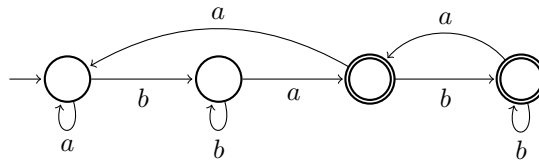# Discrete Event Systems
Solution to Exercise Sheet 2

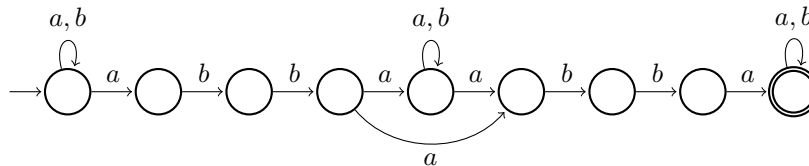# 1 Filter for an Input Stream [exam problem]

The following figure gives an example with four states.
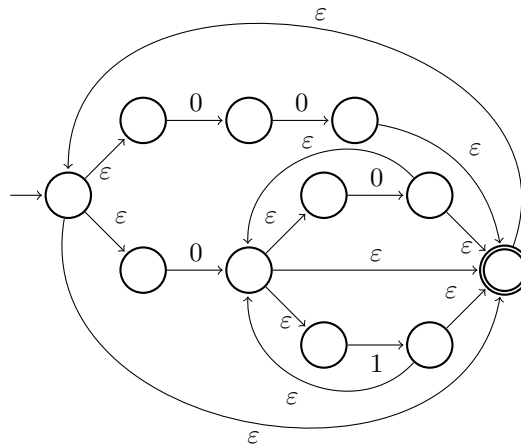

The main idea here is to use two different accepting states after having read a: one for zero b's and one for more than zero b's.

2 Nondeterministic Finite Automata

a) The following automaton also accepts strings containing $a\,b\,b\,a\,b\,b\,a$ as a substring.

b) The following automaton is obtained using the transformations presented in the lecture.


This automaton, however, is not minimal. If you take a closer look at the regular expression $(00 \cup (0(0 \cup 1)^*))^*$, you will see that it can be simplified to $(0(0 \cup 1)^*)^*$ and this in turn to $0(0 \cup 1)^*$. A minimal automaton for this regular expression is given by
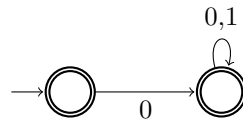
Note: The absence of a transition with label 1 in the left state means that if a 1 is read, the automaton goes into a non-accepting state and stays there for the rest of the execution. NFAs may be under-determined in this notion, but FAs must provide a transition for every symbol of the alphabet and potentially a crash state to "capture" non-accepting executions.

**c)** Three states are enough if we can use NFAs. With FAs, we needed at least four states.
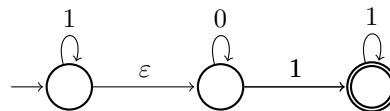
d) A deterministic machine for which every state is an accepting state, accepts *every* string of the corresponding alphabet. However, this does not hold for a nondeterministic automaton, namely if it is under-determined.

3 De-randomization

a) The automaton accepts strings adhering to the regular expression $0^*1^*2^*$. Without ε-transitions we get the following automaton.
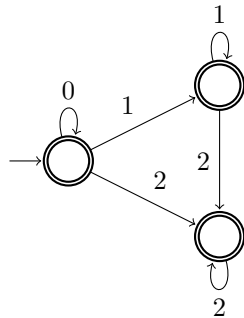

b) The deterministic automaton can be found by applying the power set construction presented in the lecture followed by the state minimization algorithm. However, it is obvious that the automaton shown below does the job.
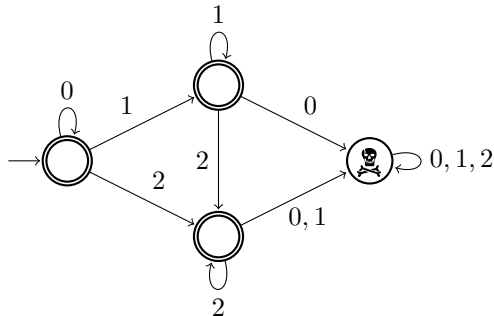
4 States Minimization

State q_0 can be omitted as it is not reachable. Moreover, states q_1 and q_5 can be merged, as there is no input sequence which will show a difference between these two states. A regular expression of the language is $((ba \cup ab)a^*c)^*$.
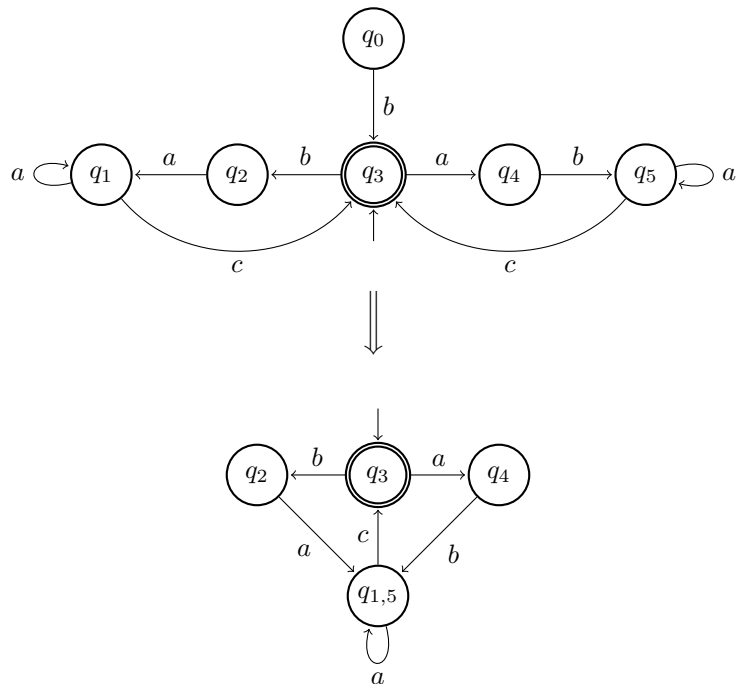
5 "Regular" Operations in UNIX

In UNIX, the special symbol "$" stands for the end of a line. We have:

```
egrep '(password|passwort)(a|e|i|o|u|A|E|I|O|U)*$' <file>
```