Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing

HS 2010 Prof. R. Wattenhofer / J. Smula, T. Langner

Discrete Event Systems

Exercise Sheet 5

1 Counter Automaton

A push-down automaton is basically a finite automaton augmented by a stack. Consider a finite automaton that (instead of a stack) has an additional *counter* C, i.e., a register that can hold a single integer of arbitrary size. Initially, $C = 0$. We call such an automaton a *Counter Automaton* M. M can only increment or decrement the counter, and test it for 0. Since theoretically, all possible data can be coded into one single integer, a counter automaton has unbounded memory. Further, let L_{count} be the set of languages recognized by counter automata.

a) Let L_{reg} be the set of regular languages. Prove that $L_{reg} \subseteq L_{count}$.

b) Prove that the opposite is not true, that is, $L_{count} \nsubseteq L_{reg}$. Do so by giving a language which is in L_{count}, but not in L_{reg}. Characterize (with words) the kind of languages a counter automaton can recognize, but a finite automaton cannot.

c) Which automaton is stronger? A counter automaton or a push-down automaton? Explain your decision.

2 Tandem Pumping

For the following languages, determine whether they are context free or not. Prove your claims!

a) $L = \{a^i b^j c^k \mid 0 < i < j < k\}$

b) $L = \{x \mid x \in \{0,1\}^*$, and x contains an even number of '0' and an even number of '1'$\}$

c) $L = \{x\#y \mid x, y \in \{0,1\}^*$, and x is a permutation of $y\}$

3 Context Free Grammars

Are the following languages context-free? If so, give a CFG describing L.

a) $L = \{w\#x\#y\#z \mid w, x, y, z \in \{a,b\}^*$ and $|w| = |z|, |x| = |y|\}$
What changes if $|w| = |y|$, $|x| = |z|$?

b) $L = \{x\#y \mid x, y \in \{a,b\}^*$ and $|x| \bmod 3 = 0$ and $(|x| + |y|) \bmod 2 = 0\}$

4 Push Down Automata

For each of the following context free languages, draw a PDA that accepts L.

a) $L = \{a^i b^j a^j b^i \mid i, j > 0\}$

b) $L = \{u \mid u \in \{0,1\}^* \text{ and } u^{reverse} = u\} = \{u \mid \text{“}u \text{ is a palindrome”}\}$

c) $L = \{u \mid u \in \{0,1\}^* \text{ and } u^{reverse} \neq u\} = \{u \mid \text{“}u \text{ is no palindrome”}\}$

5 Designing Turing Machines

Alice is very happy because she was accepted for an internship at Tintel, one of the world's leading processor manufacturers. Unfortunately, she has only attended the famous DES lecture during her studies at ETH and knows nothing about electronic circuits. Therefore, she wants to solve her first assignment using a Turing Machine – please assist her:

a) Mister Intal (Alice's supervisor) wants Alice to design a Turing Machine to compute the sum $a + b$ of two binary numbers a and b with equal length. The two numbers a and b are written on the machine tape in the form $a_n a_{n-1} \dots a_1 a_0 + b_n b_{n-1} \dots b_1 b_0$, where a_n is the MSB (most significant bit) of a. Note that the two numbers are separated by the '+' symbol, that they have an equal number of bits[1], and that $\Gamma = \{0, 1, +\}$. Initially, the head of the TM points to a_n. At the end, it should point to the MSB of the result.

Provide a plain text description of your TM as well as a finite state machine controlling the tape head. Use the following notation for transitions:

'$\alpha \rightarrow \beta \mid \gamma$'	read α from the tape at the current position, then write a β and finally move left if $\gamma = L$ or move right if $\gamma = R$.
'$\alpha \mid \gamma$'	abbreviation for transitions of the form $\alpha \rightarrow \alpha \mid \gamma$ (these transitions do not modify the content of the tape).

Hint: You might want to extend the alphabet Γ to put temporary symbols on the tape.

b) In her second assignment, Alice is asked to implement a *binary to unary converter*. This converter takes a number a in binary (alphabet $\{0, 1\}$) and converts it to a unary number u (alphabet $\{1\}$). Initially, the TM head points to the MSB of a. At the end, the head should point to the right-most digit of u. Proceed as in the previous exercise.

Hint: The number n in unary representation consists of n ones.

[1] Your machine may crash or produce a wrong result if this condition does not hold.