Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Computer Engineering and
Networks Laboratory
——— TEC, NSG, DISCO ———

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Discrete Event Systems

Exam

Friday, 3rd February 2017, 14:00–16:00.

Do not open until told to by the supervisors!

The exam lasts 120 minutes, and there is a total of 120 points. The maximal number of points for each question is indicated in parentheses. Your answers must be in English. Be sure to always justify your answers.

Give your solutions on the page corresponding to the exam question and/or the empty one(s) following it. In case you run out of space, we also added extra pages – in case you use them, please indicate which question you are solving! Should even this not be enough, please contact a supervisor.

Please write down your name and Legi number (student ID) in the boxes below. Once the exam starts, also write your name on every page in the top right corner.

Name	**Legi-Nr.**

Points

Question	**Topic**	**Achieved Points**	**Maximal Points**
1	Regular Languages		25
2	Context-free Languages		15
3	Electronics Store		15
4	Online Gas Station Search		25
5	True or False – Automata & Petri Nets		10
6	Binary Decision Diagram		10
7	Petri Net		20
Total			**120**

1 Regular Languages (25 points)

1.1 Draw an automaton [9 points]

Consider the alphabet $\Sigma = \{0, 1\}$. For each of the following languages, draw a DFA recognizing it:

a) [2] all the strings whose length is not divisible by 3. For example, strings with length of 0, 3, 6, 9 characters should not be accepted. The designed DFA should have at *most* 3 states.

b) [3] all the strings that contain 1 in every even position. For example, the strings ϵ, `0`, `1`, `01`, `010`, should be accepted, while the string `0100`, should not. The designed DFA should have at *most* 3 states.

c) [4] all the strings whose length is not divisible by 3 and that contain 1 in every even position. The designed DFA should have at *most* 9 states.

Additional space for question 1.1

1.2 Regular or not? [16 points]

Are the following languages regular? If so, exhibit a finite automaton (deterministic or not) or a regular expression for it. If not, prove it formally using the pumping lemma or the closure properties of regular languages.

a) [4] $L = \{w \mid w \in \{a, b\}^*$ such that *if* w contains the substring ab then it should contain the substring $ba\}$.

b) [4] $L = \{w \mid w \in \{0, 1\}^*$ and w is not a palindrome$\}$.

c) [4] $L = \{wtw \mid t, w \in \{0, 1\}^+\}$.

d) [4] $L = \{xwtw \mid x, w, t \in \{0, 1\}^+\}$.

Additional space for question 1.2

2 Context-free Languages (15 points)

2.1 Write me a grammar [5]

Give a context-free grammar (the production rules) for the following language:

$$L = \{a^n b^m c^m d^{3n} \mid n \geqslant 0, m > 0\}$$

2.2 Ambiguous or Not? [5]

Let G be the context-free grammar given by the following production rules:

$$S \rightarrow aSA \mid \epsilon$$
$$A \rightarrow bAc \mid \epsilon$$

Is G ambiguous grammar? Explain your answer.

2.3 Draw me a PDA [5]

Let $L = \{w \in \{a, b\}^* \mid \text{the length of } w \text{ is an } \textit{odd} \text{ number and the first, middle, and last character of } w \text{ are identical}\}$. Design a PDA that accepts L. The designed PDA should have at *most* 7 states.

3 Electronics Store (15 points)

A new electronics store has opened its doors in Infinityland. It consists of a showroom and a checkout area. People arrive at the showroom according to a Poisson process with parameter λ. The time that a person spends in the showroom is exponentially distributed with parameter μ. The showroom has unlimited capacity, i.e., there can be arbitrarily many people in the showroom at the same time. The population of Infinityland is infinite.

a) [4] Model the showroom as a Continuous Time Markov Chain.

b) [4] Derive a formula for π_k, i.e., the steady state probability that exactly k people are in the showroom. The formula should not depend on π_0.
Hint:

$$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x.$$

Once a customer has put all items in his/her cart, he/she proceeds to the checkout counters. Assume now that on average one customer per minute arrives at the checkout counters. There are m checkout counters, each one with its own waiting queue. Customers choose one of the checkout counters independently and uniformly at random. The service time at each checkout counter is exponentially distributed with an average of 20 customers per hour.

c) `[2]` How many checkout counters are necessary so that the waiting queues do not grow indefinitely?

d) `[5]` Calculate the expected time a customer spends at the checkout counter (waiting time and service time combined).

4 Online Gas Station Search (25 points)

You rent a car to drive from A to B. You start at A and must return the car fully fueled at B. You want to refuel at the cheapest price. You only know that the highest gas price can be up to twice as high as the lowest one. Having no more information, you plan to not deviate from the path, but instead to decide online whether you should refuel at the current gas station.

You know that there are exactly three gas stations ahead of you. If you are at a gas station, you will either refuel the whole tank or nothing at all. Assume that the fuel consumption between the gas stations and B is negligible. Consider the following deterministic strategy:

- You check the price at the first gas station, but do not refuel. If the price at the second gas station is better, you refuel there. Otherwise, you refuel at the third gas station.

a) [5] Is this strategy c-competitive for some constant c? Compute the competitive ratio.

In order to improve the competitive ratio, you are allowed to use randomization. Consider the following randomized strategy:

- You toss a fair coin at the first gas station. If you get heads, you refuel. If not, you proceed to the next gas station, repeat the coin toss and refuel at heads. If the coin tosses were tails at both stations, you refuel at the third gas station.

b) [5] Compute the *expected* competitive ratio for this strategy.

c) [5] Let p_1 be the probability with which you refuel at the first gas station, and p_2 at the second. How would you choose p_1 and p_2 to get the best expected price? Compute the expected competitive ratio and show that your choice of p_1 and p_2 is best possible.

It turns out that the first gas station did not exist and there are exactly *two* gas stations left. To get the best price, you decide not to gamble. Instead, you are now allowed to refuel parts of the total amount at any gas station you are passing.

d) [5] Describe a *deterministic* online algorithm you should apply in order to get the best out of your situation when allowed to fill your tank partially. Give the competitive ratio of your algorithm.

e) [5] Prove that there is no better deterministic strategy for two stations than the one you described in **d)**.

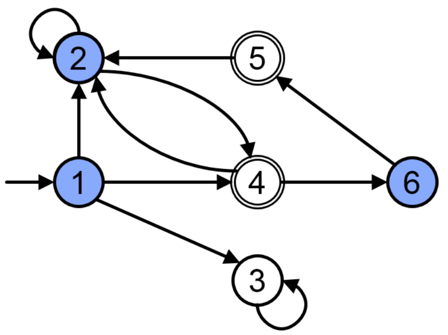
5 True or False – Automata & Petri Nets (10 points)

For each of the following statements, assess if it is true or false and tick the corresponding box. No justification is needed.
Every correct answer grants one point. Leaving a statement blank gives 0 point. Every incorrect answer looses one point on the total, with a minimum of 0 point for the whole question.

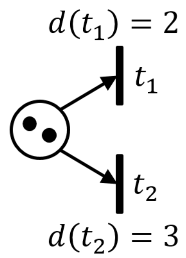
Notes:

- Automaton **A** is depicted in Fig.1(a).
- Petri net **P** is depicted in Fig.1(b).
- $[\![\, p \,]\!]$ denotes the set of states which satisfies property p.
  For example for automaton **A**, $[\![\, p \,]\!] = \{1, 2, 6\}$.

	Statement	True	False
1	If an automaton contains a finite number of states, computing the subset of reachable states always takes a finite number of steps.		
2	A given Binary Decision Diagram can represent different functions i.e., functions for which a same input returns different outputs.		
3	For automaton **A**, $[\![\, \text{EG } p \,]\!] = \{1, 2\}$.		
4	For automaton **A**, $[\![\, \text{EX } (\, \overline{p} \texttt{ OR } \text{AG } p \,) \,]\!] = \{1, 2, 3, 6\}$.		
5	Automaton **A** satisfies AX p.		
6	The execution of a Petri net (i.e., the token game) is deterministic.		
7	A Petri net never fires multiple transitions simultaneously.		
8	A Petri net without deadlock is called a safe Petri net.		
9	If the coverability graph of a Petri net contains the state $(1, 0, w)$, then for any positive integer p, there exists a positive integer q such that $(1, 0, p + q)$ is a reachable marking of the Petri net.		
10	Assuming the token game starts at time 0, transition t_2 of Petri net **P** fires at time 3.		



(a) Automaton **A** – 1 is the initial state. 4 and 5 are accepting (or final) states. Property p is true only in states 1, 2, and 6.



(b) Timed Petri net **P** – The delay functions of transitions are constant and indicated by the transitions.

Figure 1: Automaton **A** (1(a)) and Timed Petri net **P** (1(b))

6 Binary Decision Diagram (10 points)

a) [6] Given the boolean expression of function f and the ordering of variables $x_1 \prec x_2 \prec x_3 \prec x_4$ (i.e., x_1 is the first variable), construct the BDD (Binary Decision Diagram) of f. Merge all equivalent nodes, including the leaves.
Note: Use solid lines for *True* arcs and dashed lines for *False* arcs.

$$f : \overline{(x_1 \cdot \overline{x_2} + x_3) \cdot x_4} + \overline{x_2} \cdot \overline{x_3} \cdot \overline{x_4} + x_2 \cdot x_4$$

b) [2] Consider the BDD of a boolean function g in Fig. 2. Simplify the BDD of g when x_2 is evaluated to 0.

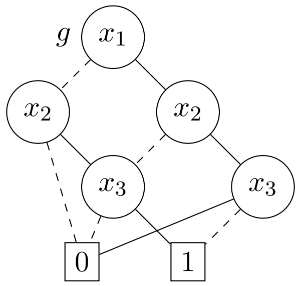


Figure 2: BDD of the boolean function g

c) [2] Express g in Fig. 2 as a boolean function.

Additional space for question 6

7 Petri net (20 points)

Throughout this question, we consider the Petri net **P** in Fig. 3. This question contains 3 independent sub-questions.

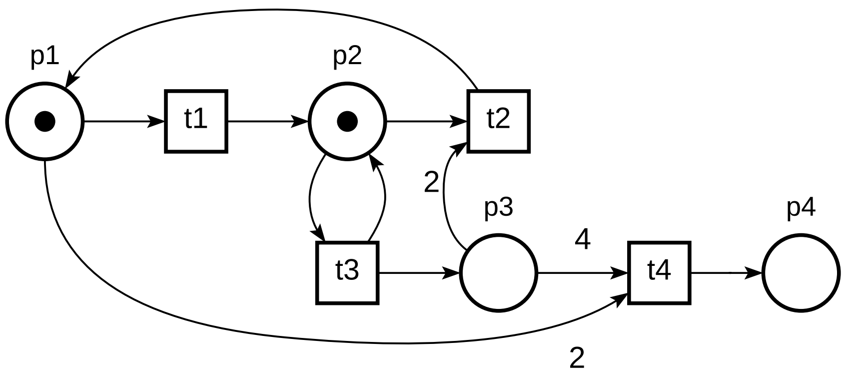


Figure 3: Petri net **P** – Circles, dots and squares represent places, tokens and transitions, respectively. Arcs' weights are marked on the arcs when they are different from 1.

Notes

- X^t denotes the transpose of vector X.
- $M^t = [p_1, p_2, p_3, p_4]$ and $T^t = [t_1, t_2, t_3, t_4]$ are marking and firing vectors of **P** respectively.
  p_i denotes the number of tokens in place i.
  t_i denotes the number of firings of transition i.

7.1 Reachability [8 points]

a) [2] Derive the incidence matrix A of the Petri net **P**.

b) Consider the firing sequence $S = t_1 \cdot t_3 \cdot t_3 \cdot t_3 \cdot t_3 \cdot t_3 \cdot t_4 \cdot t_3 \cdot t_2$ and the corresponding firing vector $T_S^t = [1,\ 1,\ 6,\ 1]$.

(i) [2] Assume that S is a valid firing sequence from the initial marking $M_0^t = [1\,, 1\,, 0\,, 0]$. Apply the incidence matrix and the state equation of the Petri net **P** to compute the marking M_1 obtained from M_0 after firing S.

(ii) [1] Is S a valid firing sequence with respect to the marking M_0? Why?

c) [2] Apply the state equation of the Petri net **P** to derive the initial marking M_2 for which the marking $M_3^t = [1\,, 1\,, 1\,, 1]$ is reached after firing the sequence S.

d) [1] Assume M_2 is a valid marking. Is it sufficient to conclude that S is a valid firing sequence with respect to M_2? Why?

Additional space for question 7.1

Additional space for question 7.1

7.2 Coverability `[4 points]`

Construct the coverability graph of the Petri net **P**.

Note: The coverability graph is obtained from the coverability tree by merging nodes with the same marking.

Additional space for question 7.2

7.3 Verification [8 points]

We define that the Petri net **P** is in an *accepting state* if and only if there is at least one token in the place p_4. Furthermore, we assume that the only observable events are the firing of transitions.
For convenience, we denote with "t_i" the event that transition t_i fires. For example, "t_2" means *"transition t_2 fires"*.

We define the following properties:

A No matter what happens, eventually an accepting state is reached.

B No matter what happens, at any point in the execution, there exists a path where eventually an accepting state is reached.

a) [4] Write **A** and **B** as CTL formulas.

b) [4] Does the Petri net **P** with initial marking $M_0^t = [1\,,1\,,0\,,0]$ satisfy **A**? Does it satisfy **B**? Justify your answers.
Note: You may use the results from question 7.2 to help justifying your answers.

Additional space for question 7.3