# Discrete Event Systems
## Exercise Sheet 4

## 1  Regular and Context-Free Languages

**a)** Consider the context-free grammar $G$ with the production $S \to SS \mid 1S2 \mid 0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.

**b)** The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language $L$ that is regular.

## 2  Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

**a)** $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$

**b)** $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

## 3  Pushdown Automata

Consider the following context-free grammar $G$ with non-terminals $S$ and $A$, start symbol $S$, and terminals "(", ")", and "0":

$$
\begin{aligned}
S &\to SA \mid \varepsilon \\
A &\to AA \mid (S) \mid 0
\end{aligned}
$$

**a)** What are the eight shortest words produced by $G$?

**b)** Context-free grammars can be ambiguous. Prove or disprove that $G$ is unambiguous.

**c)** Design a push-down automaton $M$ that accepts the language $L(G)$. If possible, make $M$ deterministic.

## 4  Pumping Lemma Revisited

**a)** Determine whether the language $L = \{1^{n^2} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

**b)** Determine whether the language $L = \{1^{\lfloor \sqrt{n} \rfloor} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

**c)** Consider a regular language $L$ and a pumping number $p$ such that every word $u \in L$ can be written as $u = xyz$ with $|xy| \le p$ and $|y| \ge 1$ such that $xy^i z \in L$ for all $i \ge 0$.

Can you use the pumping number $p$ to give a bound on the minimum number of states needed for the corresponding DFA? What about the minimum number of states of the corresponding NFA?