



# Computational Thinking

## Exercise 5 (Cryptography)

### 1 Nonce Reuse

In the ElGamal digital signature scheme, why should the same random nonce never be reused for 2 different messages with the same public/secret keypair?

### 2 Cryptographic Hash Functions

Let  $h_1, h_2 : \{0, 1\}^* \rightarrow \{0, 1\}^n$  be two collision resistant functions. Are the following hash functions also collision resistant? Explain<sup>1</sup>.

- $h_3(x) = h_1(x) \oplus h_2(x)$
- $h_4(x) = x_0; h_1(x)$

*Hint:* Try to find a collision or reduce the collision-resistance of the constructed hash functions to collision-resistance of  $h_1$  and  $h_2$ .

### 3 ElGamal Encryption

In the lecture we have shown that:  $\text{CDH} \leq \text{Breaking-ElGamal-Encryption}$ . Show that  $\text{Breaking-ElGamal-Encryption} \leq \text{CDH}$ .

### 4 Active Adversary in ElGamal Encryption

Alice wants to bid an amount of money ( $2k\$$ ) in an auction<sup>2</sup>. To do this, Alice sends the amount of money she is bidding securely by using the ElGamal-Encryption scheme.

- Show that ElGamal Encryption scheme is homomorphic.
- Use this property to reduce the amount of money that Alice is bidding by half (i.e. to  $k\$$ ).
- How can Alice prevent this attack?

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<sup>1</sup> $x_0$  means the first bit of the message  $x$ , and as in the lecture, concatenation of messages is denoted by ;

<sup>2</sup>For example, in Ebay.