ETH

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Mobile Computing
Exercise 3

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1 Walsh Codes

In the lecture you have learned about Walsh codes and how they are recursively constructed (see page 31 of the notes of lecture 2). In this exercise you will prove two fundamental properties of those codes.

1.1 Orthogonality

Prove that the code words of a Walsh code are pairwise orthogonal (as mentioned in the lecture). As an example, consider the Walsh code $\mathcal{C}_2$ of length 4. We have

$$\mathcal{C}_2 = \{(+1,+1,+1,+1),(+1,+1,-1,-1),(+1,-1,+1,-1),(+1,-1,-1,+1)\}\,.$$

For all 6 possible pairs of code words, we can easily verify that their inner product is 0.

1.2 Balance of the Code Words

From the recursive construction of Walsh codes, it is obvious that the word of all ones is always a code word ($(+1,+1,\ldots,+1) \in \mathcal{C}$). Prove that for all other code words, half of the components are $+1$ and half of the components are -1, i.e. prove that the code words of a Walsh code are balanced.

2 Random White Noise and Orthogonal Codes

Let $\mathcal{C}$ be an orthogonal code consisting of m code words of length m ($\mathcal{C} \subset \{-1,+1\}^m$, $|\mathcal{C}| = m$, and $\forall c_i, c_j \in \mathcal{C} : c_i \cdot c_j = 0$ if $i \neq j$). Assume that a distinct station S uses the code word $s \in \mathcal{C}$ to send a 1 and $\bar{s}$ to send a 0. The other stations $S_i \neq S$ use other code words $s_i \in \mathcal{C}$ and the respective complements $\bar{s_i}$ to send their bits. Additionally we add a random noise vector N to the signals of the stations. To receive the bit of station S, the received signal

$$r = \pm s + \sum_i \pm s_i + N$$

has to be multiplied by s, resulting in

$$X = s \cdot r = s \cdot \pm s + \underbrace{s \cdot \sum \pm s_i}_{=0} + s \cdot N = \pm m + s \cdot N.$$

We assume that the total noise vector N is the sum of k statistically independent noise vectors N_i ($1 \leq i \leq k$) and that the components of N_i are independent $\mathcal{N}(0, \sigma^2)$-distributed random variables:

$$N = \sum_{i=1}^{k} N_i, \qquad N_i = \left(\mathcal{N}(0,\sigma^2), \mathcal{N}(0,\sigma^2), \ldots, \mathcal{N}(0,\sigma^2)\right).$$

1. Compute the distribution of X.
2. Set $m = 128$ and $\sigma^2 = 1$. How many noise vectors N_i can be tolerated such that at least 99% of the received signals are decoded correctly (i.e. how big can k get)?

Hint: Use the fact that the sum of two independent normally distributed random variables is a normally distributed random variable again:

$$X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2), X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2) \quad \Longrightarrow \quad X_1 + X_2 \sim \mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2).$$

3 Slotted Aloha

In this exercise we want to analyse 'Slotted Aloha' for the case that the number of stations n is not exactly known. We assume that in each time slot each station transmits with probability p. In the lecture you saw that the probability that the slot can be used (i.e. the probability that exactly one station transmits) is

$$\Pr(\text{success}) = n \cdot p(1-p)^{n-1}.$$

If n is fixed, we can maximize the above expression and get the optimal p as shown in the lecture. Now assume that the only thing we know about n is $A \leq n \leq B$.

1. Which value p maximizes $\Pr(\text{success})$ for the worst $n \in [A, B]$?
2. What is this 'worst case optimal' value for p if $A = 100$ and $B = 200$?