

Discrete Event Systems

Exercise Sheet 4

1 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

- a) $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$
- b) $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

2 Regular and Context-Free Languages

- a) Consider the context-free grammar G with the production $S \rightarrow SS \mid 1S2 \mid 0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.
- b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language L that is regular.

3 Push Down Automata

For each of the following context free languages, draw a PDA that accepts L .

- a) $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{\text{reverse}} = u\} = \{u \mid \text{"}u \text{ is a palindrome"}\}$
- b) $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{\text{reverse}} \neq u\} = \{u \mid \text{"}u \text{ is no palindrome"}\}$

4 Ambiguity

Consider the following context-free grammar G with non-terminals S and A , start symbol S , and terminals “(”, “)”, and “0”:

$$\begin{aligned} S &\rightarrow SA \mid \varepsilon \\ A &\rightarrow AA \mid (S) \mid 0 \end{aligned}$$

- a) What are the eight shortest words produced by G ?
- b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.
- c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.

5 Counter Automaton

A push-down automaton is basically a finite automaton augmented by a stack. Consider a finite automaton that (instead of a stack) has an additional *counter* C , i.e., a register that can hold a single integer of arbitrary size. Initially, $C = 0$. We call such an automaton a *Counter Automaton* M . M can only increment or decrement the counter, and test it for 0. Since theoretically, all possible data can be coded into one single integer, a counter automaton has unbounded memory. Further, let \mathcal{L}_{count} be the set of languages recognized by counter automata.

- a) Let \mathcal{L}_{reg} be the set of regular languages. Prove that $\mathcal{L}_{reg} \subseteq \mathcal{L}_{count}$.
- b) Prove that the opposite is not true, that is, $\mathcal{L}_{count} \not\subseteq \mathcal{L}_{reg}$. Do so by giving a language which is in \mathcal{L}_{count} , but not in \mathcal{L}_{reg} . Characterize (with words) the kind of languages a counter automaton can recognize, but a finite automaton cannot.
- c) Which automaton is stronger? A counter automaton or a push-down automaton? Explain your decision.

6 Inequality-checking with PDAs [Exam HS20]

Draw a PDA that recognizes $L = \{x\#y \mid x, y \in \{0, 1\}^+, x \neq y\}$. Use at most 12 states.

Hint: Can two strings x and y with $|x| < |y|$ ever be equal?