ETH
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Distributed Computing

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Computational Thinking
Exercise 3

1 Hamiltonian

Consider a graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. A Hamiltonian path in G is a path that visits every vertex in V exactly once. Similarly, a Hamiltonian cycle is a cycle that visits every vertex of the graph exactly once. The decision problems *Hamiltonian Path* and *Hamiltonian Cycle* ask whether such a path or cycle exists in a given graph.

a) Show that Hamiltonian Cycle $\leq$ Hamiltonian Path.

b) Show that Hamiltonian Path $\leq$ Hamiltonian Cycle.

2 Circuit Complexity

In this task, we revisit the complexity classes AC^i and NC^i discussed in the lecture (check Definition 2.46 and the following remark in the lecture notes.) Let us first look into how these classes relate to each other.

a) Show that $\mathsf{NC}^0 \subset \mathsf{AC}^0 \subset \mathsf{NC}^1 \subset \mathsf{AC}^1 \ldots$.

Consider now the parity problem:

Definition 2.1 (PARITY) *Given a binary string, decide if it contains an even number of* 1*s.*

b) Show that PARITY is in NC^1.

c) Show that PARITY is not in NC^0.

Next, we will consider the complexity of the binary sum problem.

Definition 2.2 (SUM) *Given two non-negative integers as binary strings of length n, compute their sum as a binary string of length $n + 1$.*

We will assume that the input to a circuit is the two strings concatenated.

d) Show that SUM is in AC^0. What is the size of the circuit that you found?

e) Show that SUM is not in NC^0.