Discrete Event Systems

Exercise Session 1

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Formal Definition of a Finite Automaton

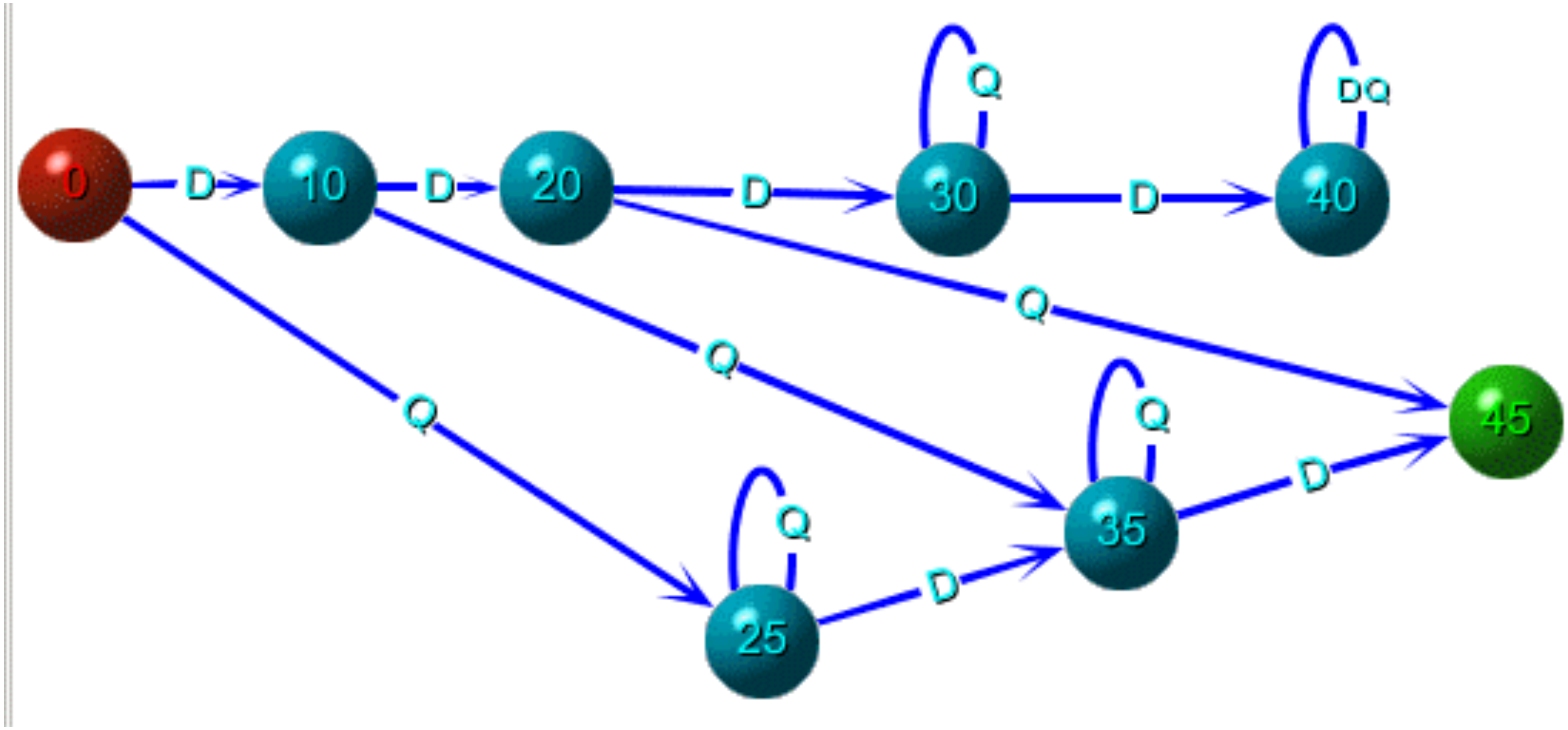
A finite automaton (FA) is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$,where

- Q is a finite set called the states
- Σ is a finite set called the alphabet
- $\delta: Q \times \Sigma \rightarrow Q$ is the transformation function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states (a.k.a. final states).

Vending Machine Java Code

```
Soda vend(){
   int total = 0, coin;
   while (total != 45){
       receive(coin);
       if ((coin==10 && total==40)
       ||(coin==25 && total>=25))
               reject(coin);
       else
               total += coin;
   }
   return new Soda();
}
```

Vending Machine "Logics"



Cartesian Product Construction

- We want to construct a finite automaton M that recognizes any strings belonging to L_1 or L_2.

- Idea: Build M such that it simulates *both* M_1 and M_2 simultaneously and accept if either of the automatons accepts

Formal Definition

- Given two automata
  $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

- Define the unioner of M_1 and M_2 by:
  $M_\cup = (Q_1 \times Q_2, \Sigma, \delta_1 \times \delta_2, (q_1, q_2), F_\cup)$

  - where the accept state (q_1, q_2) is the combined start state of both automata

  - where $F_\cup$ is the set of ordered pairs in $Q_1 \times Q_2$ with at least one state an accept state. That is: $F_\cup = Q_1 \times F_2 \cup F_1 \times Q_2$

  - where the transition function δ is defined as
  $\delta\big((q_1, q_2), j\big) = \big(\delta_1(q_1, j), \delta_2(q_2, j)\big) = \delta_1 \times \delta_2$

Other constructions: Intersector

- Other constructions are possible, for example an intersector:

- Accept only when both ending states are accept states. So the only difference is in the set of accept states. Formally the intersector of M_1 and M_2 is given by $M_\cap = (Q_1 \times Q_2, \Sigma, \delta_1 \times \delta_2, (q_{0,1}, q_{0,2}), F_\cap)$, where $F_\cap = F_1 \times F_2$.

Complement

- How about the complement? The complement is only defined with respect to some universe.
- Given the alphabet Σ, the *default universe* is just the set of all possible strings Σ^*. Therefore, given a language L over Σ, i.e. $L \subseteq \Sigma^*$ the complement of L is $\Sigma^* - L$
- Note: Since we know how to compute set difference, and we know how to construct the automaton for Σ^* we can construct the automaton for $\overline{L}$.
- Question: Is there a simpler construction for $\overline{L}$?
- Answer: Just switch accept-states with non-accept states.