Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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Discrete Event Systems

Exercise Sheet 9

1 Probability of Arrival

In the script, there is a lemma saying that the probability of arrival in a markov chain can be computed using the formula

$$f_{ij} = p_{ij} + \sum_{k:k \neq j} p_{ik} f_{kj} \quad .$$

Prove this lemma.

2 Basketball [Exam]

Mario, Luigi and Trudy meet to play basketball. To improve their scoring abilities, Mario suggests the following game: Each player has to score m times. After each miss, he has to perform 10 push-ups.

a) Assume that Mario always scores with a constant probability p. How many push-ups does he do in expectation in his game?

b) Luigi wants to show that he is better and wants to score m times in sequence. After each miss, he performs 10 push-ups as well, and then tries again to score m times in a row.

(i) How many push-ups does Luigi in expectation, assuming he also scores with a constant probability p?

(ii) How many shots will Luigi do in expectation? (Tricky bonus exercise)
Hint: Show first that for the number of shots S in an unsuccesful round, we have $\mathbf{E}[S] \leq \frac{1}{1-p}$ and then use this fact.

c) Trudy accepts Luigi's game and tries to score $m = 3$ times in a row. But Trudy is a bit lazy and gives up as soon as she has missed two times in a row. Trudy scores with constant probability $p = 0.5$.

(i) What is the probability that Trudy scores $m = 3$ times in a row? What is the probability that she gives up?

(ii) How many push-ups does Trudy do in expectation?

Hint:
$$\sum_{i=1}^{\infty} i \cdot q^{i-1} = \frac{1}{(1-q)^2} \qquad \text{for } |q| < 1$$

3 Night Watch [Exam]

After spending all their money on ice cream in Rome, Rachel and Hector need to start working at nights in order to improve their poor financial situation. Their task is to guard a famous Swiss bank which, from an architectonic perspective, looks as follows:

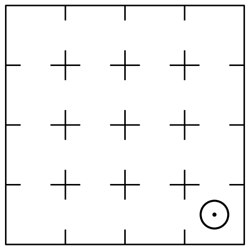

Figure 1: Offices of a Swiss bank.

Thus, there are 4×4 rooms, all connected by doors as indicated in the figure.

At first, Hector and Rachel always stay together. They start in the room on the upper left. Every minute, they change to the next room, which is chosen uniformly at random from all possible (adjacent) rooms.

a) Compute the probability (in the steady state) that Rachel and Hector are in the room where the thief enters the bank (indicated with $\odot$)!

b) After a few weeks of martial arts training, Hector and Rachel are so strong that they can easily catch a thief on their own. Thus, they decide that it would be smarter to patrol individually: After every minute, each of them chooses the next room *independently*. What is now the probability that at least one of them is in the room where the thief enters?