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Computational Thinking
Exercise 14

1 PCP warm-up

Do the following PCPs have a solution?

a) Domino set $\left[\frac{a}{aaab}\right], \left[\frac{abba}{ba}\right], \left[\frac{aa}{aba}\right], \left[\frac{bbab}{bb}\right]$.

b) Domino set $\left[\frac{ab}{abb}\right], \left[\frac{aaba}{abb}\right], \left[\frac{baa}{aaa}\right]$.

c) Domino set $\left[\frac{abbb}{b}\right], \left[\frac{b}{bca}\right], \left[\frac{cac}{ca}\right], \left[\frac{aa}{cb}\right], \left[\frac{bb}{bbb}\right]$.

d) Domino set $\left[\frac{ad}{dda}\right], \left[\frac{bc}{ca}\right], \left[\frac{c}{a}\right], \left[\frac{d}{db}\right], \left[\frac{ab}{bc}\right]$.

2 PCP variants

Are the following variants of the PCP problem decidable or undecidable?

a) ab^* PCP: each word α and each word β has the following form: it starts with a single letter a, and then an arbitrary number of letters b. Some examples for valid words are a, abb or $abbbbbb$.

b) Limited-use PCP: given an integer parameter k in the input, we only accept domino sequences that contain each domino at most k times.

c) Unique-triplet PCP: we only accept domino sequences where no consecutive triplet of dominoes appears two times, i.e. there are no distinct indices i, j such that each of the following three pairs of dominoes are the same: those at positions i and j, those at positions $(i+1)$ and $(j+1)$, and those at positions $(i+2)$ and $(j+2)$.

d) Two-color PCP: besides the two words (α, β), dominoes also have a color: each domino is painted red or blue. We only accept domino sequences that are alternating, i.e. a red domino is always followed by a blue domino, and vice versa.

e) Half-used PCP: given the input set of dominoes S, we only accept domino sequences that use at most half of the domino types (possibly with repetitions), i.e. there are at least $\frac{1}{2} \cdot |S|$ input dominoes that never occur in the sequence.

f) Silly PCP: for each domino (α, β) of the input set, the two words have the same length, i.e. we have $|\alpha| = |\beta|$.

g) Almost-silly PCP: for some constant integer $c > 1$, the length of each word α and each word β has to be a multiple of c.

h) Binary PCP: the size of the alphabet is restricted to two characters, i.e. $\Sigma = \{0, 1\}$.