Chapter 7
GEOMETRIC ROUTING

Distributed Computing Group

Mobile Computing

Summer 2003

Overview

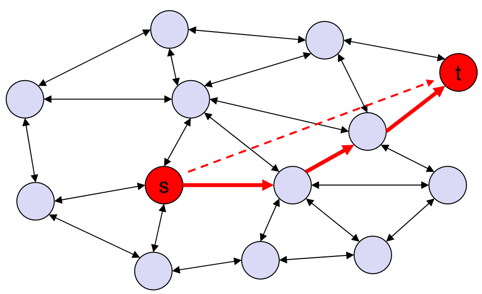
- Geometric routing
- Greedy geometric routing

- Euclidean and planar graphs
- Unit disk graph
- Gabriel graph and other planar graphs

- Face Routing
- Adaptive Face Routing
- Lower bound
- Greedy (Other) Adaptive Face Routing

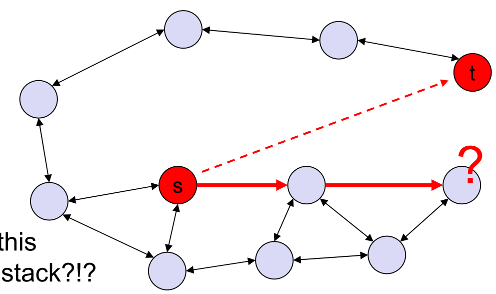
Geometric (Directional, Position-based) routing

- …even with all the tricks there will be flooding every now and then.
- In this chapter we will assume that the nodes are location aware (they have GPS, Galileo, or an ad-hoc way to figure out their coordinates), and that we know where the destination is.
- Then we simply route towards the destination



Geometric routing

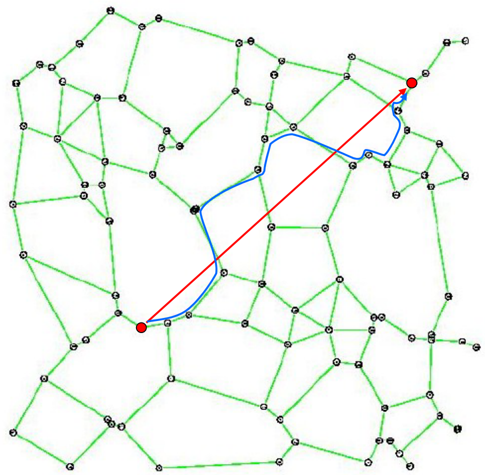
- Problem: What if there is no path in the right direction?
- We need a guaranteed way to reach a destination even in the case when there is no directional path…
- Hack: as in flooding nodes keep track of the messages they have already seen, and then they backtrack* from there



*backtracking? Does this mean that we need a stack?!?

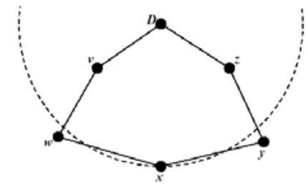
Greedy routing

- Greedy routing looks promising.
- Maybe there is a way to choose the next neighbor and a particular graph where we always reach the destination?

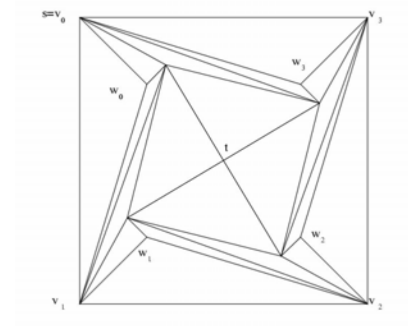


Examples why greedy algorithms fail

- We greedily route to the neighbor which is closest do the destination: But both neighbors of x are not closer to destination D

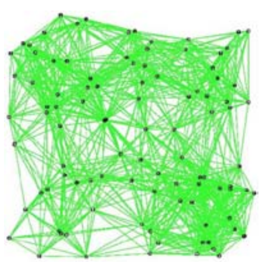


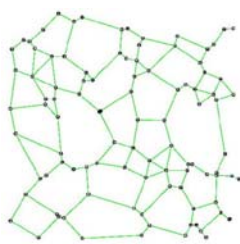
- Also the best angle approach might fail, even in a triangulation: if, in the example on the right, you always follow the edge with the narrowest angle to destination t, you will forward on a loop $v_0, w_0, v_1, w_1, \ldots, v_3, w_3, v_0, \ldots$



Euclidean and Planar Graphs

- Euclidean: Points in the plane, with coordinates
- Planar: can be drawn without "edge crossings" in a plane

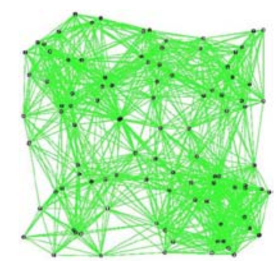




- Euclidean planar graphs (planar embedding) simplify geometric routing.

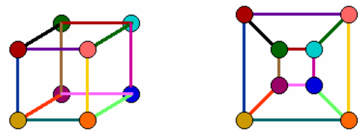
Unit disk graph

- We are given a set *V* of nodes in the plane (points with coordinates).
- The unit disk graph *UDG*(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is an edge between two nodes *u*,*v* iff the Euclidean distance between *u* and *v* is at most 1.
- Think of the unit distance as the maximum transmission range.
- We assume that the unit disk graph *UDG* is connected (that is, there is a path between each pair of nodes)
- The unit disk graph has many edges.
- Can we drop some edges in the *UDG* to reduced complexity and interference?

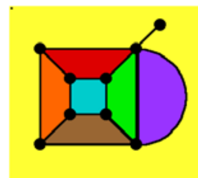


Planar graphs

- Definition: A planar graph is a graph that can be drawn in the plane such that its edges only intersect at their common end-vertices.

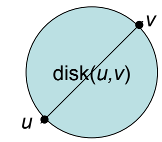


- Kuratowski's Theorem: A graph is planar iff it contains no subgraph that is edge contractible to K_5 or $K_{3,3}$.
- Euler's Polyhedron Formula: A connected planar graph with *n* nodes, *m* edges, and *f* faces has $n - m + f = 2$.
- Right: Example with 9 vertices,14 edges, and 7 faces (the yellow "outside" face is called the infinite face)
- Theorem: A simple planar graph with n nodes has at most 3n–6 edges, for $n \geq 3$.

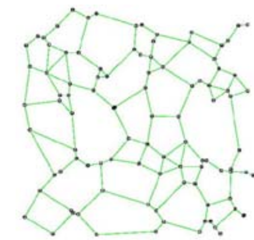


Gabriel Graph

- Let disk(*u,v*) be a disk with diameter (*u,v*) that is determined by the two points *u,v*.
- The Gabriel Graph GG(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is an edge between two nodes *u,v* iff the disk(*u,v*) including boundary contains no other points.

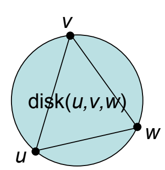


- As we will see the Gabriel Graph has interesting properties.

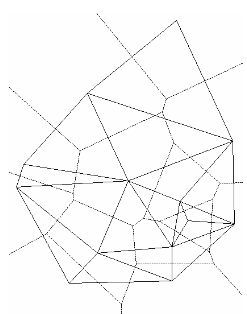


Delaunay Triangulation

- Let disk(*u,v,w*) be a disk defined by the three points *u,v,w*.
- The Delaunay Triangulation (Graph) DT(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is a triangle of edges between three nodes *u,v,w* iff the disk(*u,v,w*) contains no other points.

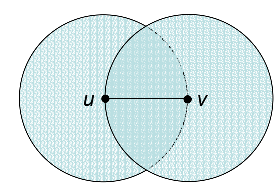


- The Delaunay Triangulation is the dual of the Voronoi diagram, and widely used in various CS areas; the DT is planar; the distance of a path (s,…,t) on the DT is within a constant factor of the s-t distance.

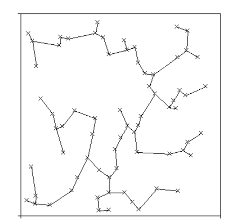


Other planar graphs

- Relative Neighborhood Graph RNG(V)
- An edge e = (u,v) is in the RNG(V) iff there is no node w with (u,w) < (u,v) and (v,w) < (u,v).



- Minimum Spanning Tree MST(V)
- A subset of *E* of *G* of minimum weight which forms a tree on *V*.

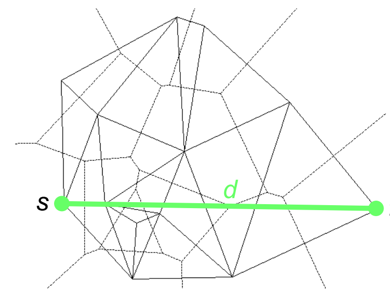


Properties of planar graphs

- Theorem 1:
  $MST(V) \subseteq RNG(V) \subseteq GG(V) \subseteq DT(V)$
- Corollary:
  Since the MST(V) is connected and the DT(V) is planar, all the planar graphs in Theorem 1 are connected and planar.
- Theorem 2:
  The Gabriel Graph contains the Minimum Energy Path (for any path loss exponent $\alpha \geq 2$)
- Corollary:
  $GG(V) \cap UDG(V)$ contains the Minimum Energy Path in UDG(V)

Routing on Delaunay Triangulation?

- Let d be the Euclidean distance of source s and destination t
- Let c be the sum of the distances of the links of the shortest path in the Delaunay Triangulation
- It was shown that $c = \Theta(d)$

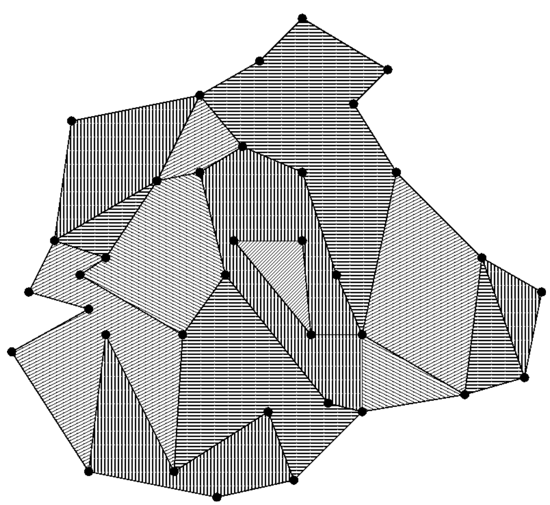


- Two problems:
1) How do we find this best route in the DT? With flooding?!?
2) How do we find the DT at all in a distributed fashion?

… and even worse: The DT contains edges that are not in the UDG, that is, nodes that cannot hear each other are “neighbors” on DT

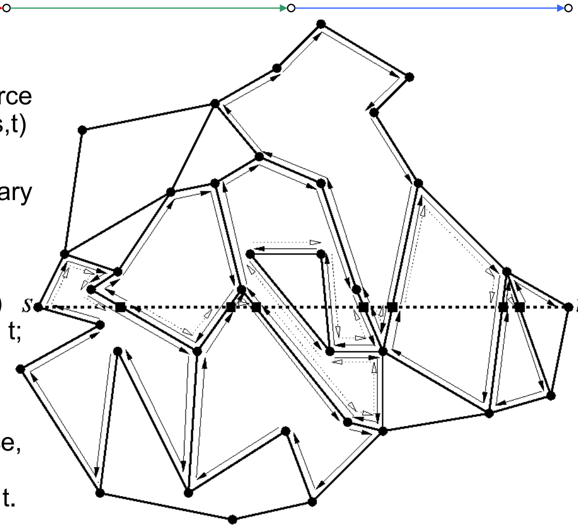
Breakthrough idea: route on faces

- Remember the faces…
- Idea:
  Route along the boundaries of the faces that lie on the source–destination line

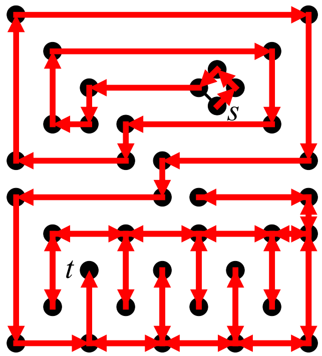


Face Routing

0. Let f be the face incident to the source s, intersected by (s,t)
1. Explore the boundary of f; remember the point p where the boundary intersects with (s,t) which is nearest to t; after traversing the whole boundary, go back to p, switch the face, and repeat 1 until you hit destination t.



Face Routing Works on *Any* Graph



Face routing is correct

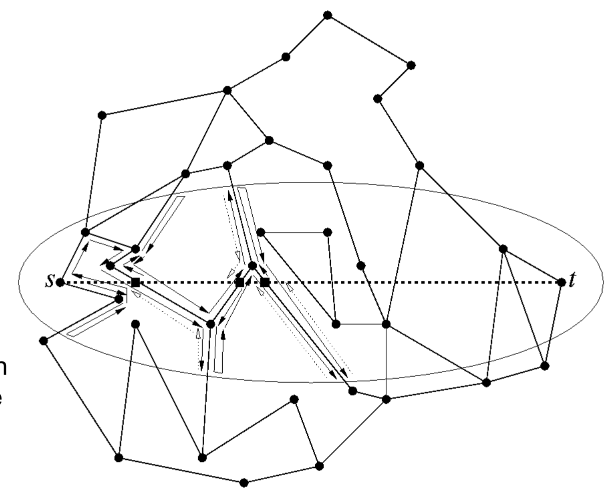
- Theorem: Face routing terminates on any simple planar graph in O(n) steps, where n is the number of nodes in the network
- Proof: A simple planar graph has at most 3n–6 edges. You leave each face at the point that is closest to the destination, that is, you never visit a face twice, because you can order the faces that intersect the source—destination line on the exit point. Each edge is in at most 2 faces. Therefore each edge is visited at most 4 times. The algorithm terminates in O(n) steps.

Is there something better than Face Routing?

- How to improve face routing? Face Routing 2 ☺
- Idea: Don’t search a whole face for the best exit point, but take the first (better) exit point you find. Then you don’t have to traverse huge faces that point away from the destination.
- Efficiency: Seems to be practically more efficient than face routing. But the theoretical worst case is worse – $O(n^2)$.
- Problem: if source and destination are very close, we don’t want to route through all nodes of the network. Instead we want a routing algorithm where the cost is a function of the cost of the best route in the unit disk graph (and independent of the number of nodes).

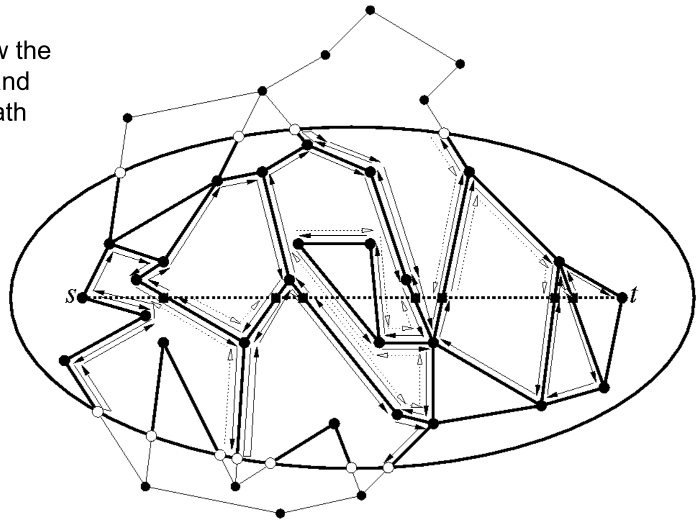
Adaptive Face Routing (AFR)

- Idea: Use face routing together with ad-hoc routing trick 1!!
- That is, don’t route beyond some radius r by branching the planar graph within an ellipse of exponentially growing size.



AFR Example Continued

- We grow the ellipse and find a path



AFR Pseudo-Code

0. Calculate G = GG(V) ∩ UDG(V)
   Set c to be twice the Euclidean source—destination distance.

1. Nodes w ∈ W are nodes where the path s-w-t is larger than c. Do face routing on the graph G, but without visiting nodes in W. (This is like pruning the graph G with an ellipse.) You either reach the destination, or you are stuck at a face (that is, you do not find a better exit point.)

2. If step 1 did not succeed, double c and go back to step 1.

- Note: All the steps can be done completely locally, and the nodes need no local storage.

The Ω(1) Model

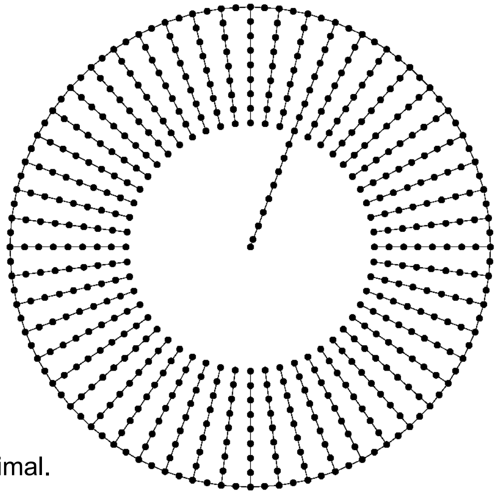
- We simplify the model by assuming that nodes are sufficiently far apart; that is, there is a constant d_0 such that all pairs of nodes have at least distance d_0. We call this the Ω(1) model.
- This simplification is natural because nodes with transmission range 1 (the unit disk graph) will usually not “sit right on top of each other”.
- Lemma: In the Ω(1) model, all natural cost models (such as the Euclidean distance, the energy metric, the link distance, or hybrids of these) are equal up to a constant factor.
- Remark: The properties we use from the Ω(1) model can also be established with a backbone graph construction.

Analysis of AFR in the Ω(1) model

- Lemma 1: In an ellipse of size c there are at most $O(c^2)$ nodes.
- Lemma 2: In an ellipse of size c, face routing terminates in $O(c^2)$ steps, either by finding the destination, or by not finding a new face.
- Lemma 3: Let the optimal source—destination route in the UDG have cost c*. Then this route c* must be in any ellipse of size c* or larger.
- Theorem: AFR terminates with cost $O(c^{*2})$.
- Proof: Summing up all the costs until we have the right ellipse size is bounded by the size of the cost of the right ellipse size.

Lower Bound

- The network on the right constructs a lower bound.
- The destination is the center of the circle, the source any node on the ring.
- Finding the right chain costs $\Omega(c^{*2})$, even for randomized algorithms
- Theorem:
  AFR is asymptotically optimal.



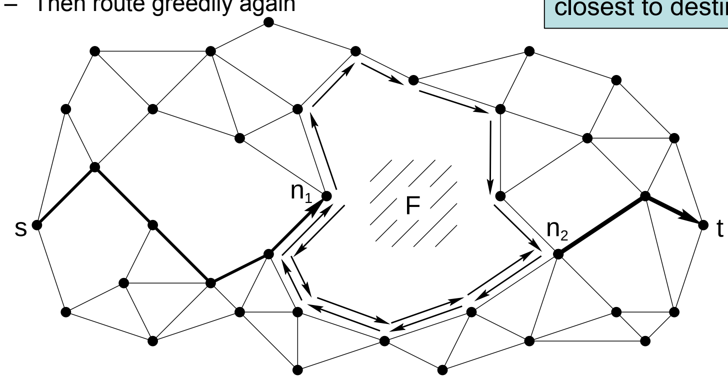
Non-geometric routing algorithms

- In the $\Omega(1)$ model, a standard flooding algorithm enhanced with trick 1 will (for the same reasons) also cost $O(c^{*2})$.
- However, such a flooding algorithm needs O(1) extra storage at each node (a node needs to know whether it has already forwarded a message).
- Therefore, there is a trade-off between O(1) storage at each node or that nodes are location aware, and also location aware about the destination. This is intriguing.

GOAFR – Greedy Other Adaptive Face Routing

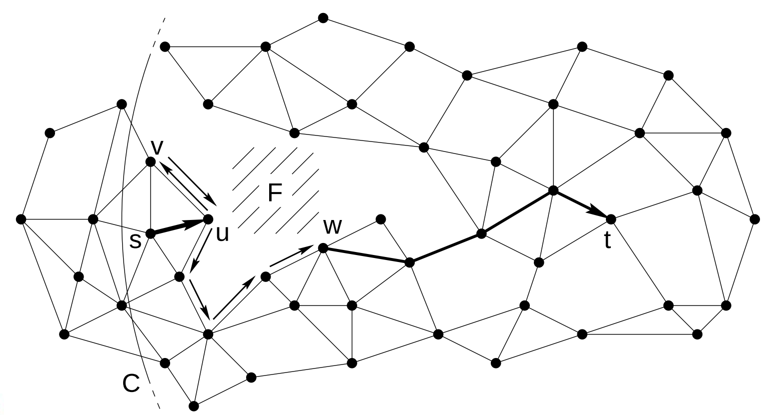
- Back to geometric routing…
- AFR Algorithm is not very efficient (especially in dense graphs)
- Combine Greedy and (Other Adaptive) Face Routing
  - Route greedily as long as possible
  - Circumvent "dead ends" by use of face routing
  - Then route greedily again

Other AFR: In each face proceed to node closest to destination



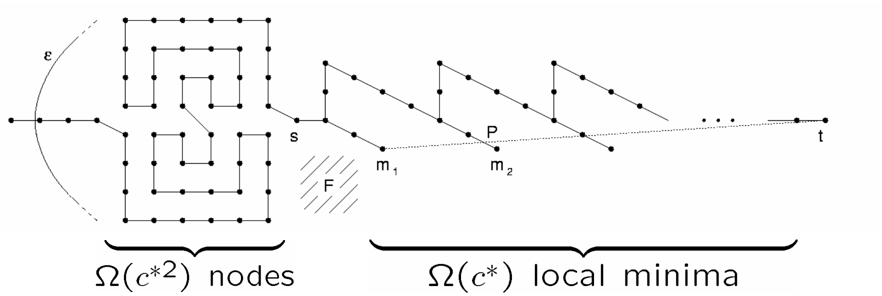
GOAFR+

- GOAFR+ improvements:
  - Early fallback to greedy routing
  - (Circle centered at destination instead of ellipse)



GOAFR+ — Early Fallback

- We could fall back to greedy routing as soon as we are closer to t than the local minimum
- But:



- "Maze" with $\Omega(c^{*2})$ edges is traversed $\Omega(c^{*})$ times $\rightarrow$ $\Omega(c^{*3})$ steps

GOAFR – Greedy Other Adaptive Face Routing

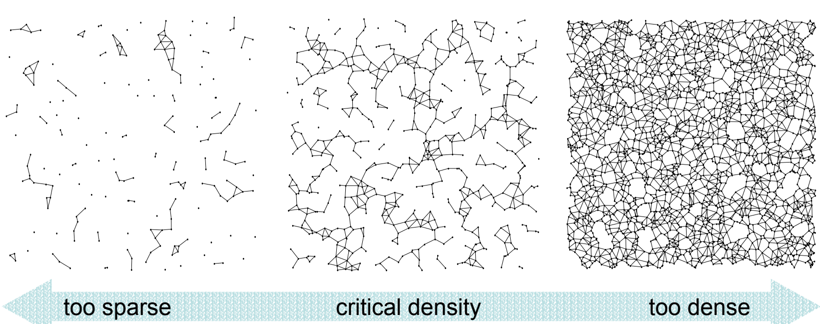
- Early fallback to greedy routing:
  - Use counters p and q. Let u be the node where the exploration of the current face F started
    - p counts the nodes closer to t than u
    - q counts the nodes *not* closer to t than u
  - Fall back to greedy routing as soon as $p > \sigma \cdot q$ (constant $\sigma > 0$)

Theorem: GOAFR is still asymptotically worst-case optimal…
…*and* it is efficient in practice, in the average-case.

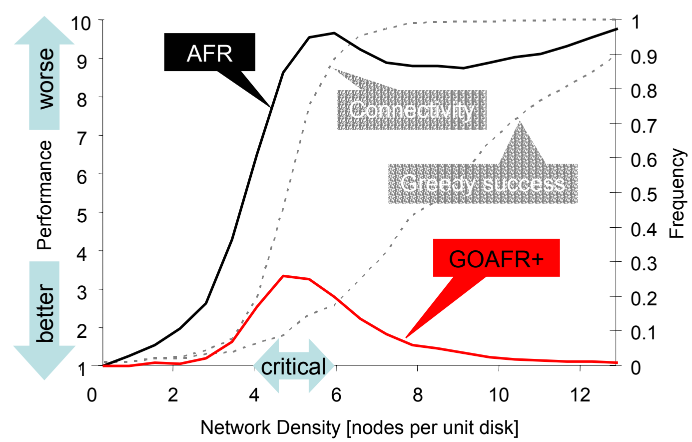
- What does "practice" mean?
  - Usually nodes placed uniformly at random

Average Case

- Not interesting when graph not dense enough
- Not interesting when graph is too dense
- Critical density range ("percolation")
  - Shortest path is significantly longer than Euclidean distance



Simulation on Randomly Generated Graphs



A Word on Performance

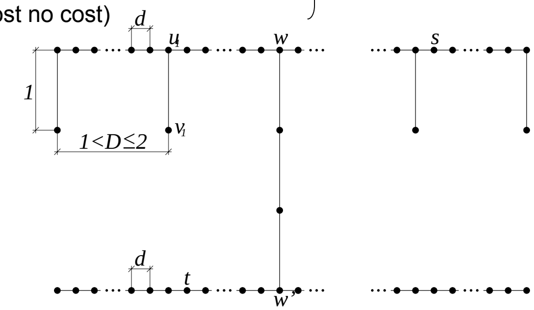
- What does a performance of 3.3 in the critical density range mean?
- If an optimal path (found by Dijkstra) has cost c, then GOAFR+ finds the destination in 3.3·c steps.
- It does *not* mean that the *path* found is 3.3 times as long as the optimal path! The path found can be much smaller…
- Remarks about cost metrics
  - In this lecture “cost” c = c hops
  - There are other results, for instance on distance/energy/hybrid metrics
  - In particular: With energy metric there is no competitive geometric routing algorithm

Energy Metric Lower Bound

Example graph: k “stalks”, of which only one leads to t

- any deterministic (randomized) geometric routing algorithm A has to visit all k (at least k/2) “stalks”
- optimal path has constant cost c^* (covering a constant distance at almost no cost)

$$\lim_{k \to \infty} \frac{c(A)}{c^*} = \infty$$



→ With energy metric there is no competitive geometric routing algorithm

Milestones in Geometric Routing

Kleinrock et al.	Various 1975ff	MFR et al.	Geometric Routing proposed
Kranakis, Singh, Urrutia	CCCG 1999	Face Routing	First correct algorithm
Bose, Morin, Stojmenovic, Urrutia	DialM 1999	GFG	First average-case efficient algorithm (simulation but no proof)
Karp, Kung	MobiCom 2000	GPSR	A new name for GFG
Kuhn, Wattenhofer, Zollinger	DialM 2002	AFR	First worst-case analysis. Tight $\Theta(c^2)$ bound.
Kuhn, Wattenhofer, Zollinger	MobiHoc 2003	GOAFR	Worst-case optimal and average-case efficient, percolation theory
Kuhn, Wattenhofer, Zhang, Zollinger	PODC 2003	GOAFR+	Currently best algorithm, other cost metrics, etc.