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Computational Thinking
Exercise 14

1 Pattern Generation with Cellular Automata

Suppose we are given a grid with tiles only placed in the top most row. Our goal is to define tiling rules to tile the entire plane below in a specific pattern. A tiling rule defines the color of the tile based on the color of the tile above, and the tiles to the right and left of that tile. In particular, the color the tile at position (i, j) is specified by the the colors of the tiles at positions $(i-1, j-1), (i-1, j), (i-1, j+1)$. For example, given a line of alternating black and white tiles, then the two tiling rules

$$(white, black, white) \rightarrow white$$
$$(black, white, black) \rightarrow black$$

will cover the entire plane in a chess board pattern. Here, the first transision rule would mean that if position $(i-1, j-1)$ is white, position $(i-1, j)$ is black, and position $(i-1, j+1)$ is white again, then the tile at position (i, j) will be white. Given a line of white tiles with only a single black tile in the middle. Design a set of transition rules, such that the tiles form the Sierpinski triangle, seen in Figure 1.

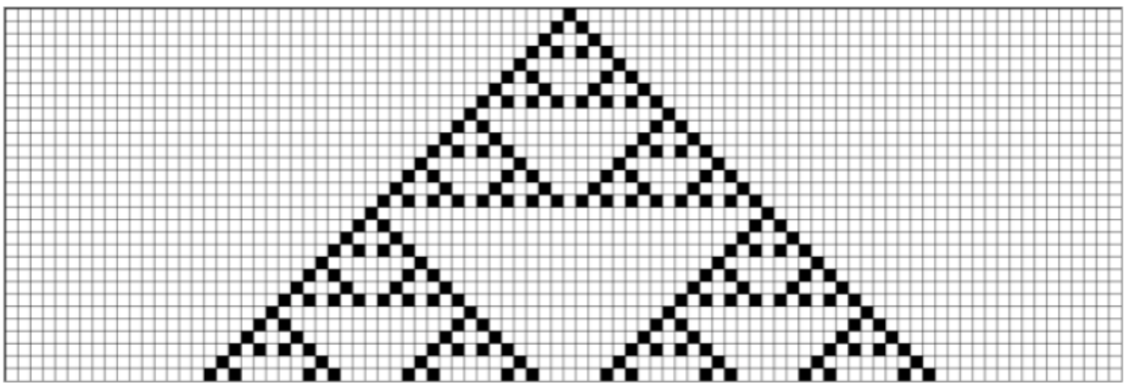

Figure 1: Am image of the Sierpinski triangle.

2 PCP warm-up

Do the following PCPs have a solution?

a) Domino set $\left[\frac{a}{aaab}\right], \left[\frac{abba}{ba}\right], \left[\frac{aa}{aba}\right], \left[\frac{bbab}{bb}\right]$.

b) Domino set $\left[\frac{ab}{abb}\right], \left[\frac{aaba}{abb}\right], \left[\frac{baa}{aaa}\right]$.

c) Domino set $\left[\frac{abbb}{b}\right], \left[\frac{b}{bca}\right], \left[\frac{cac}{ca}\right], \left[\frac{aa}{cb}\right], \left[\frac{bb}{bbb}\right]$.

d) Domino set $\left[\frac{ad}{dda}\right], \left[\frac{bc}{ca}\right], \left[\frac{c}{a}\right], \left[\frac{d}{db}\right], \left[\frac{ab}{bc}\right]$.

3 PCP variants

Are the following variants of the PCP problem decidable or undecidable?

a) ab^* PCP: each word α and each word β has the following form: it starts with a single letter a, and then an arbitrary number of letters b. Some examples for valid words are a, abb or $abbbbbb$.

b) Limited-use PCP: given an integer parameter k in the input, we only accept domino sequences that contain each domino at most k times.

c) Unique-triplet PCP: we only accept domino sequences where no consecutive triplet of dominoes appears two times, i.e. there are no distinct indices i, j such that each of the following three pairs of dominoes are the same: those at positions i and j, those at positions $(i + 1)$ and $(j + 1)$, and those at positions $(i + 2)$ and $(j + 2)$.

d) Two-color PCP: besides the two words (α, β), dominoes also have a color: each domino is painted red or blue. We only accept domino sequences that are alternating, i.e. a red domino is always followed by a blue domino, and vice versa.

e) Half-used PCP: given the input set of dominoes S, we only accept domino sequences that use at most half of the domino types (possibly with repetitions), i.e. there are at least $\frac{1}{2} \cdot |S|$ input dominoes that never occur in the sequence.

f) Silly PCP: for each domino (α, β) of the input set, the two words have the same length, i.e. we have $|\alpha| = |\beta|$.

g) Almost-silly PCP: for some constant integer $c > 1$, the length of each word α and each word β has to be a multiple of c.

h) Binary PCP: the size of the alphabet is restricted to two characters, i.e. $\Sigma = \{0, 1\}$.