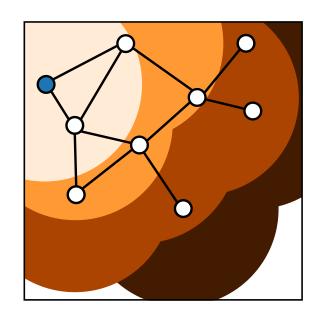
## Discrete Event Systems Verification of Finite Automata (Part 2)



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December 2, 2021

Most materials from Lothar Thiele

### Thank you for your feedback!

- Slightly too fast
- Reachability was covered too quickly
- More examples would be nice
- More interaction would be nice



# Last week in Discrete Event Systems

#### Verification Scenarios

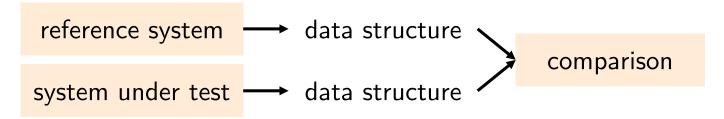
#### Example

$$y = (x_1 + x_2) \cdot x_3$$

$$x_1 \circ \longrightarrow + \longrightarrow y$$

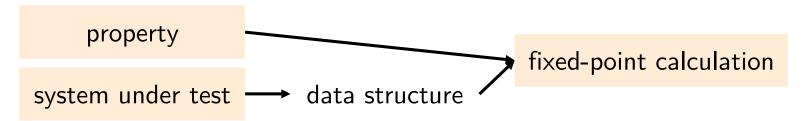
$$x_3 \circ \longrightarrow + \longrightarrow y$$

Comparison of specification and implementation



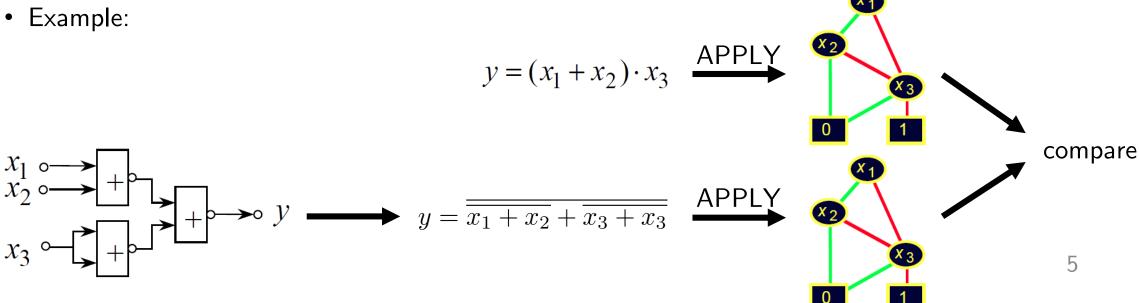
"The device can always be switched off."

Proving properties



### Comparison using BDDs

- Boolean (combinatorial) circuits: Compare specification and implementation, or compare two implementations.
- Method:
  - Representation of the two systems in ROBDDs, e.g., by applying the APPLY operator repeatedly.
  - Compare the structures of the ROBDDs.

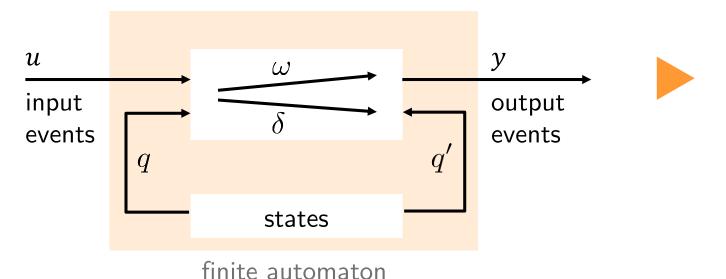


### Sets and Relations using Boolean Expressions

- Representation of a relation  $R \subseteq A \times B$ 
  - Binary encoding  $\sigma(a)$ ,  $\sigma(b)$  of all elements  $a \in A$ ,  $b \in B$
  - Representation of *R*

$$(a,b) \in R \iff \psi_R(\sigma(a),\sigma(b))$$

• Example finite automaton:



characteristic function of the relation R

$$\psi_{\delta}(u, q, q') = 1$$

$$\psi_{\omega}(u, q, y) = 1$$

we remove the binary encoding for convenience in our notation; but u, q, q' are actually represented as binary vectors <sup>6</sup>

### Reachability of States – State Diagram

Question

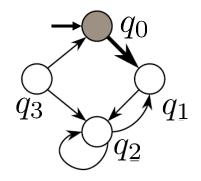
Is a state  $q \in Q$  reachable by a sequence of state transitions?

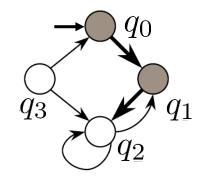
$$Q_0 = \{q_0\}$$

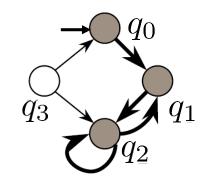
$$Q_1 = Q_0 \cup \{q_1\}$$

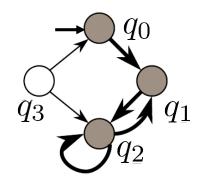
$$Q_0 = \{q_0\}$$
  $Q_1 = Q_0 \cup \{q_1\}$   $Q_2 = Q_1 \cup \{q_1, q_2\}$   $Q_3 = Q_2 \cup \{q_1, q_2\}$ 

$$Q_3 = Q_2 \cup \{q_1, q_2\}$$









Problem

Drawing state diagrams is not feasible in general.

### Reachability of States – Boolean Expressions

Fixed-point computation

- Start with the initial state
- Determine the set of states that can be reached in one
- Take the union and iterate until a fixed-point is reached

$$Q_0 = \{q_0\}$$
 
$$Q_{i+1} = Q_i \cup Suc(Q_i, \delta) \qquad \text{until } Q_{i+1} = Q_i$$
 
$$\psi_{Q_{i+1}}(q') = \psi_{Q_i}(q') + (\exists q: \psi_{Q_i}(q) \cdot \psi_{\delta}(q, q')) \qquad \text{Test by comparing the}$$

 $Q_R$ : set of reachable states

$$Q_R = Q_0 \cup_{i \geq 0} Suc(Q_i, \delta)$$
 Finite union if model is finite 
$$\psi_{Q_R}(q') = \psi_{Q_0}(q') \sum_{i \geq 0} (\exists q: \psi_{Q_i}(q) \cdot \psi_{\delta}(q, q'))$$
 8

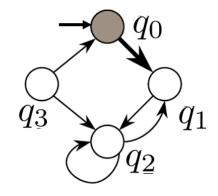
ROBDDs of  $Q_{i+1} = Q_i$ 

### Reachability of States – Example

State encoding

$$(x_1, x_0) = \sigma(q)$$

$\sigma(q)$	$x_1$	x <sub>0</sub>
$q_0$	0	0
$q_1$	0	1
$q_2$	1	0
$q_3$	1	1



Transition relation encoding

$$\psi_{\delta}(q,q')$$

entries where  $\psi_\delta(q,q')=1$  only

$x_1$	$x_0$	x <sub>1</sub> '	$x_0$
0	0	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	1	0
1	1	0	0

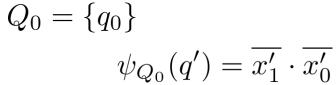
e.g. 
$$q_0 \rightarrow q_1$$

$$q_2 \rightarrow q_2$$

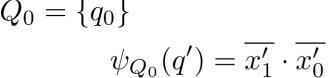
As a Boolean function

$$\psi_{\delta}(q,q') = \overline{x_0'} \cdot (x_0 \cdot (x_1 + x_1') + x_1 \cdot x_1') + \overline{x_0} \cdot x_0' \cdot \overline{x_1'}$$

$$q_3$$
  $q_1$   $q_2$ 



$$\psi_{Q_{i+1}}(q') = \psi_{Q_i}(q') + (\exists q : \psi_{Q_i}(q) \cdot \psi_{\delta}(q, q'))$$



States 
$$\sigma(q) \times_1$$

$\sigma(q)$	$x_1$	$x_0$
$q_0$	0	0
$q_1$	0	1
$q_2$	1	0
$q_3$	1	1

$Q_1 = Q_0 \cup \{q_1\}$
$\psi_{O_1}(q')$

$$\psi_{Q_1}(q') = \overline{x_1'} \cdot \overline{x_0'} + \overline{x_1'} \cdot x_0' = \overline{x_1'}$$

#### Transitions

$x_1$	$x_0$	x <sub>1</sub> '	x <sub>0</sub> '
0	0	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	1	0
1	1	0	0

$$Q_2 = Q_1 \cup \{q_1, q_2\}$$

$$\psi_{Q_2}(q') = \overline{x_1'} + (x_1' \cdot \overline{x_0'} + \overline{x_1'} \cdot x_0') = \overline{x_1'} + \overline{x_0'}$$

$$q_0$$
 $q_3$ 
 $q_1$ 

$$Q_3 = Q_2 \cup \{q_1, q_2\}$$

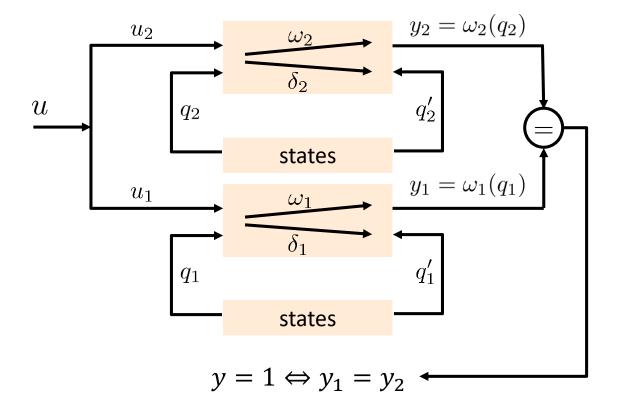
$$\psi_{Q_3}(q') = (\overline{x_1'} + \overline{x_0'}) + (\overline{x_1'} + \overline{x_0'}) = \overline{x_1'} + \overline{x_0'}$$

### Comparison of Finite Automata

For simplicity, we only consider Moore automata, i.e., the output depends on the current state only. The output function is  $\omega: Q \to \Sigma$  and  $y = \omega(q)$ .

Strategy

- 1. Compute the set of jointly reachable states.
- 2. Compare the output values of the two finite automata.



## This week in Discrete Event Systems

Efficient state representation

- Set of states as Boolean function
- Binary Decision Diagram representation

Computing reachability

- Leverage efficient state representation
- Explore successor sets of states

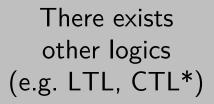
Today

Proving properties

- Temporal logic (CTL)
- Encoding as reachability problem

### Temporal logics

- Verify properties of a finite automaton, for example
  - Can we always reset the automaton?
  - Is every request followed by an acknowledgement?
  - Are both outputs always equivalent?
- Specification of the query in a formula of temporal logic.
- We use a simple form called Computation Tree Logic (CTL).
- Let us start with a minimal set of operators.
  - Any atomic proposition is a CTL formula.
  - CTL formula are constructed by composition of other CTL formula.



Formula	Examples
Atomic proposition	The printer is busy. The light is on.
Boolean logic	$\phi_1 + \phi_2$ ; $\neg \phi_1$
CTL logic	$EX\ \phi_1$



### Formulation of CTL properties

Based on atomic propositions  $(\phi)$  and quantifiers

```
A\phi \rightarrow \text{«All }\phi\text{»}, \qquad \phi \text{ holds on all paths}
```

 $\mathsf{E}\phi \to \mathsf{wExists}\; \phi \mathsf{w}, \qquad \phi \; \mathsf{holds} \; \mathsf{on} \; \mathsf{at} \; \mathsf{least} \; \mathsf{one} \; \mathsf{path}$ 

```
X\phi \rightarrow \text{«NeXt }\phi\text{»}, \qquad \phi \text{ holds on the next state}
```

$$F\phi \rightarrow \text{«Finally } \phi\text{»}, \quad \phi \text{ holds at some state along the path}$$

$$G\phi \rightarrow \text{ "Globally } \phi$$
",  $\phi$  holds on all states along the path

$$\phi_1 \cup \phi_2 \rightarrow (\phi_1 \cup \text{ntil } \phi_2), \quad \phi_1 \text{ holds until } \phi_2 \text{ holds}$$

implies that  $\phi_2$  has to hold eventually

Quantifiers over paths

Path-specific quantifiers

#### Formulation of CTL properties

CTL quantifiers works in pairs

$$\{A,E\}\ \{X,F,G,U\}\phi$$

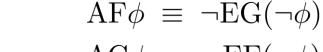
You need one of each!

Can be more than one pair

AG 
$$\phi_1$$
 where  $\phi_1 = \mathsf{EF} \; \phi_2 \equiv \mathsf{AG} \; \mathsf{EF} \; \phi_2$ 

E,G,X,U are sufficient to define the whole logic.

A and F are convenient, but not necessary



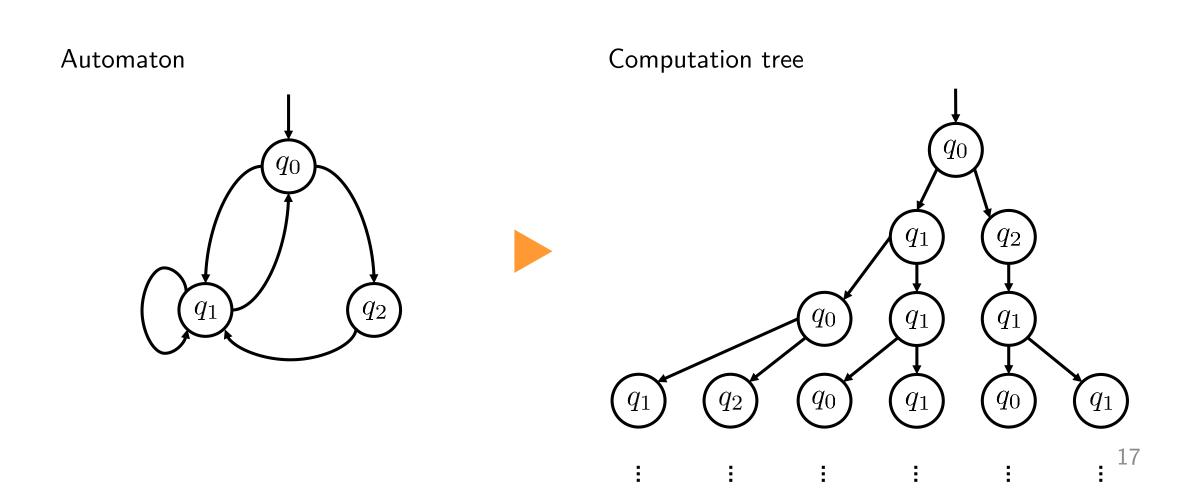
$$AG\phi \equiv \neg EF(\neg \phi)$$

$$AX\phi \equiv \neg EX(\neg \phi)$$

$$EF\phi \equiv true EU\phi$$

No need to know that one  $\phi_1 AU \phi_2 \equiv \neg[(\neg \phi_1)EU \neg(\phi_1 + \phi_2)] + EG(\neg \phi_2)$ 

## CTL works on computation trees

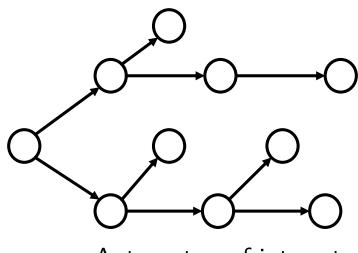


### CTL works on computation trees

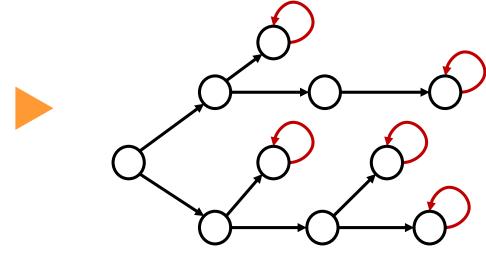
M satisfies  $\phi \Leftrightarrow q_0 \models \phi$ where  $q_0$  is the initial state of M

Required fully-defined transition functions

Each state has at least one successor (can be itself)





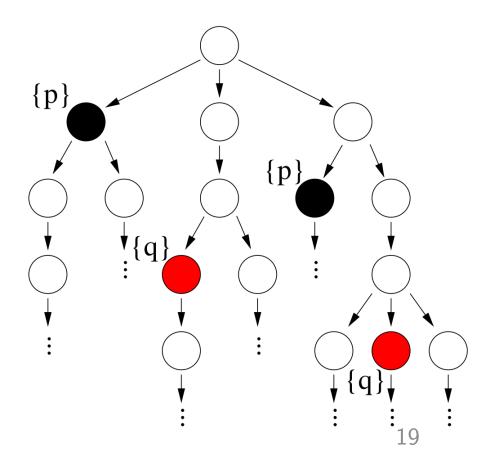


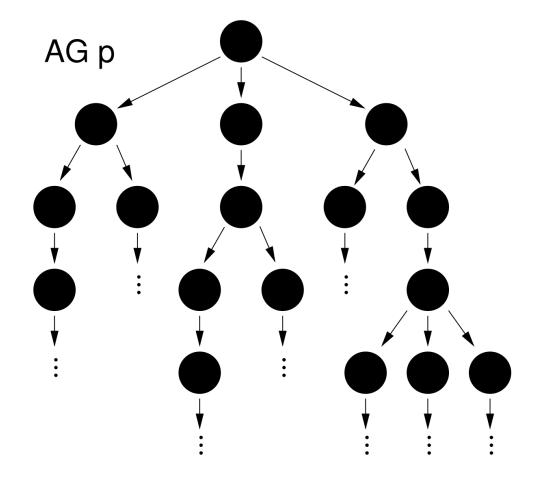
Automaton to work with

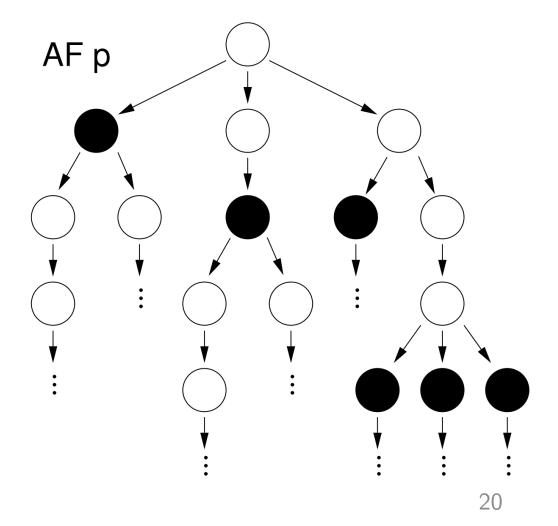
 We use this computation tree as a running example.

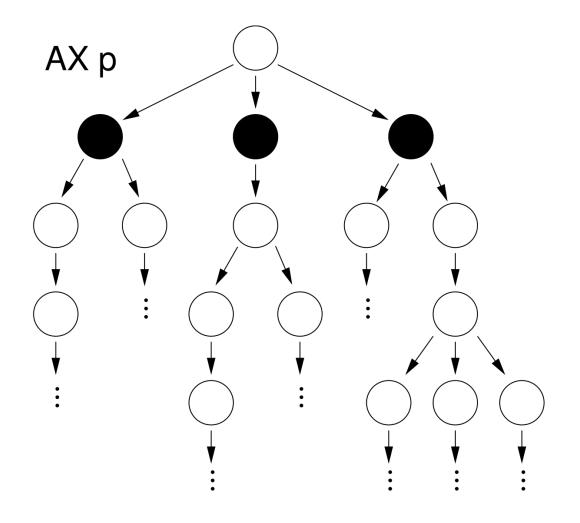
 We suppose that the black and red states satisfy atomic properties p and q, respectively.

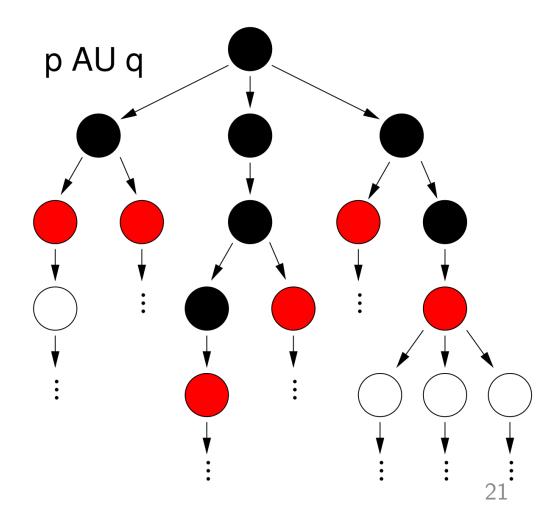
The topmost state is the initial state;
 in the examples, it always satisfies the given formula.

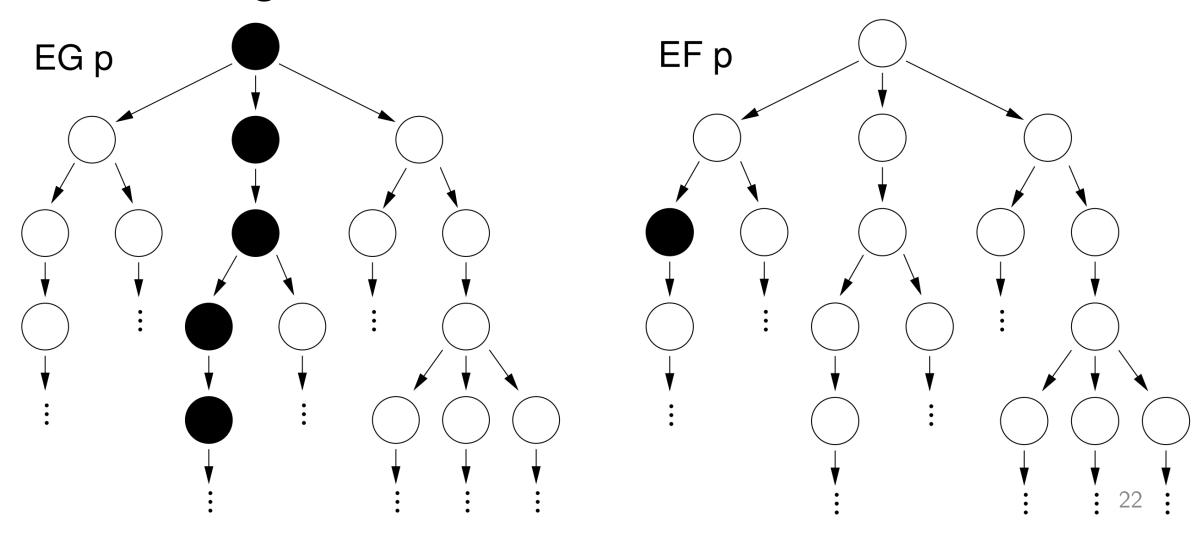


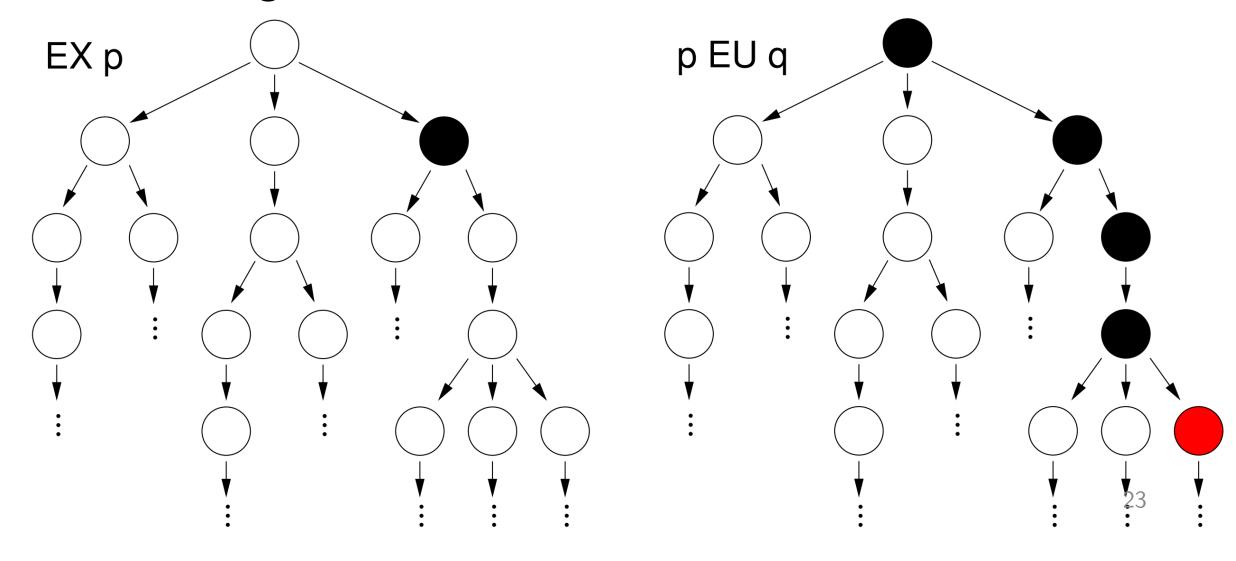




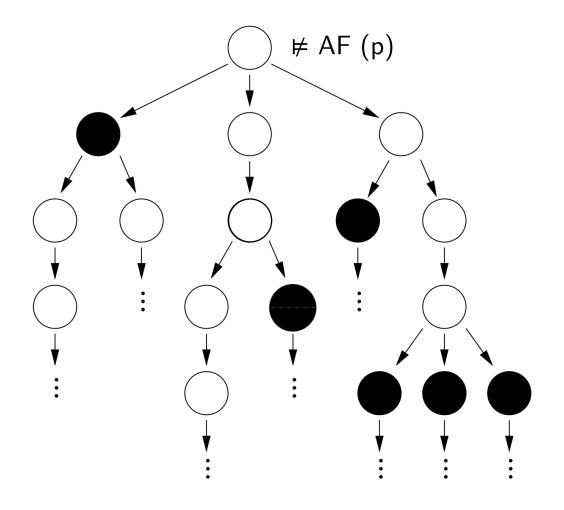


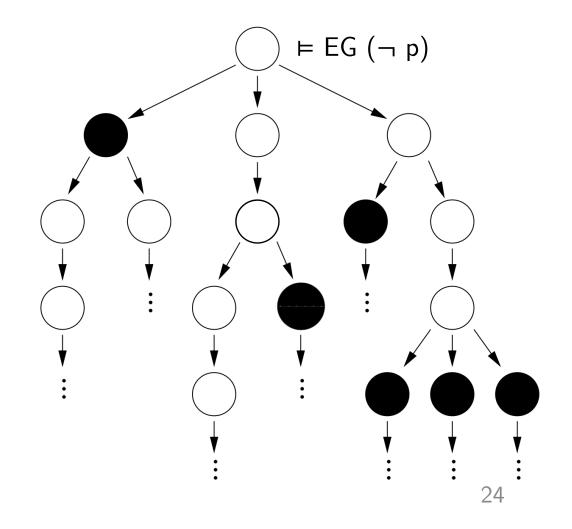






## Intuition for "AF $p = \neg EG (\neg p)$ "





## Interpreting CTL formula

Encoding	Proposition
р	I like chocolate
q	It's warm outside

- AG p
- EF p
- AF EG p
- EG AF p

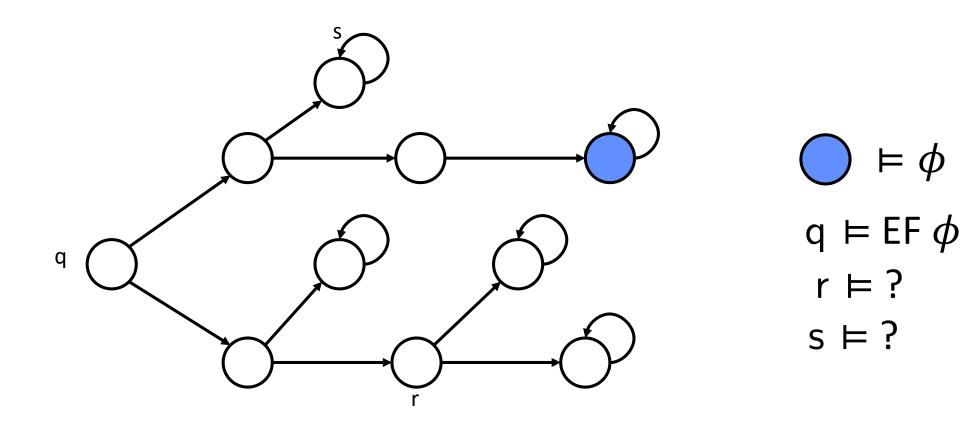
p AU q

## Interpreting CTL formula

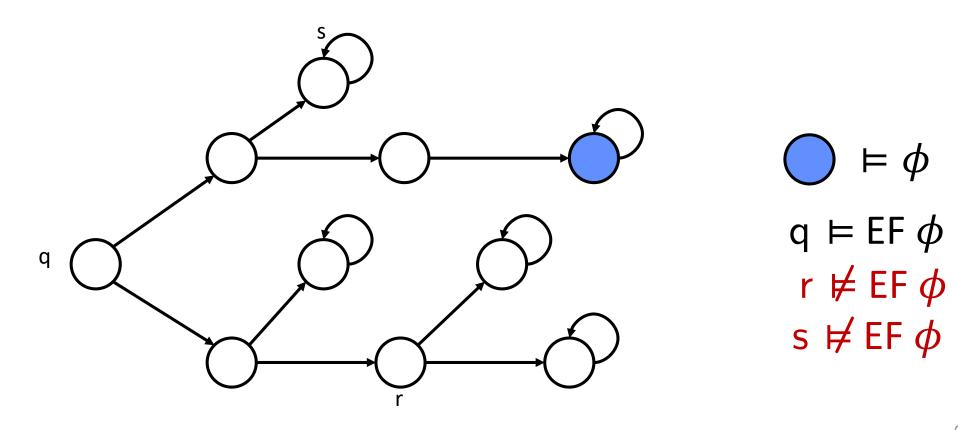
Encoding	Proposition
р	I like chocolate
q	It's warm outside

- AG p
   I will like chocolate from now on, no matter what happens.
- EF p It's possible I may like chocolate someday, at least for one day.
- AF EG p There will be always sometime in the future (AF) that I may suddenly start liking chocolate for the rest of time (EG).
- EG AF p This is a critical time in my life. Depending on what happens (E), it's possible that for the rest of time (G), there will always be some time in the future (AF) when I will like chocolate. However, if the wrong thing happens next, then all bets are off and there's no guarantee about whether I will ever like chocolate.
- P AU q No matter what happens, I will like chocolate from now on. But when it gets warm outside, I don't know whether I still like it. And it will get warm outside someday.

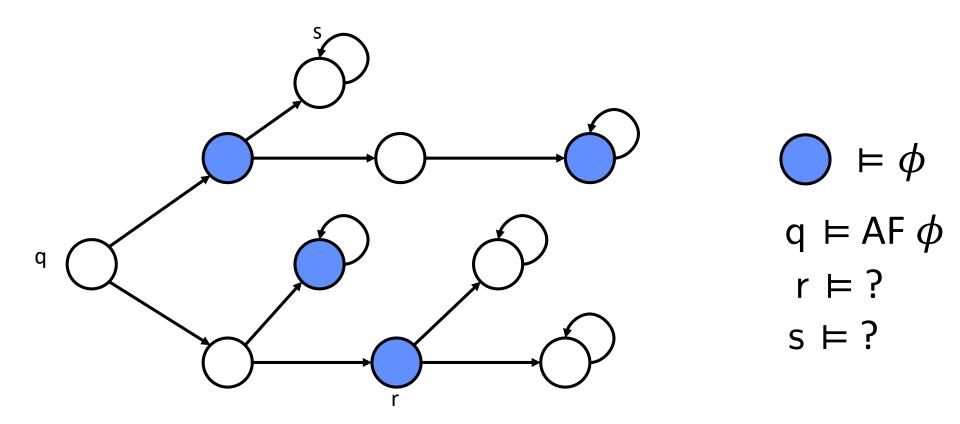
## EF $\phi$ : "There exists a path along which at some state $\phi$ holds."



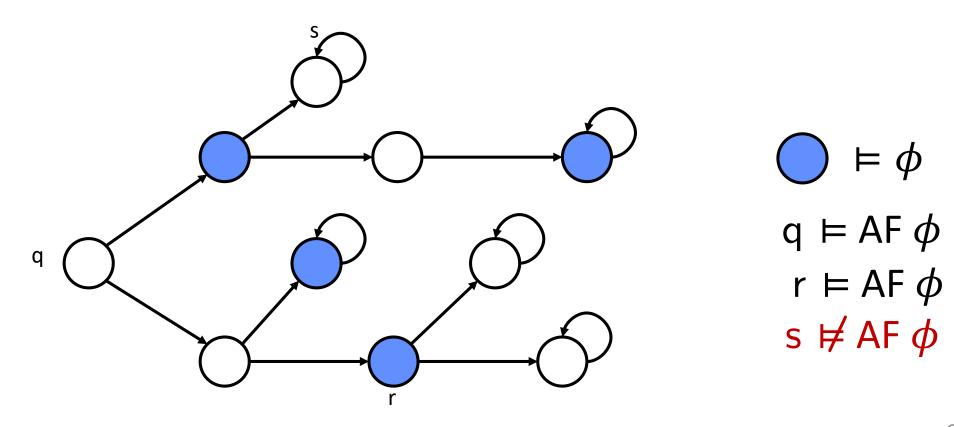
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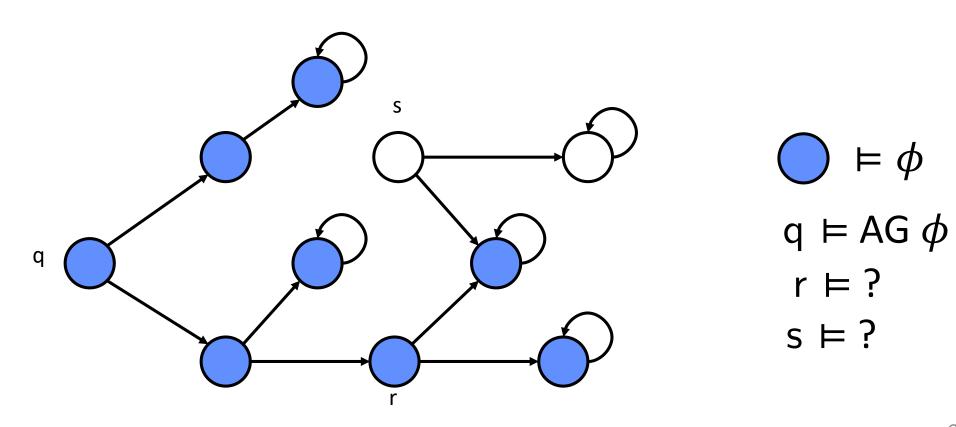
## $\mathsf{AF}\ \phi$ : "On all paths, at some state $\phi$ holds ."



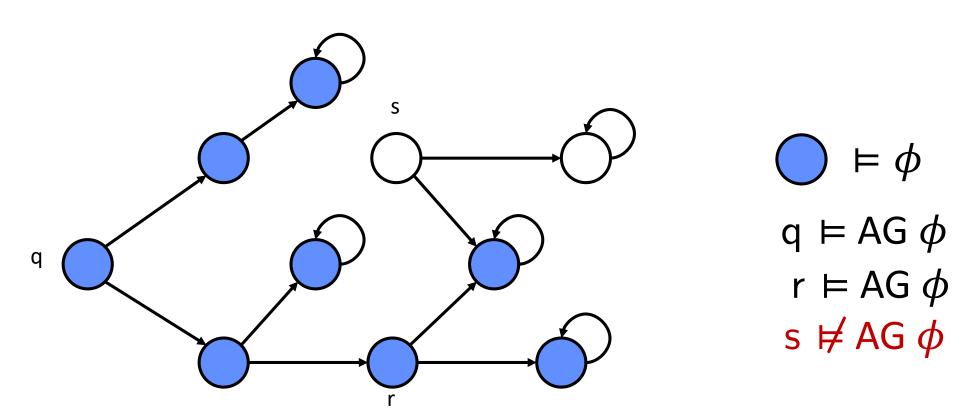
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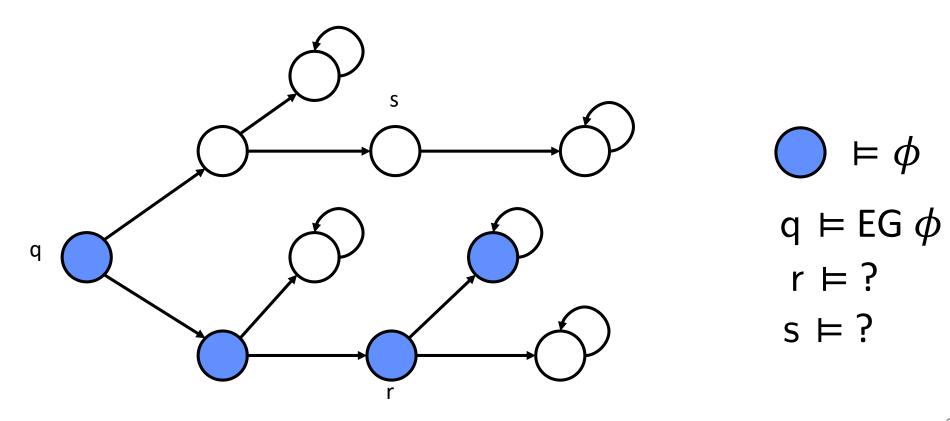
## $\mathsf{AG}\ \phi$ : "On all paths, for all states $\phi$ holds."



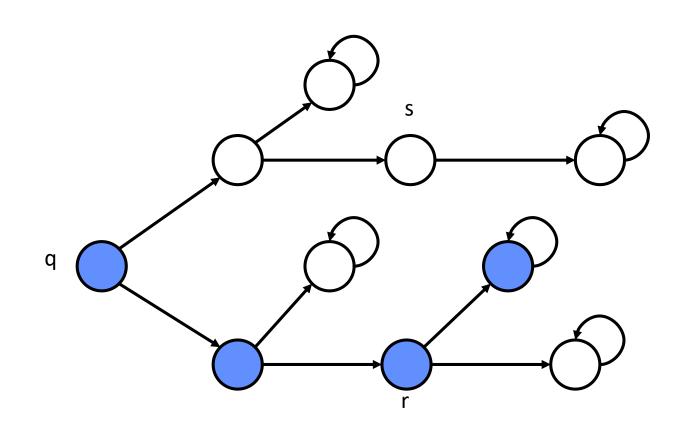
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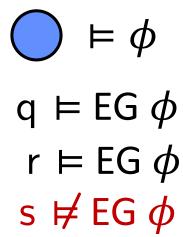


## EG $\phi$ : "There exists a path along which for all states $\phi$ holds."

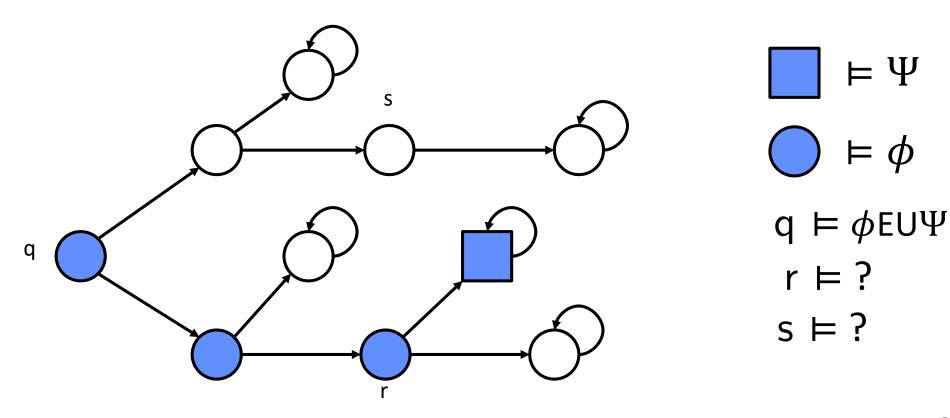


## EG $\phi$ : "There exists a path along which for all states $\phi$ holds."

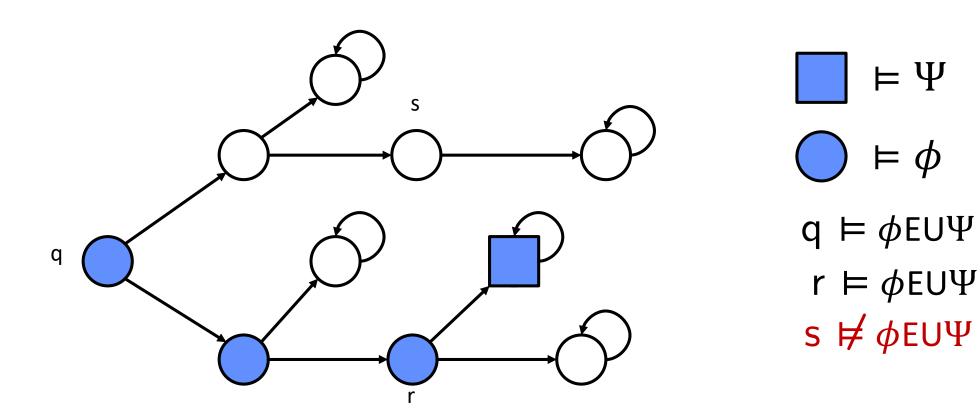




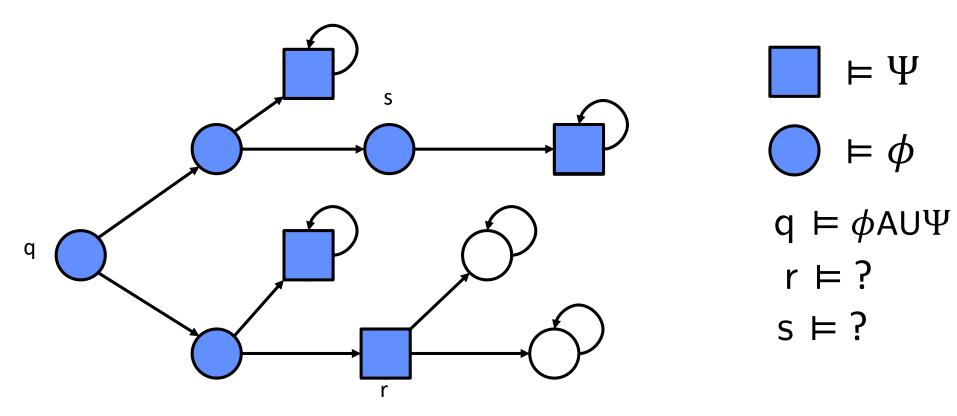
# φEUΨ: "There exists a path along which φ holds until Ψ holds."



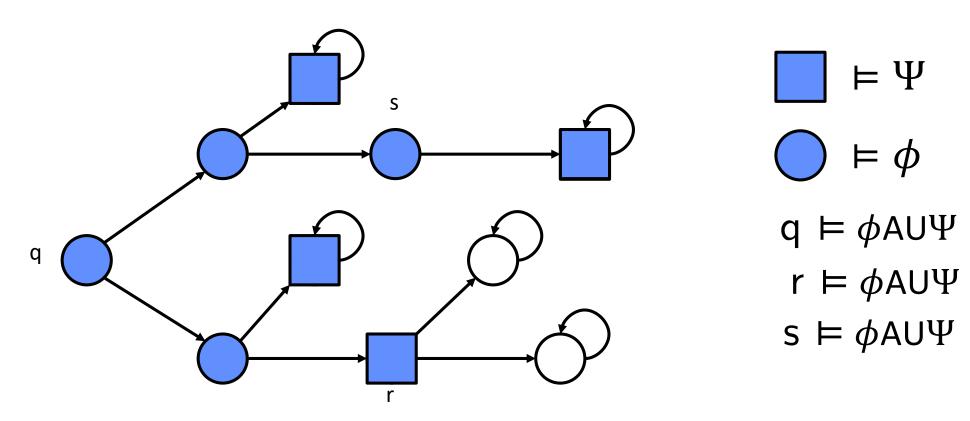
# $\phi$ EU $\Psi$ : "There exists a path along which $\phi$ holds until $\Psi$ holds."



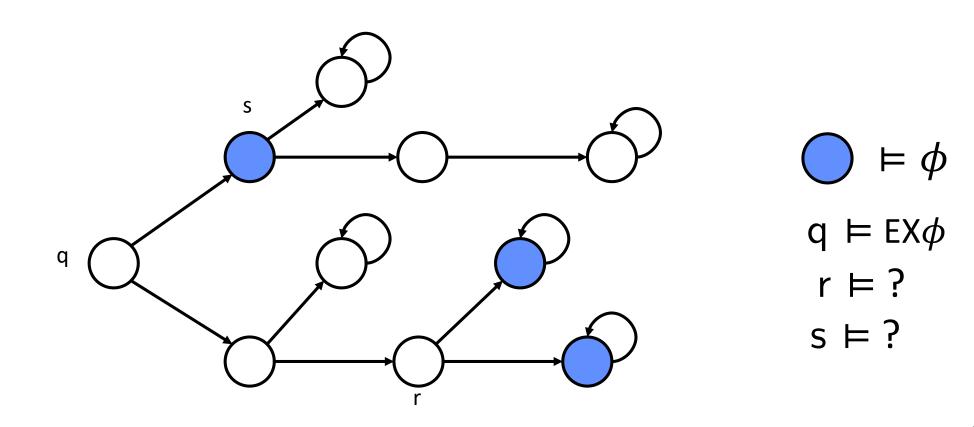
### $\phi AU\Psi$ : "On all paths, $\phi$ holds until $\Psi$ holds."



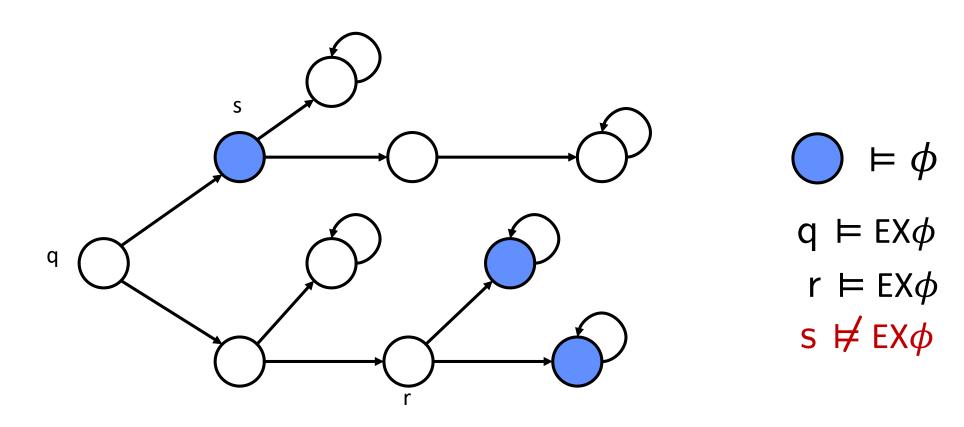
### $\phi$ AU $\Psi$ : "On all paths, $\phi$ holds until $\Psi$ holds."



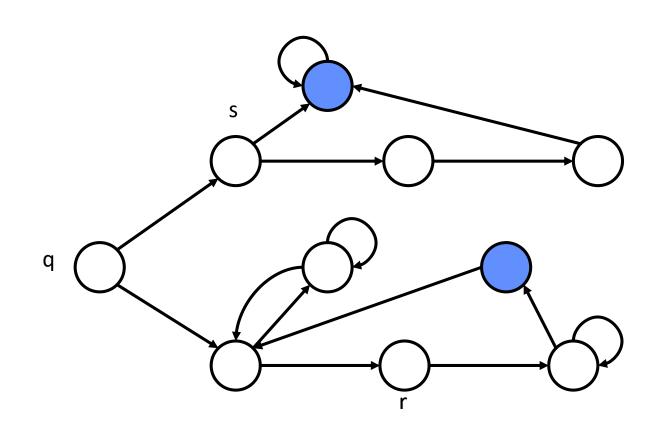
# $\mathsf{EX}\phi$ : "There exists a path along which the next state satisfies $\phi$ ."



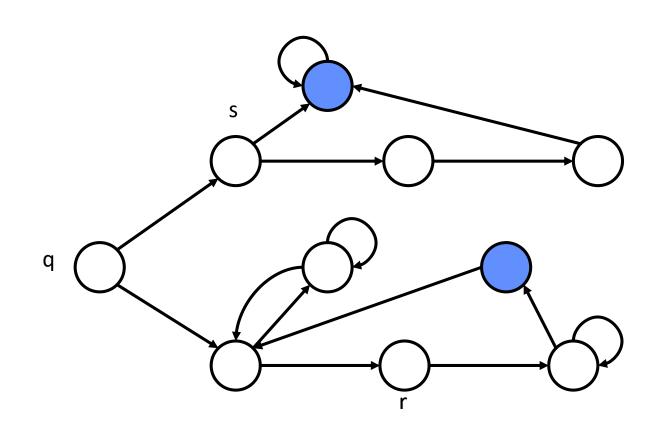
# $\mathsf{EX}\phi$ : "There exists a path along which the next state satisfies $\phi$ ."



# AG EF $\phi$ : "On all paths and for all states, there exists a path along which at some state $\phi$ holds."



# AG EF $\phi$ : "On all paths and for all states, there exists a path along which at some state $\phi$ holds."

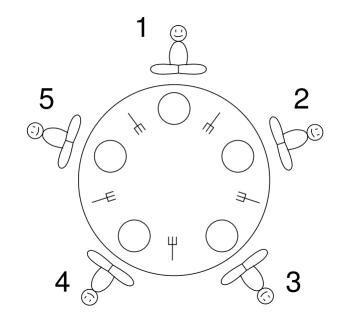


### Specifying using CTL formula

## Famous problem

#### Dining Philosophers

- Five philosophers are sitting around a table, taking turns at thinking and eating.
- Each needs two forks to eat.
- They put down forks.
   only once they have eaten.
- There are only five forks.



Atomic proposition

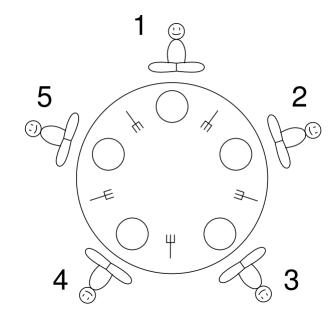
 $e_i$ : Philosopher i is currently eating.

### Specifying using CTL formula

"Philosophers 1 and 4 will never eat at the same time."

"Every philosopher will get infinitely many turns to eat."

"Philosopher 2 will be the first to eat."



#### Specifying using CTL formula

"Philosophers 1 and 4 will never eat at the same time."

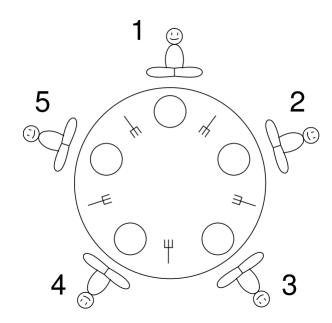
$$AG \neg (e_1 \cdot e_4)$$

"Every philosopher will get infinitely many turns to eat."

$$AG(AFe_1 \cdot AFe_2 \cdot AFe_3 \cdot AFe_4 \cdot AFe_5)$$

"Philosopher 2 will be the first to eat."

$$\neg (e_1 + e_3 + e_4 + e_5) \text{ AU } e_2$$



• In order to compute CTL formula, we first define  $[\![\phi]\!]$  as the set of all initial states of the finite automaton for which CTL formula  $\phi$  is true. Then we can say that a finite automaton with initial state  $q_0$  satisfies  $\phi$  iff

$$q_0 \in \llbracket \phi \rrbracket$$

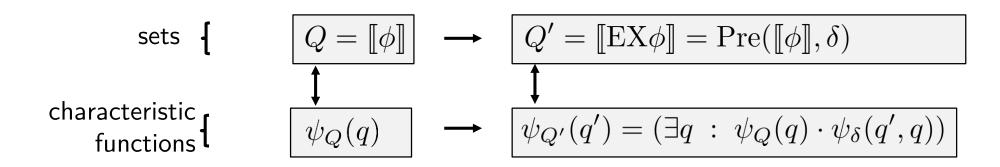
- Now, we can use our "trick": computing with sets of states!
  - $\psi_{\llbracket \phi \rrbracket}(q)$  is true if the state q is in the set  $\llbracket \phi \rrbracket$ , i.e., it is a state for which the CTL formula is true.
  - Therefore, we can also say

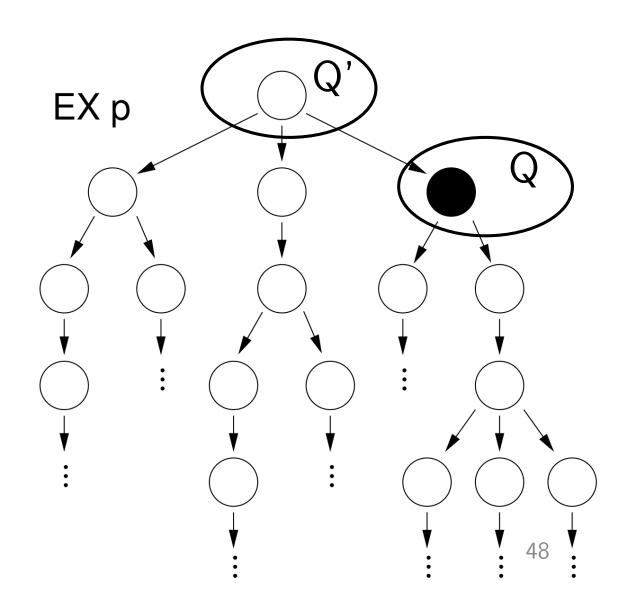
- When we compute the CTL-formulas, we start from the innermost terms.
- Remember: We suppose that every state has at least one successor state (could be itself).

- We now show how to compute some operators in CTL. All others can be determined using the equivalence relations between operators that we listed earlier.
  - EX  $\phi$ : Let us first define the set of predecessor states of Q, i.e., the set of states that lead in one transition to a state in Q:

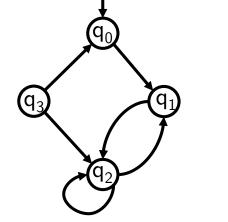
$$Q' = Pre(Q, \delta) = \{q' \mid \exists q : \psi_{\delta}(q', q) \cdot \psi_{Q}(q)\}$$

Suppose that Q is the set of initial states for which the formula  $\phi$  is true. Then we can write





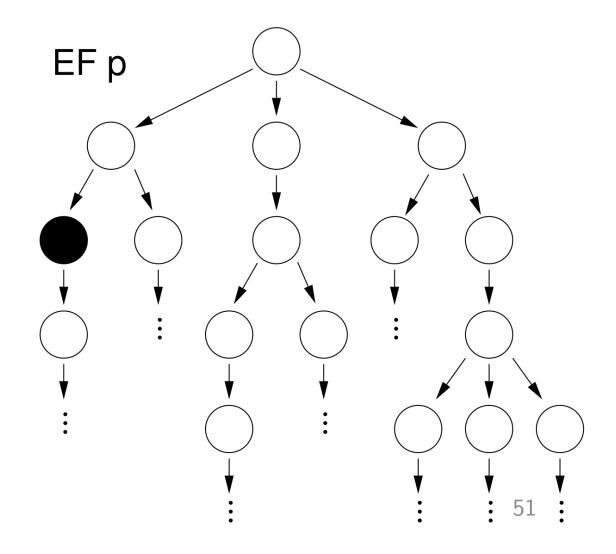
• Example for EX  $\phi$  : Compute EX  $q_2$ 



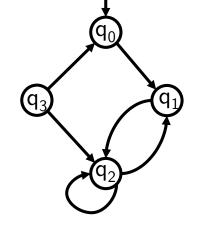
As  $q_0 \notin \llbracket EX \ q_2 \rrbracket = \{q_1, q_2, q_3\}$ , the CTL formula EX  $q_2$  is not true.

• EF  $\phi$ : The idea here is to start with the set of initial states for which the formula  $\phi$  is true. Then we add to this set the set of predecessor states. For the resulting set of states we do the same, ..., until we reach a fixed-point. The corresponding operations can be done using BDDs (as described before).

$$Q_0 = \llbracket \phi \rrbracket$$
 
$$Q_i = Q_{i-1} \cup \operatorname{Pre}(Q_{i-1}, \delta) \qquad \text{for all } i > 1 \text{ until a fixed-point } Q' \text{ is reached}$$
 
$$\llbracket \operatorname{EF} \phi \rrbracket = Q'$$



 $oldsymbol{\cdot}$  Example for EF $oldsymbol{\phi}$ : Compute EF  $q_2$ 



$$Q_0 = [\![q_2]\!] = \{q_2\}$$

$$Q_1 = \{q_2\} \cup \text{Pre}(\{q_2\}, \delta) = \{q_1, q_2, q_3\}$$

$$Q_2 = \{q_1, q_2, q_3\} \cup \text{Pre}(\{q_1, q_2, q_3\}, \delta) = \{q_0, q_1, q_2, q_3\}$$

$$Q_3 = \{q_0, q_1, q_2, q_3\} \cup \text{Pre}(\{q_0, q_1, q_2, q_3\}, \delta) = \{q_0, q_1, q_2, q_3\}$$

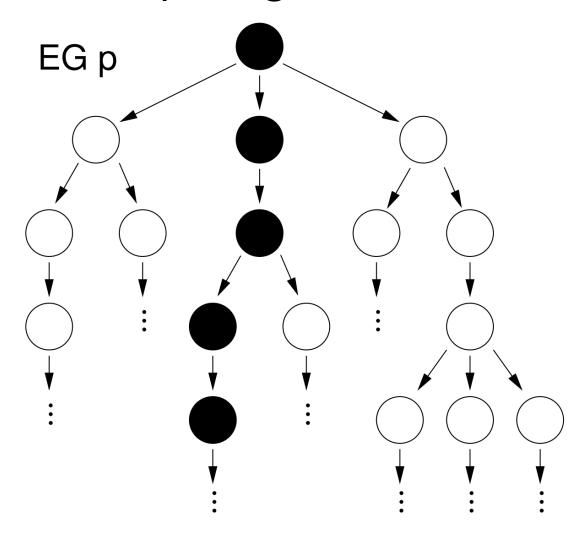
$$\llbracket EFq_2 \rrbracket = Q_3 = \{q_0, q_1, q_2, q_3\}$$

As  $q_0 \in \llbracket \mathrm{EF} q_2 
rbracket = \{q_0, q_1, q_2, q_3\}$ , the CTL formula EF  $\mathsf{q}_2$  is true.

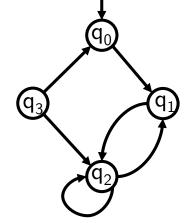
 $\{q' \mid \exists q \text{ with } \psi_Q(q) \cdot \psi_{\delta}(q',q)\} = \{q_1, q_2, q_3\}$ 

• EG  $\phi$ : The idea here is to start with the set of initial states for which the formula  $\phi$  is true. Then we cut this set with the set of predecessor states. For the resulting set of states we do the same, ..., until we reach a fixed-point. The corresponding operations can be done using BDDs (as described before).

$$Q_0 = \llbracket \phi \rrbracket$$
 
$$Q_i = Q_{i-1} \cap \operatorname{Pre}(Q_{i-1}, \delta) \quad \text{for all } i > 1 \text{ until a fixed-point is reached}$$



• Example for EG  $\phi$ : Compute EG  $q_2$ 



$$Q_0 = [q_2] = \{q_2\}$$
 $Q_1 = \{q_2\} \cap \text{Pre}(\{q_2\}, \delta) = \{q_2\}$ 
 $[EGq_2] = Q_2 = \{q_2\}$ 

As  $q_0 \not\in \llbracket \mathrm{EG} q_2 
rbracket = \{q_2\}$ , the CTL formula EG  $\mathsf{q}_2$  is not true.

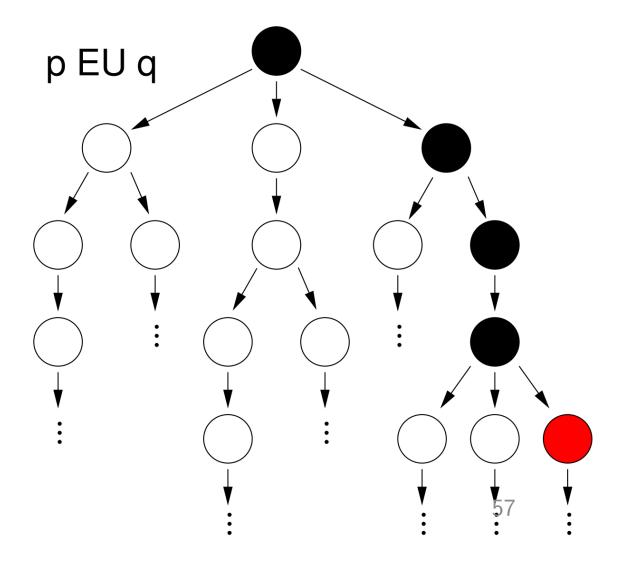
 $\{q' \mid \exists q \text{ with } \psi_Q(q) \cdot \psi_{\delta}(q',q)\} = \{q_1, q_2, q_3\}$ 

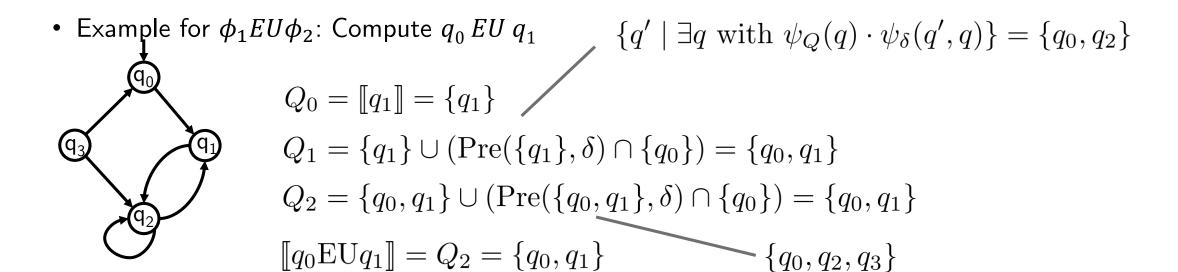
•  $\phi_1 E U \phi_2$ : The idea here is to start with the set of initial states for which the formula  $\phi_2$  is true. Then we add to this set the set of predecessor states for which the formula  $\phi_1$  is true. For the resulting set of states we do the same, ..., until we reach a fixed-point. The corresponding operations can be done using BDDs (as described before).

$$Q_0 = \llbracket \phi_2 \rrbracket$$

$$Q_i = Q_{i-1} \cup (\operatorname{Pre}(Q_{i-1}, \delta) \cap \llbracket \phi_1 \rrbracket) \quad \text{for all } i > 1 \text{ until a fixed-point is reached}$$

Like EF  $\phi_2$ , the only difference is that on our path backwards, we always make sure that also  $\phi_1$  holds.





As  $q_0 \in \llbracket q_0 \mathrm{EU} q_1 
rbracket = \{q_0, q_1\}$ , the CTL formula  $\mathsf{q_0}$  EG  $\mathsf{q_1}$  is true.

#### So... what is model-checking exactly?

Model-checking is an algorithm which takes two inputs ... Petri nets Kripke machine ... a DES model M CTL, LTL, ...

It explores the state space of M such as to either

- prove that  $M \models \phi$ , or
- return a trace where the formula does not hold in M.

Finite automato

#### So... what is model-checking exactly?

Petri nets Model-checking is an algorithm Kripke machine which takes two inputs a DES model **M** a formula  $\phi$ CTL, LTL, ...

It explores the state space of M such as to either

- prove that  $M \models \phi$ , or
- return a trace where the formula does not hold in M. a counter-example

- Extremely useful! Debugging the model
  - Searching a specific execution sequence

Finite automato

#### Let's see how it works in practice...

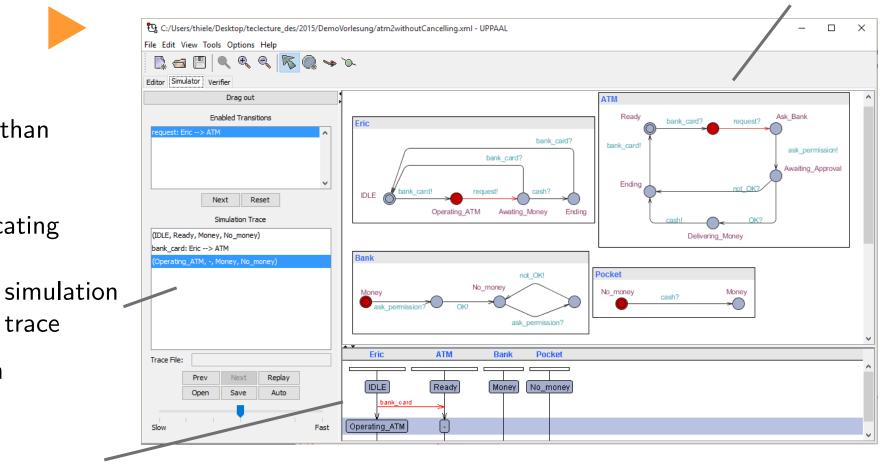
#### communicating finite automata

#### UPPAAL model-checker

- free for academia
- (much) more general than what we show here
- can verify the timed behavior of communicating finite automata

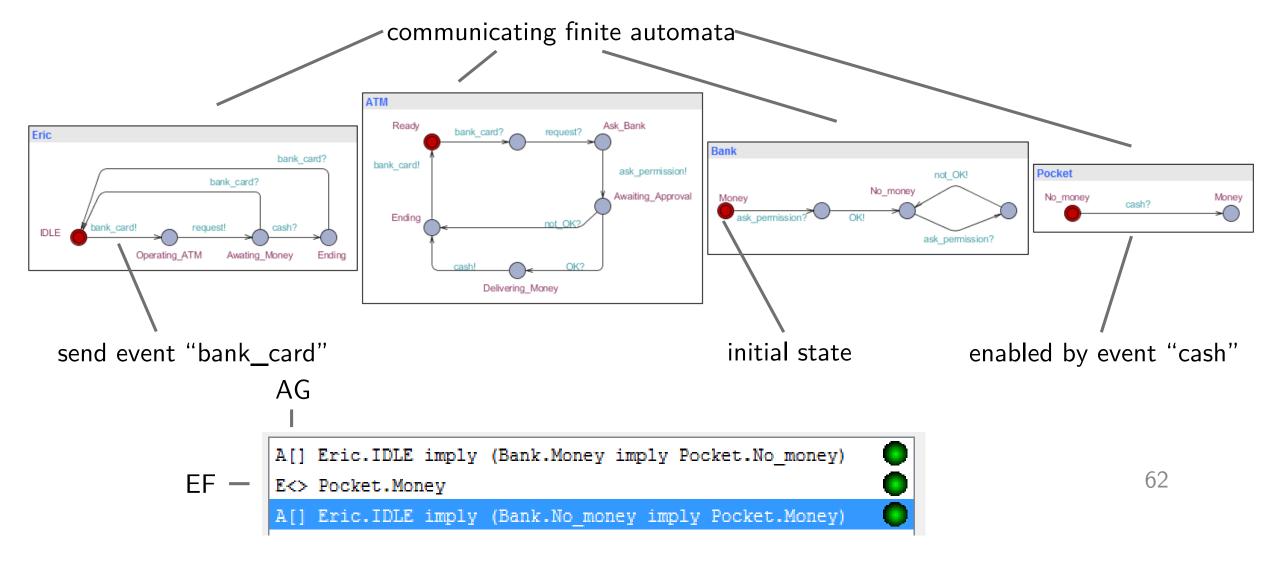
#### Example

Modeling and verification of a simple protocol for ATM-Money-Withdrawal

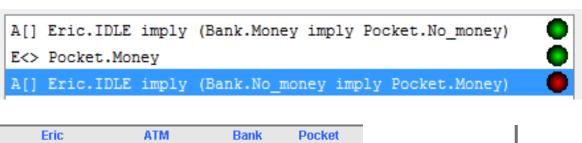


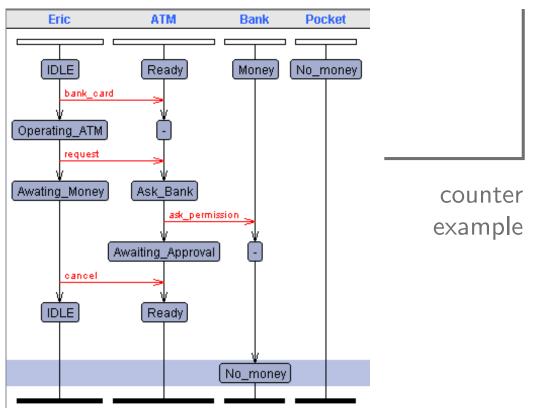
trace

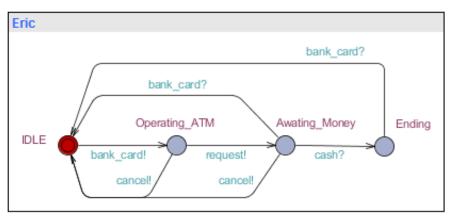
### Step 1. ATM without Cancel

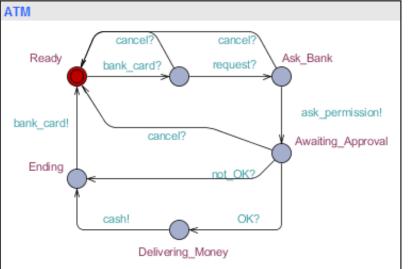


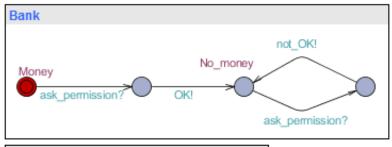
### Step 2. ATM with Cancel











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# Your turn to practice! after the break

- 1. Familiarise yourself with CTL logic and how to compute sets of states satisfying a given formula
- 2. Convert a concrete problem into a state reachability question (adapted from state-of-the-art research!)

#### Conclusion and perspectives

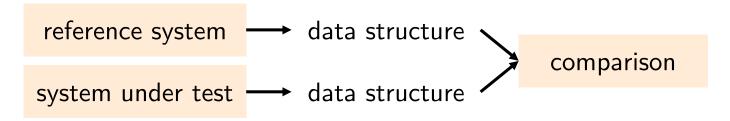
#### Example

$$y = (x_1 + x_2) \cdot x_3$$

$$x_1 \circ \longrightarrow + \longrightarrow y$$

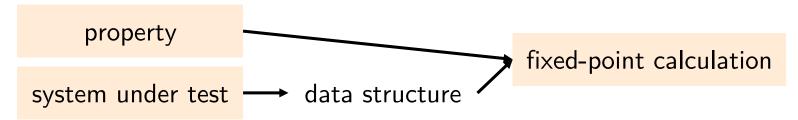
$$x_3 \circ \longrightarrow + \longrightarrow y$$

Comparison of specification and implementation



"The device can always be switched off."

Proving properties



#### Conclusion and perspectives

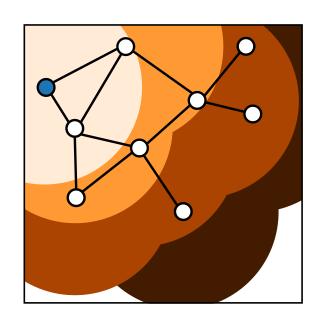
Next week(s) Petri Nets

- asynchronous DES model
- tailored model concurrent distributed systems
- capture an infinite state space with a finite model

How they work? How to use them for modeling systems? How to verify them? a computer

a network

#### See you next week! in Discrete Event Systems



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December 2, 2021

Most materials from Lothar Thiele