



Computational Thinking

Exercise 5 (Cryptography)

1 Nonce Reuse

In the ElGamal digital signature scheme, why should the same random nonce never be reused for 2 different messages with the same public/secret keypair?

2 Cryptographic Hash Functions

Let $h_1, h_2 : \{0, 1\}^* \rightarrow \{0, 1\}^n$ be two collision resistant functions. Are the following hash functions also collision resistant? Explain¹.

- $h_3(x) = h_1(x) \oplus h_2(x)$
- $h_4(x) = x_0; h_1(x)$

Hint: Try to find a collision or reduce the collision-resistance of the constructed hash functions to collision-resistance of h_1 and h_2 .

3 IND-CPA

Let $h : \{0, 1\}^* \rightarrow \{0, 1\}^n$ be a collision resistant hash function, and let (Generate, Encrypt, Decrypt) be a correct and IND-CPA secure PK encryption scheme, as defined in the lecture. We define another PK encryption scheme (Generate', Encrypt', Decrypt') as follows:

- $\text{Generate}'() = \text{Generate}() = (k_p, k_s)$
→ that is, the keys are generated in the same way
- $\text{Encrypt}'(m, k_p) = (c_1, c_2) = (h(m), \text{Encrypt}(m, k_p))$
→ In other words, $h(m)$ is appended to the encrypted message.
- $\text{Decrypt}'((c_1, c_2), k_s) = \text{Decrypt}(c_2, k_s)$

- a) Show that the new scheme is a correct encryption scheme. That is, show that for any m $\text{Decrypt}'(\text{Encrypt}'(m, k_p), k_s) = m$.
- b) Show that (Generate', Encrypt', Decrypt') is not IND-CPA secure.

Hint: Think about the IND-CPA game and how can the adversary win with non-negligible probability.

¹ x_0 means the first bit of the message x , and as in the lecture, concatenation of messages is denoted by ;