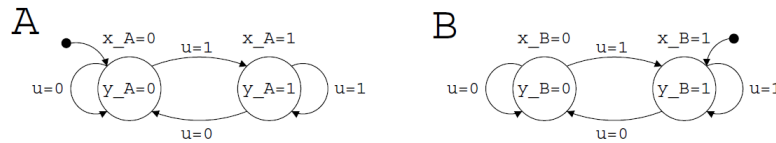


# Discrete Event Systems

## Exercise Sheet 10

### 1 Comparison of Finite Automata

Here are two simple finite automata:

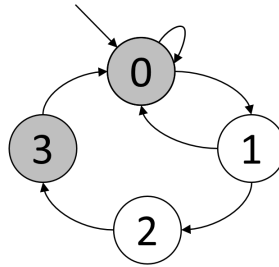


For each, we have a one bit encoding for the states ( $x_A$  and  $x_B$ ), one binary output ( $y_A$  and  $y_B$ ), and one common binary input ( $u$ ). We want to verify whether or not these two automata are equivalent. This can be done through the following steps:

- a) Express the characteristic function of the transition relation for both automaton,  $\psi_r(x, x', u)$ .
- b) Express the joint transition function,  $\psi_f$ .  
**Reminder:**  $\psi_f(x_A, x'_A, x_B, x'_B) = (\exists u : \psi_A(x_A, x'_A, u) \cdot \psi_B(x_B, x'_B, u))$ .
- c) Express the characteristic function of the reachable states,  $\psi_X(x_A, x_B)$ .
- d) Express the characteristic function of the reachable output,  $\psi_Y(y_A, y_B)$ .
- e) Are the two automata equivalent? **Hint:** Evaluate, for example,  $\psi_Y(0, 1)$ .

## 2 Temporal Logic

- a) We consider the following automaton. The property  $a$  is true on the colored states (0 and 3).



For each of the following CTL formula, list all the states for which it holds true.

- (i)  $EF a$
  - (ii)  $EG a$
  - (iii)  $EX AX a$
  - (iv)  $EF ( a \text{ AND } EX \text{ NOT}(a) )$
- b) Given the transition function  $\psi_f(x, x')$  and the characteristic function  $\psi_Z(x)$  for a set  $Z$ , write a small pseudo-code which returns the characteristic function of  $\psi_{AF Z}(x)$ . It can be expressed as symbolic boolean functions, like  $\overline{x_A}x'_A\overline{x_B}x'_B + \overline{x_A}x'_Ax_Bx'_B$ .  
**Hint:** To do this, simply use the classic boolean operators AND, OR, NOT and  $! =$ . You can also use the operator  $\text{PRE}(Q, f)$ , which returns the predecessor of the set  $Q$  by the transition function  $f$ . That is,

$$\text{PRE}(Q, f) = \{q' : \exists x, \psi_f(q', q) \cdot \psi_Q(q) = 1\}$$

**Hint:** It can be useful to reformulate  $AF Z$  as another CTL formula.