ETH
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Distributed Computing

HS 2020

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Computational Thinking
Exercise 3

1 Hamiltonian

Consider a graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. A Hamiltonian path in G is a path that visits every vertex in V exactly once. Similarly, a Hamiltonian cycle is a cycle that visits every vertex of the graph exactly once. The decision problems *Hamiltonian Path* and *Hamiltonian Cycle* ask whether such a path or cycle exists in a given graph.

a) Show that Hamiltonian Cycle $\leq$ Hamiltonian Path.

b) Show that Hamiltonian Path $\leq$ Hamiltonian Cycle.

2 Circuit Complexity

In this task, we revisit the complexity classes AC^i and NC^i that were discussed in the lecture (check Definition 2.43 and the following remark in the lecture notes.) Let us first look into how these classes relate to each other.

a) Show that $\mathsf{NC}^0 \subset \mathsf{AC}^0 \subset \mathsf{NC}^1 \subset \mathsf{AC}^1 \ldots$.

Consider now the parity problem:

Definition 2.1 (PARITY) *Given a binary string, decide if it contains an even number of* 1*s.*

b) Show that PARITY is in NC^1.

c) Show that PARITY is not in NC^0.

Next, we will consider the complexity of the binary sum problem.

Definition 2.2 (SUM) *Given two non-negative integers as binary strings of length n, compute their sum as a binary string of length $n + 1$.*

We will assume that the input to a circuit is the two strings concatenated.

d) Show that SUM is in AC^0. What is the size of the circuit that you found?

e) Show that SUM is not in NC^0.