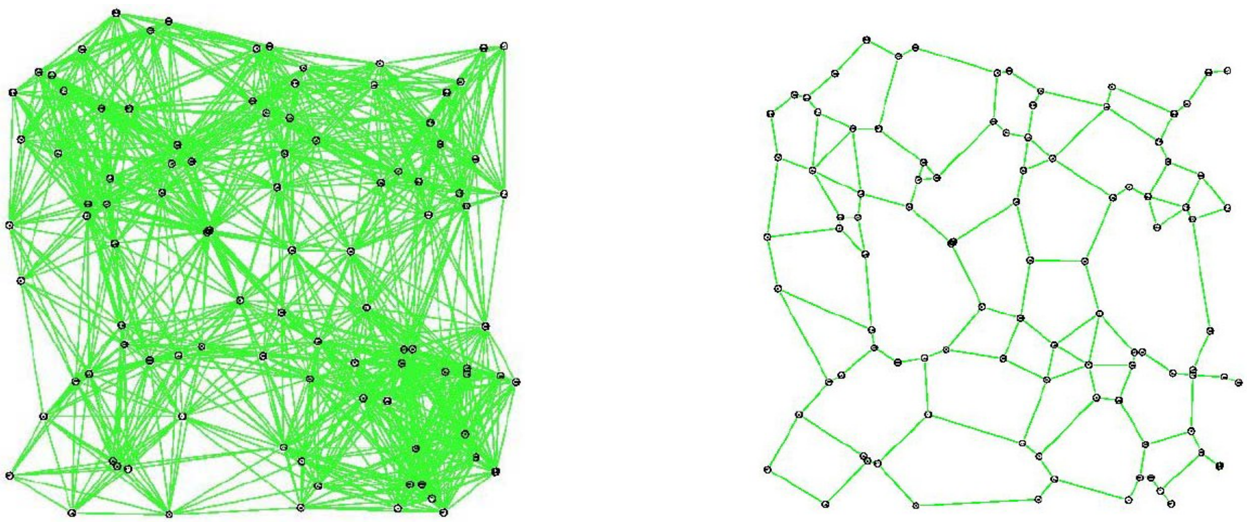


- Also the best angle approach might fail, even in a triangulation: if, in the example on the right, you always follow the edge with the narrowest angle to destination t, you will forward on a loop $v_0, w_0, v_1, w_1, \ldots, v_3, w_3, v_0, \ldots$



Euclidean and Planar Graphs

- Euclidean: Points in the plane, with coordinates
- Planar: can be drawn without "edge crossings" in a plane

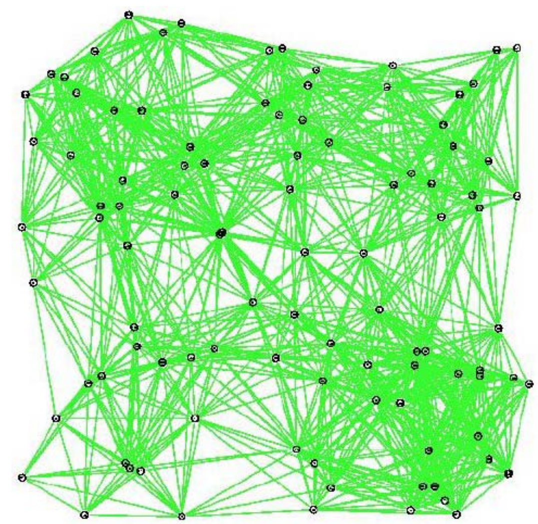


- Euclidean planar graphs (planar embedding) simplify geometric routing.

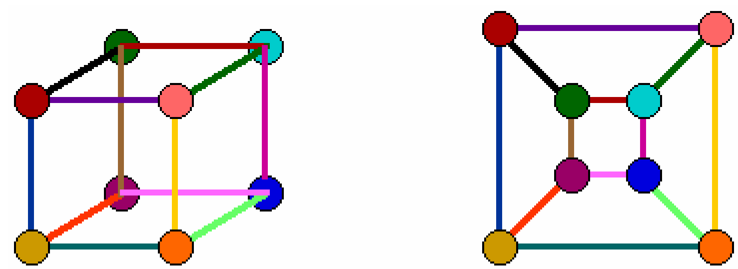
Unit disk graph

- We are given a set *V* of nodes in the plane (points with coordinates).
- The unit disk graph *UDG*(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is an edge between two nodes *u*,*v* iff the Euclidean distance between *u* and *v* is at most 1.
- Think of the unit distance as the maximum transmission range.

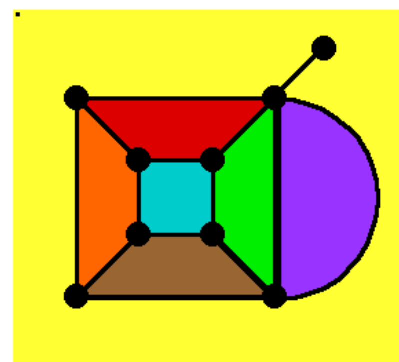
- We assume that the unit disk graph *UDG* is connected (that is, there is a path between each pair of nodes)
- The unit disk graph has many edges.
- Can we drop some edges in the *UDG* to reduced complexity and interference?



Planar graphs

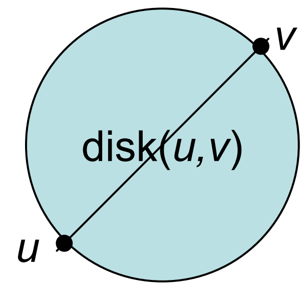


- Definition: A planar graph is a graph that can be drawn in the plane such that its edges only intersect at their common end-vertices.
- Kuratowski's Theorem: A graph is planar iff it contains no subgraph that is edge contractible to K_5 or $K_{3,3}$.
- Euler's Polyhedron Formula: A connected planar graph with *n* nodes, *m* edges, and *f* faces has $n - m + f = 2$.
- Right: Example with 9 vertices,14 edges, and 7 faces (the yellow "outside" face is called the infinite face)
- Theorem: A simple planar graph with n nodes has at most 3n–6 edges, for n≥3.

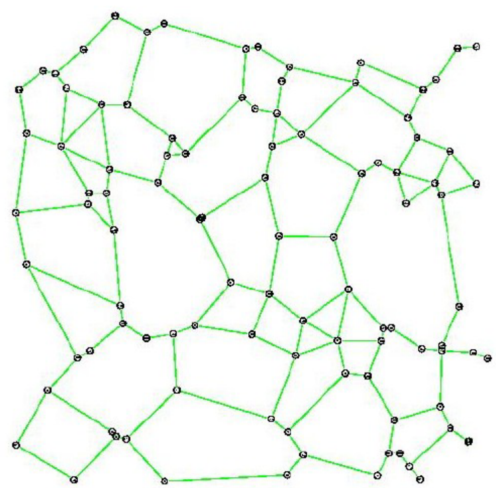


Gabriel Graph

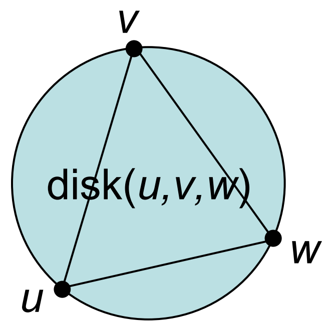
- Let disk(*u,v*) be a disk with diameter (*u,v*) that is determined by the two points *u*,*v*.
- The Gabriel Graph GG(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is an edge between two nodes *u*,*v* iff the disk(*u,v*) including boundary contains no other points.



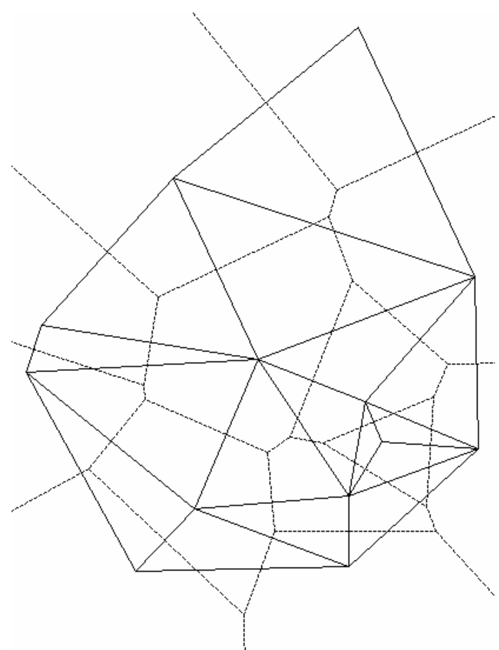
- As we will see the Gabriel Graph has interesting properties.



Delaunay Triangulation



- Let disk(*u,v,w*) be a disk defined by the three points *u*,*v*,*w*.
- The Delaunay Triangulation (Graph) DT(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is a triangle of edges between three nodes *u*,*v*,*w* iff the disk(*u,v,w*) contains no other points.

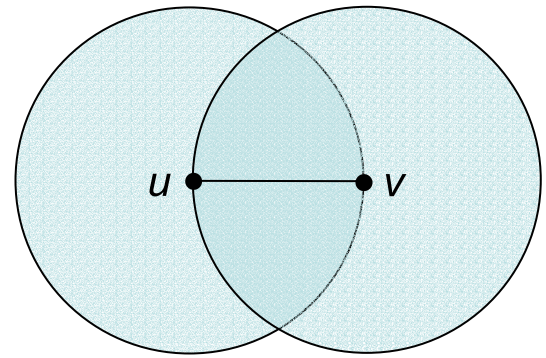


- The Delaunay Triangulation is the dual of the Voronoi diagram, and widely used in various CS areas; the DT is planar; the distance of a path (s,…,t) on the DT is within a constant factor of the s-t distance.

Other planar graphs

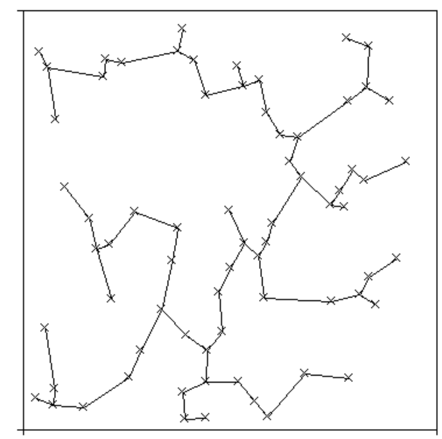
- Relative Neighborhood Graph RNG(V)

- An edge e = (u,v) is in the RNG(V) iff there is no node w with (u,w) < (u,v) and (v,w) < (u,v).



- Minimum Spanning Tree MST(V)

- A subset of *E* of *G* of minimum weight which forms a tree on *V*.



Properties of planar graphs

- Theorem 1:
  $\mathrm{MST}(V) \subseteq \mathrm{RNG}(V) \subseteq \mathrm{GG}(V) \subseteq \mathrm{DT}(V)$

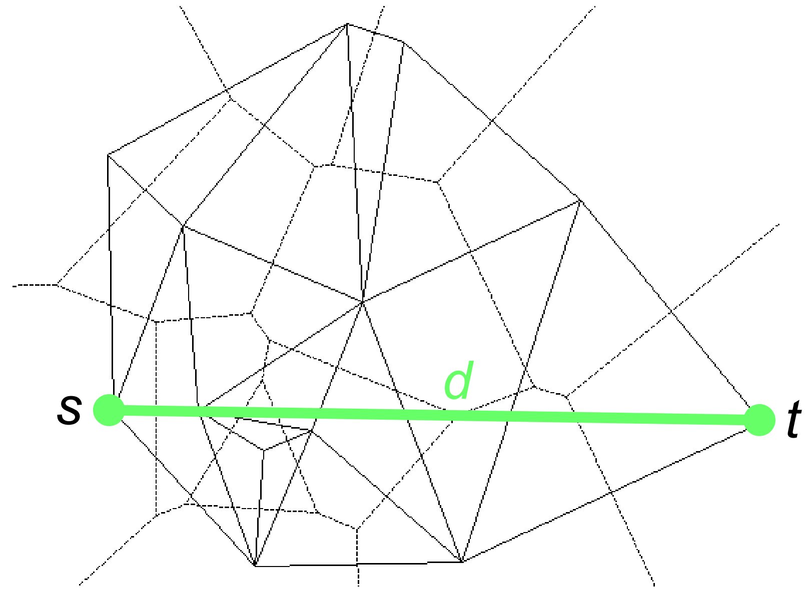
- Corollary:
  Since the MST(V) is connected and the DT(V) is planar, all the planar graphs in Theorem 1 are connected and planar.

- Theorem 2:
  The Gabriel Graph contains the Minimum Energy Path
  (for any path loss exponent $\alpha \geq 2$)

- Corollary:
  GG(V) $\cap$ UDG(V) contains the Minimum Energy Path in UDG(V)

Routing on Delaunay Triangulation?

- Let d be the Euclidean distance of source s and destination t
- Let c be the sum of the distances of the links of the shortest path in the Delaunay Triangulation
- It was shown that $c = \Theta(d)$



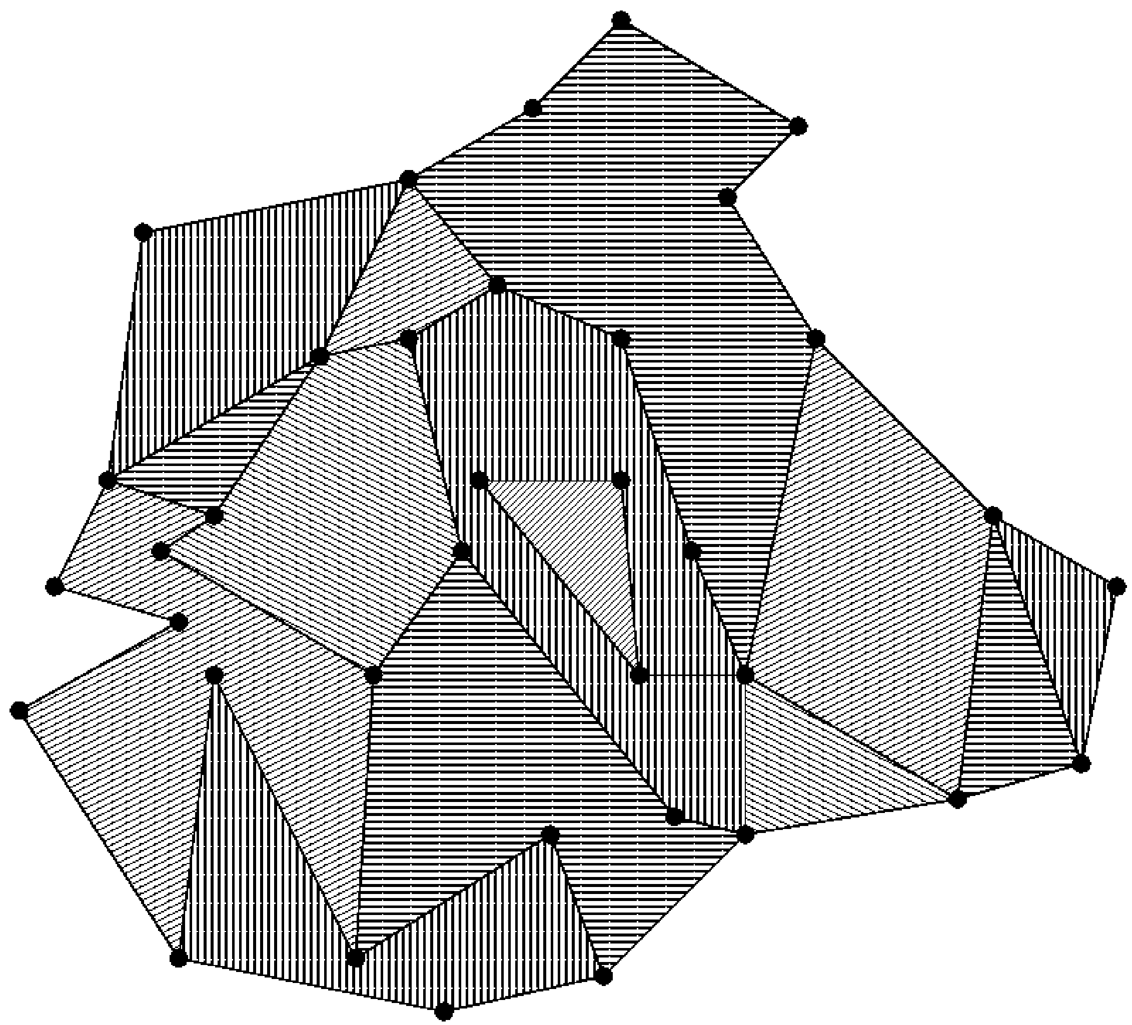
- Two problems:

1) How do we find this best route in the DT? With flooding?!?
2) How do we find the DT at all in a distributed fashion?

… and even worse: The DT contains edges that are not in the UDG, that is, nodes that cannot hear each other are "neighbors" on DT

Breakthrough idea: route on faces

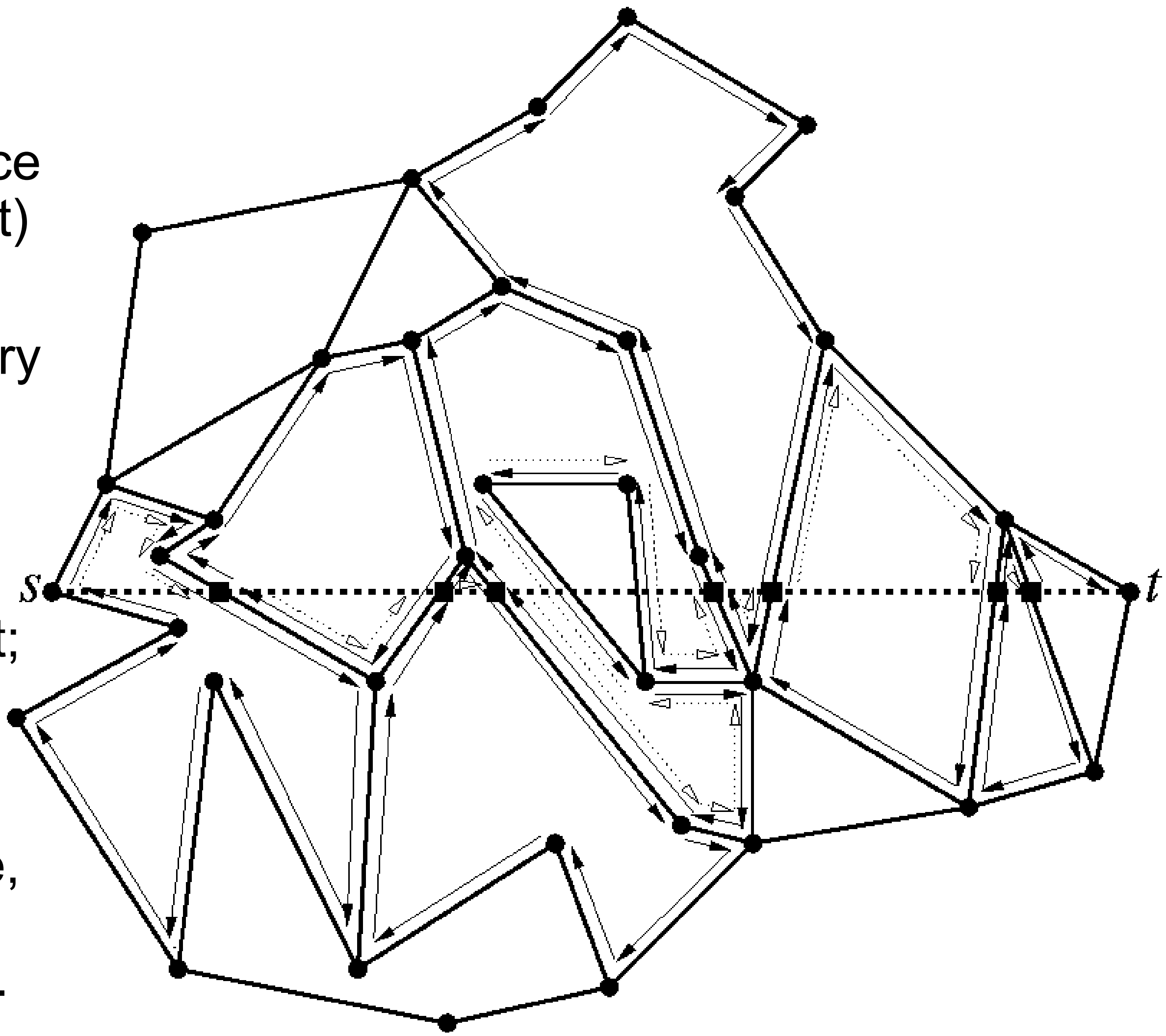
- Remember the faces…
- Idea:
  Route along the boundaries of the faces that lie on the source–destination line



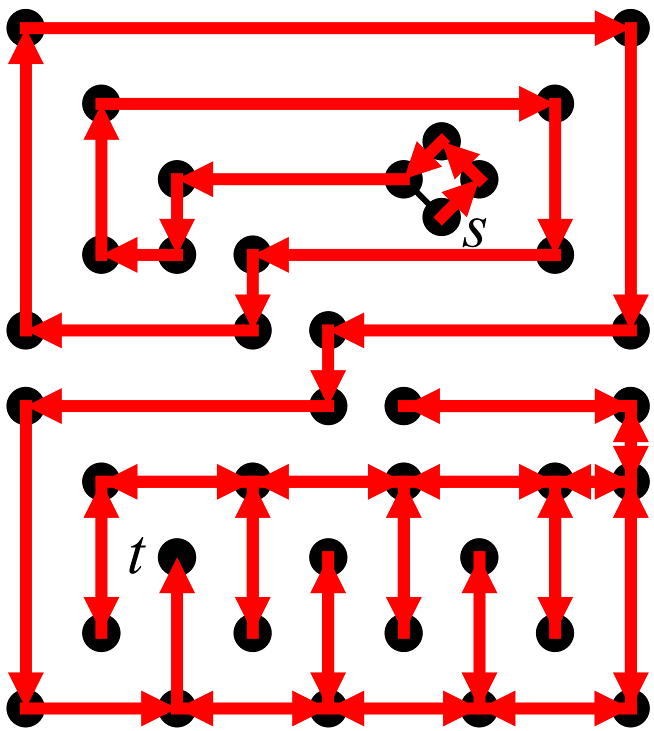
Face Routing

0. Let f be the face incident to the source s, intersected by (s,t)

1. Explore the boundary of f; remember the point p where the boundary intersects with (s,t) which is nearest to t; after traversing the whole boundary, go back to p, switch the face, and repeat 1 until you hit destination t.



Face Routing Works on *Any* Graph



Face routing is correct

- Theorem: Face routing terminates on any simple planar graph in O(n) steps, where n is the number of nodes in the network

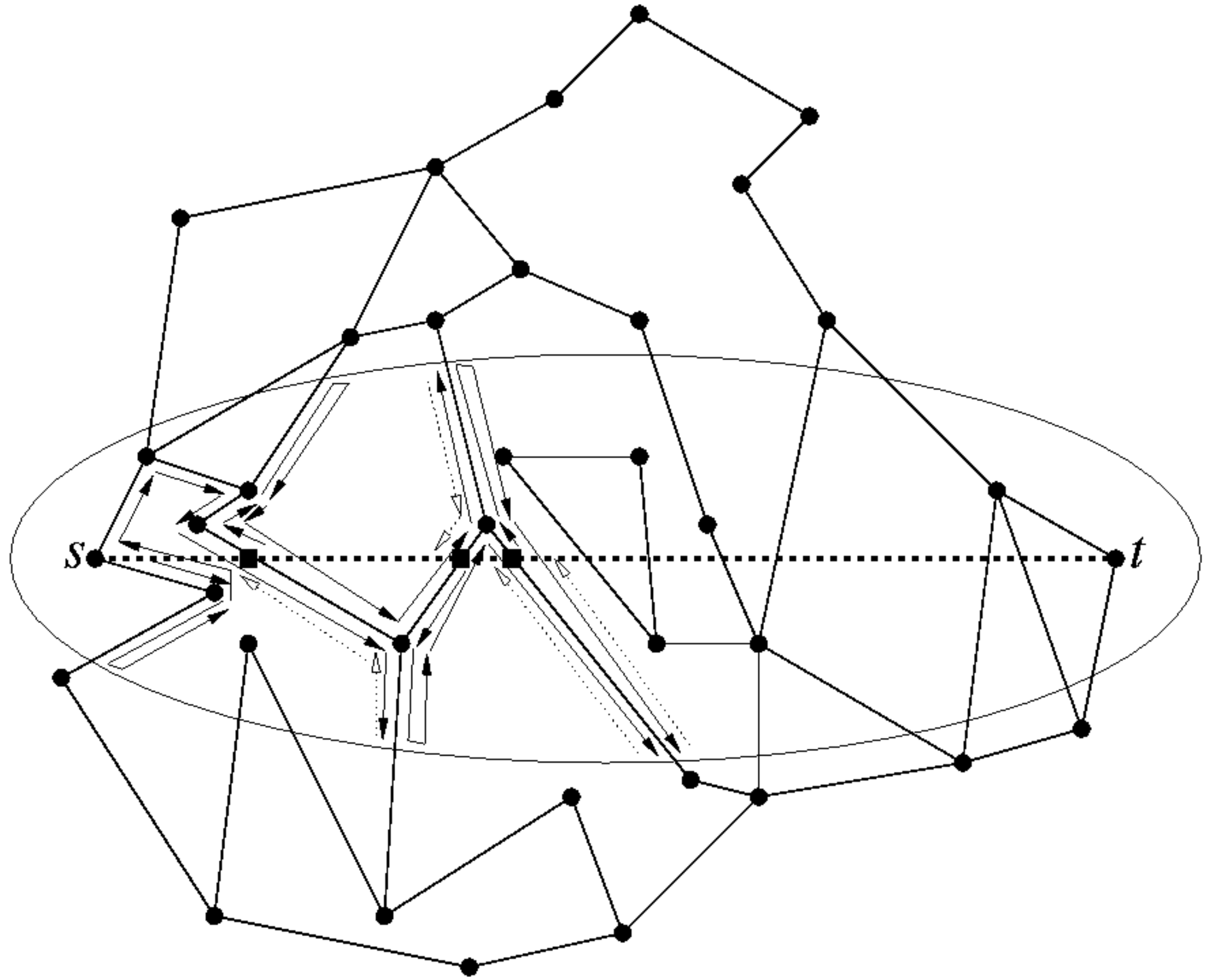
- Proof: A simple planar graph has at most 3n–6 edges. You leave each face at the point that is closest to the destination, that is, you never visit a face twice, because you can order the faces that intersect the source—destination line on the exit point. Each edge is in at most 2 faces. Therefore each edge is visited at most 4 times. The algorithm terminates in O(n) steps.

Is there something better than Face Routing?

- How to improve face routing? Face Routing 2 ☺
- Idea: Don't search a whole face for the best exit point, but take the first (better) exit point you find. Then you don't have to traverse huge faces that point away from the destination.
- Efficiency: Seems to be practically more efficient than face routing. But the theoretical worst case is worse – $O(n^2)$.
- Problem: if source and destination are very close, we don't want to route through all nodes of the network. Instead we want a routing algorithm where the cost is a function of the cost of the best route in the unit disk graph (and independent of the number of nodes).

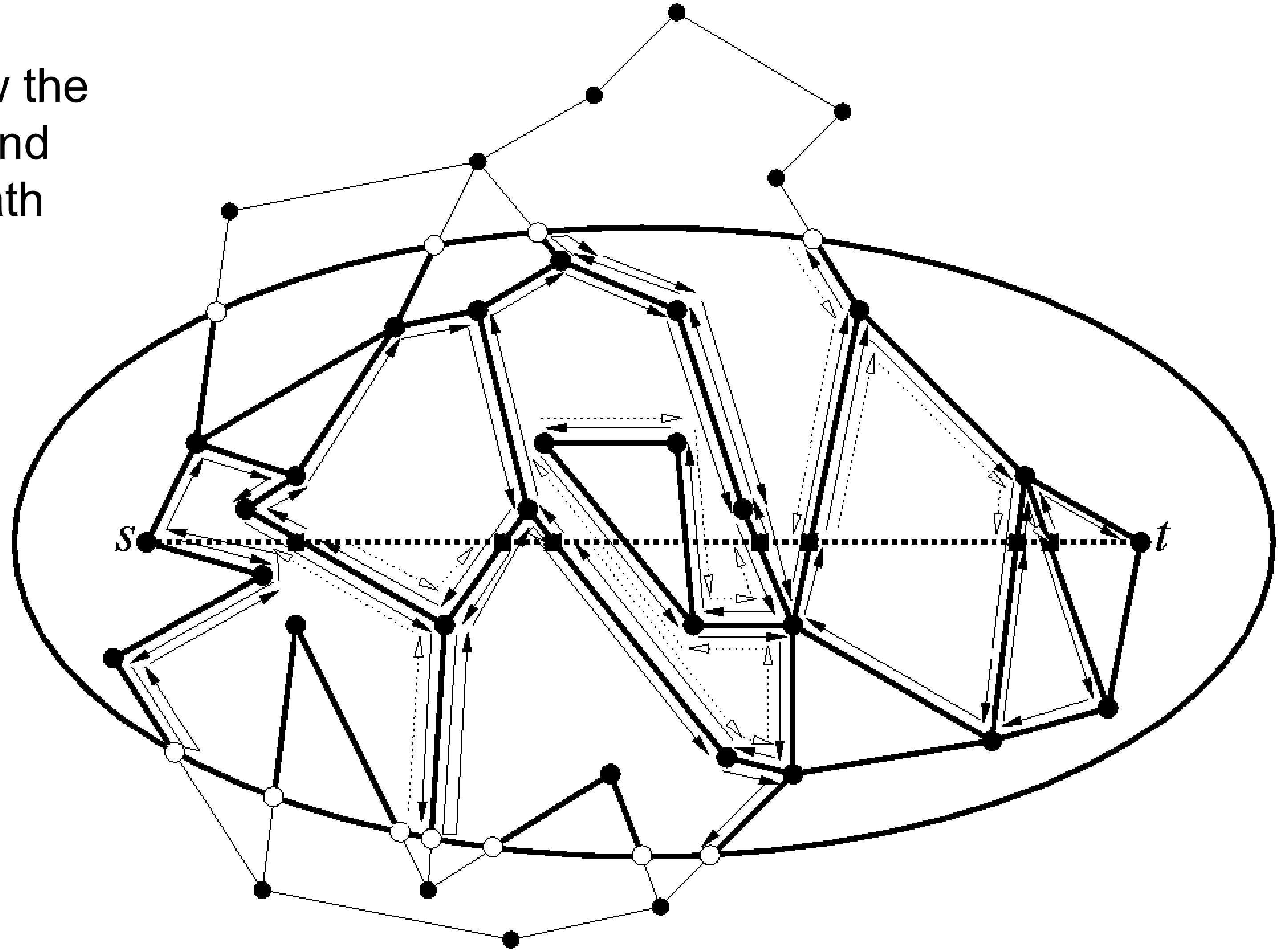
Adaptive Face Routing (AFR)

- Idea: Use face routing together with ad-hoc routing trick 1!!
- That is, don't route beyond some radius r by branching the planar graph within an ellipse of exponentially growing size.



AFR Example Continued

- We grow the ellipse and find a path



AFR Pseudo-Code

0. Calculate G = GG(V) $\cap$ UDG(V)
   Set c to be twice the Euclidean source—destination distance.

1. Nodes w $\in$ W are nodes where the path s-w-t is larger than c. Do face routing on the graph G, but without visiting nodes in W. (This is like pruning the graph G with an ellipse.) You either reach the destination, or you are stuck at a face (that is, you do not find a better exit point.)

2. If step 1 did not succeed, double c and go back to step 1.

- Note: All the steps can be done completely locally, and the nodes need no local storage.

The $\Omega(1)$ Model

- We simplify the model by assuming that nodes are sufficiently far apart; that is, there is a constant d_0 such that all pairs of nodes have at least distance d_0. We call this the $\Omega(1)$ model.

- This simplification is natural because nodes with transmission range 1 (the unit disk graph) will usually not “sit right on top of each other”.

- Lemma: In the $\Omega(1)$ model, all natural cost models (such as the Euclidean distance, the energy metric, the link distance, or hybrids of these) are equal up to a constant factor.

- Remark: The properties we use from the $\Omega(1)$ model can also be established with a backbone graph construction.

Analysis of AFR in the $\Omega(1)$ model

- Lemma 1: In an ellipse of size c there are at most $O(c^2)$ nodes.
- Lemma 2: In an ellipse of size c, face routing terminates in $O(c^2)$ steps, either by finding the destination, or by not finding a new face.
- Lemma 3: Let the optimal source—destination route in the UDG have cost c*. Then this route c* must be in any ellipse of size c* or larger.
- Theorem: AFR terminates with cost $O(c^{*2})$.
- Proof: Summing up all the costs until we have the right ellipse size is bounded by the size of the cost of the right ellipse size.