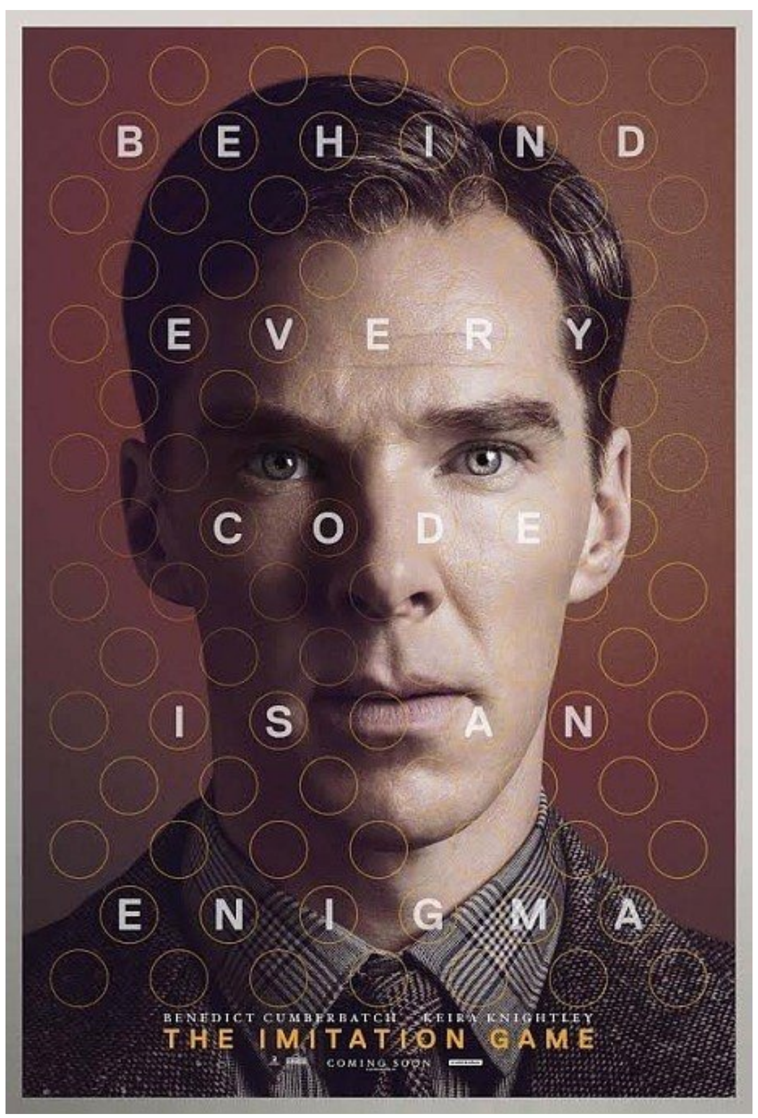
Automata & languages

A primer on the Theory of Computation



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Part 4 out of 4

Last week, we showed the

equivalence of DFA, NFA and REX

is equivalent to

$$\begin{array}{ccc} \text{DFA} & \asymp & \text{NFA} \\ & & \rotatebox{90}{$\asymp$} \\ & & \text{REX} \end{array}$$

We also started to look at
non-regular languages

Pumping lemma

If A is a regular language, then

there exist a number p s.t.

Any string in A whose length is at least p

can be divided into three pieces xyz s.t.

- $xy^iz \in A$, for each $i \geq 0$ and
- $|y| > 0$ and
- $|xy| \leq p$

To prove that a language A is not regular:

1. Assume that A is regular

2. Since A is regular, it must have a pumping length p

3. Find one string s in A whose length is at least p

4. For any split $s=xyz$,
   Show that you cannot satisfy all three conditions

5. Conclude that s cannot be pumped

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4. For any split $s=xyz$,
Show that you cannot satisfy all three conditions

5. Conclude that ***s* cannot be pumped** $\longrightarrow$ **A is not regular**

Wait…

What happens if A is a finite language?!

Pumping lemma

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- $|xy| \leq p$

Pumping lemma

If ***A* is a regular language**, then

there exist a number *p* s.t.

As we saw two weeks ago,

all finite languages are regular…

So what's ***p***?

***p* := len(longest_string) + 1**

makes the lemma hold vacuously

Non-regular languages are not closed under most operations

if L_1 and L_2 are regular,

then so are

$L_1 \cup L_2$

$L_1 . L_2$

L_1^*

if L_1 and L_2 are not regular,

then

$L_1 \cup L_2$

$L_1 . L_2$

L_1^*

may or may not be regular!

$(L_1)^C$ is not regular

non RL are closed under complement

This week is all about

Context-Free Languages

a superset of Regular Languages