

Computational Thinking

Exercise 3

1 Hamiltonian

Consider a graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. A Hamiltonian path in G is a path that visits every vertex in V exactly once. Similarly, a Hamiltonian cycle is a cycle that visits every vertex of the graph exactly once. The decision problems *Hamiltonian Path* and *Hamiltonian Cycle* ask whether such a path or cycle exists in a given graph.

- a) Show that Hamiltonian Cycle \leq Hamiltonian Path.
- b) Show that Hamiltonian Path \leq Hamiltonian Cycle.

2 Circuit Complexity

In this task, we revisit the complexity classes AC^i and NC^i that were discussed in the lecture (check Definition 2.43 and the following remark in the lecture notes.) Let us first look into how these classes relate to each other.

- a) Show that $NC^0 \subset AC^0 \subset NC^1 \subset AC^1 \dots$

Consider now the parity problem:

Definition 2.1 (PARITY) *Given a binary string, decide if it contains an even number of 1s.*

- b) Show that PARITY is in NC^1 .
- c) Show that PARITY is not in NC^0 .

Next, we will consider the complexity of the binary sum problem.

Definition 2.2 (SUM) *Given two non-negative integers as binary strings of length n , compute their sum as a binary string of length $n + 1$.*

We will assume that the input to a circuit is the two strings concatenated.

- d) Show that SUM is in AC^0 . What is the size of the circuit that you found?
- e) Show that SUM is not in NC^0 .