

Discrete Event Systems

Exercise Sheet 1

General Instructions

- This year, all the exercise sessions will be held virtually as Zoom meetings. Please connect to the meeting on time, *i.e.* at 4.15pm, just after the lecture.
- For each exercise, we will give you some time to solve it. Once you have it solved, raise your virtual hand in Zoom. When many of you have solved the exercise, or after a certain time, we will start the discussion and present a correct solution.
- Despite the session being virtual, we aim to make it interactive and ask one of you to volunteer and present the exercise “in front of” the class. To do that, we recommend that you share your screen via Zoom and explain your solution.
- In this exercise session, you will often have to draw automata. There are several online tools available to draw automata, *e.g.* <http://madebyevan.com/fsm/>. Alternatively, you may use any online whiteboard/drawing tool of your choice or draw the solution on a sheet of paper. In that case, if you present your solution to the exercise, just take a picture of your answer and show it on your screen (shared via Zoom).
- During the discussion/correction, we may annotate your answer on Zoom. Please do **not** annotate anything yourself unless we explicitly ask you to do so. After the session, we will make the sample solutions for all exercises available on the website.
- For questions during the exercise session, we recommend using the Zoom chat. You can write the question in the public chat, or you can send it privately to a TA. We will try to answer questions as soon as possible; if we can't answer a question during the exercise session, please send us an email. Note that public questions will be answered with priority.

1 Finite Automata

In this exercise you are asked to design your first finite automata. Try to minimize the number of states of your machines.

- a) Consider the alphabet $\{0,1\}$. Implement an automaton which accepts the following strings:
At first there are zero or more '1's, followed by zero or more '0's. This in turn is followed by an arbitrary (non-zero) number of '1's.
- b) Design an automaton which decides whether a number is divisible by three. Assume that the digits of the number are inserted sequentially, that is, the number '135' is inserted as '1', '3' and finally '5'. How many states do you need? (Hint: Cross sum!)

2 Vending Machine Revisited

Consider the vending machine shown at the beginning of the course. Do you see any problems with this machine? How can the machine be made more user-friendly?

3 “Mais im Bundeshuus”

It is Wednesday morning and the seven members of the Swiss Federal Council meet to decide about an important topic: In order to decrease the expenses of education, should only women be allowed to study at ETH?

- a) Assume that the seven members vote one after another. Further, assume that there is no abstention of voting. Design an automaton which accepts the ballot if and only if the majority of the members voted in favor of the proposition.
- b) Extend your automaton for the case of abstentions of voting. The automaton should accept if and only if more members voted in favor of the proposition than against it.

4 Build Finite Automata with the Construction Rules

Consider the alphabet $\{a,b\}$. For each of the following languages, implement an automata that accepts it using the constructions seen in the lecture (*e.g.*, the cartesian product construction).

- a) $\{w \mid w \text{ has exactly two } a\text{'s and at least two } b\text{'s}\}$
- b) $\{w \mid w \text{ does not contain string } baba\}$

5 Exam question [2015]

Consider the alphabet $\Sigma = \{0,1\}$ of binary strings representing numbers (with the most significant bit to the left). Draw a DFA which accepts all the strings that are divisible by 5. Strings are read from left to right. For example, the strings 0, 101, 1010 and 01010 would be accepted while 01, 10 would not. Observe that the empty string, ε , should not be accepted either.

Hint: Consider relying on the modulo operator (mod) which returns the remainder of the division of one number by another. For instance, $1 \bmod 5 = 1$ and $2 \bmod 5 = 2$.

Reminder: A bit shift to the left doubles the value of a binary number.

6 Filter for an Input Stream

We would like to construct an automaton that recognizes substrings from an input stream. The input stream consists of symbols $\{a,b\}$ and the substrings that the automaton should detect are of the form bab^* . In other words, the input of the automaton is a series of a 's and b 's. The automaton should go into an accepting state whenever the most recently received symbols form a string of the form bab^* . For example, in the input stream $b \underline{a} \underline{b} \underline{b} \underline{b} \underline{a} a a a b \underline{a} \underline{b} \underline{a} a$, the automaton should be in an accepting state exactly after the reception of an underlined symbol. Construct a deterministic finite automaton that precisely fulfils the above specification.