Discrete Event Systems
Exercise Sheet 3

1 Pumping Lemma [Exam]

Is the following language regular? Prove your claims!

$$L = \{0^a 1^b 0^c 1^d \mid a, b, c, d \geq 0 \text{ and } a = 1, b = 2 \text{ and } c = d\}$$

2 Deterministic Finite Automata [Exam]

Transform the NFA A in Figure 1 into an equivalent DFA using the powerset construction presented in the lecture, while assuming $\Sigma = \{0, 1\}$. (*Hint:* Only construct states which are necessary!)
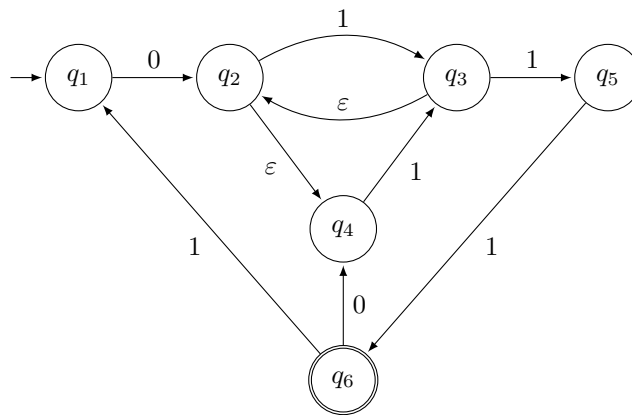

Figure 1: NFA A.

3 Transforming Automata [Exam]

Consider the DFA B in Figure 3 over the alphabet $\Sigma = \{0, 1\}$. Give a regular expression for the language L accepted by the automaton B. If you like, you can do this by ripping out states as presented in the lecture.
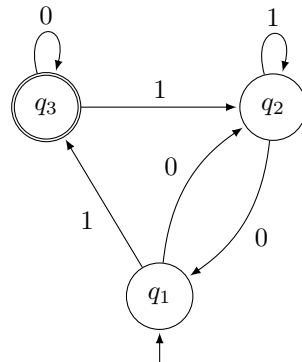

Figure 2: DFA B.

4 Regular and Context-Free Languages

a) Consider the context-free grammar G with the production $S \to SS \mid 1S2 \mid 0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.

b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language L that is regular.

5 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

a) $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$

b) $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

6 Ambiguity

Consider the following context-free grammar G with non-terminals S and A, start symbol S, and terminals "(", ")", and "0":

$$
\begin{aligned}
S &\to SA \mid \varepsilon \\
A &\to AA \mid (S) \mid 0
\end{aligned}
$$

a) What are the eight shortest words produced by G?

b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.

c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.