



# Computational Thinking

## Exercise 10

### 1 Global Minimum

You want to find the global minimum of  $f$  using gradient descent, where  $f = 3x^4 - 4x^3 - 12x^2 + 4$

- a) Given a small enough learning rate, for what range of initial values  $x_0$  will gradient descent converge to the global minimum?
- b) Let  $x_0 = 3$ . What is the optimum learning rate to reach the global minimum in the least number of steps?
- c) Does Newton's method use the optimum learning rate? Why (not)?
- d)\* What about if  $f = ax^2 + bx + c$  with  $a > 0$  and we have an arbitrary starting point  $x_0$ ?

## 2 Logistic Regression & XOR

We want to learn the “XOR” function with logistic regression. Our input space is  $\mathcal{X} = \{0, 1\}^2$  and our output space is  $\mathcal{Y} = \{0, 1\}$  and we want to learn the mapping

$$(x_1, x_2) \mapsto x_1 \oplus x_2$$

- a) Why can logistic regression not learn “XOR”?
- b) Show that logistic regression can learn “XOR” by manually adding features.
- c) How about “AND”, “OR”, “NOT AND”? Can logistic regression learn these?
- d) Show that “hierarchical” logistic regression with 2 layers can learn “XOR”. What does this remind you of?
- e) How about a decision tree, can it learn “XOR”?

### 3 Gini Impurity

**Definition 10.1** (Classification splitting criterion: Gini Impurity). *For node  $v$  containing samples  $D_v$  from  $k$  classes, the gini measure of impurity is defined as:*

$$G = 1 - \sum_{i=1}^k p_i^2$$

where

$$p_i = \frac{|\{\mathbf{x} \in D_v \mid f(\mathbf{x}) = i\}|}{|D_v|}$$

is the fraction of samples within  $D_v$  that belongs to class  $i$ .

Take a look at this data!

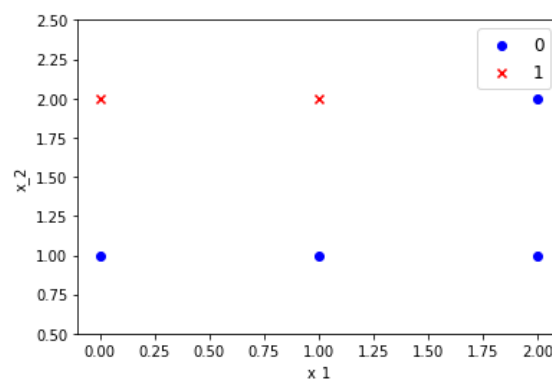


Figure 1: Some binary data

- Construct an optimal decision tree (requiring the minimum number of splits).
- Show that we find an optimal decision tree by using the CART loss function with Gini impurity.
- Give an example dataset, where CART with Gini does not find an optimal decision tree.