





HS 2021

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Computational Thinking Exercise 9

1 Linear Regression

Here is a dataset D with 3 samples. You want to fit a linear model of the form $\hat{f}(x) = w_0 + w_1 x$.

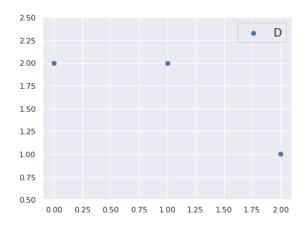


Figure 1: A dataset with 3 samples.

- a) Which weights minimize the squared error loss? What is the total absolute error? What is the total squared error?
- **b)** Which weights give a lower absolute error loss? What is the total absolute error? What is the total squared error?

2 Polynomial Regression

Here is a function. You have sampled a set of training data points with noise as shown by the dots.

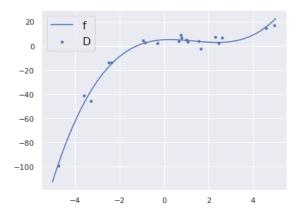


Figure 2: A function and some training data D.

Which model will give you the lowest bias:

- **a**) $\hat{f} = 3$
- **b**) $\hat{f} = w_0$
- c) $\hat{f} = w_0 + w_1 x$
- **d)** $\hat{f} = w_0 + w_1 x + w_2 x^2$
- e) $\hat{f} = w_0 + w_1 x + w_2 x^2 + w_3 x^3$

Which model will give you the lowest variance:

- a) $\hat{f} = 3$ b) $\hat{f} = w_0$ c) $\hat{f} = w_0 + w_1 x$ d) $\hat{f} = w_0 + w_1 x + w_2 x^2$
- e) $\hat{f} = w_0 + w_1 x + w_2 x^2 + w_3 x^3$

3 Ridge Regression

In the lecture we saw that linear regression without regularization has a closed form solution:

$$\boldsymbol{w}^* = \left(\mathbf{X}^T \mathbf{X} \right)^{-1} \mathbf{X}^T \boldsymbol{y}.$$

Recall from the lecture that Ridge regression minimizes:

$$\min_{\boldsymbol{w}} \left\{ \frac{1}{n} \sum_{(\boldsymbol{x}, y) \in D} (y - \boldsymbol{w}^T \boldsymbol{x})^2 + \lambda \sum_{i=0}^{d-1} w_i^2 \right\}$$

a) By differentiating the loss function, show that Ridge regression has the following closed-form solution:

$$\boldsymbol{w}_{ridge}^{*} = \left(\mathbf{X}^{T} \mathbf{X} + \lambda n \mathbf{I} \right)^{-1} \mathbf{X}^{T} \boldsymbol{y}$$

where **I** is the $d \times d$ identity matrix.

- **b)** What happens to the weights \boldsymbol{w}^*_{ridge} in the limit as $\lambda \to \infty$?
- c) What happens to the weights \boldsymbol{w}^*_{ridge} in the limit as $\lambda \to 0$?

4 Rescaling

Suppose we have a dataset D with 1000 samples and 100 features $\{x_1, x_2, \ldots, x_{100}\}$. Now, we rescale one of these feature by multiplying with 10 (say that feature is x_1).

- a) Show that the OLS weights remain unchanged for i > 1, and that $w_1^{*'} = \frac{1}{10} w_1^*$
- b) Conclude that the OLS predictions do not change
- c) What about with Lasso and Ridge regression? Do the weights change? Do the predictions change?