Topology Control

Chapter 3

ETH
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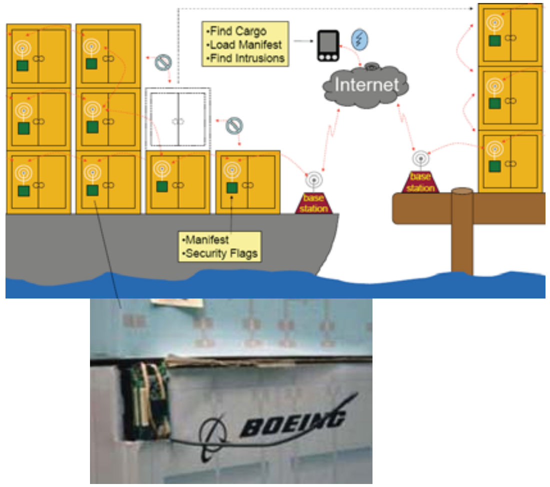
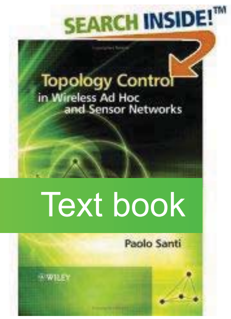
Inventory Tracking (Cargo Tracking)

- Current tracking systems require line-of-sight to satellite.
- Count and locate containers
- Search containers for specific item
- Monitor accelerometer for sudden motion
- Monitor light sensor for unauthorized entry into container



Rating



- Area maturity

First steps — Text book

- Practical importance

No apps — Mission critical

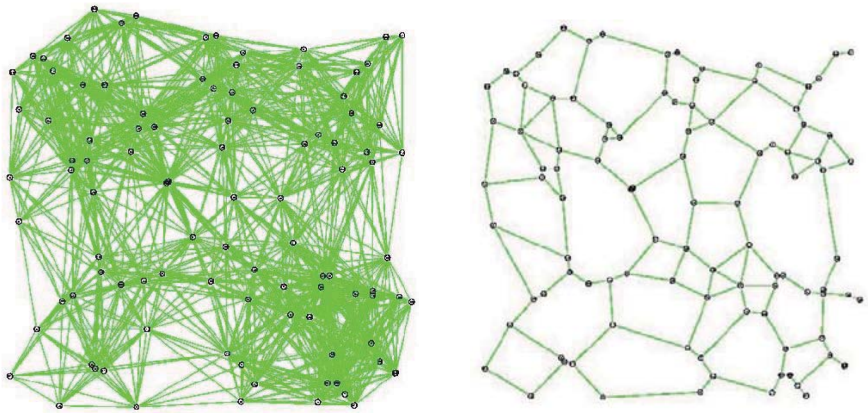
- Theory appeal

Boooooooring — Exciting

Overview – Topology Control

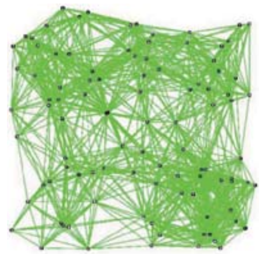
- Proximity Graphs: Gabriel Graph et al.
- Practical Topology Control: XTC
- Interference

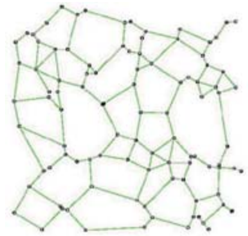
Topology Control



- Drop long-range neighbors: Reduces interference and energy!
- But still stay connected (or even spanner)

Topology Control as a Trade-Off





Network Connectivity
Spanner Property

$d_{TC}(u,v) \leq c \cdot d(u,v)$

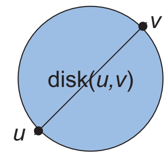
Conserve Energy
Reduce Interference
Sparse Graph, Low Degree
Planarity
Symmetric Links
Less Dynamics

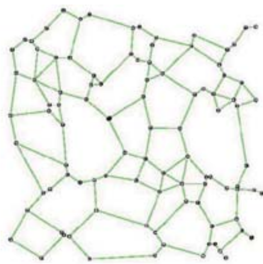
Spanners

- Let the distance of a path from node u to node v, denoted as d(u,v), be the sum of the Euclidean distances of the links of the shortest path.
  - Writing $d(u,v)^p$ is short for taking each link distance to the power of *p*, again summing up over all links.
- Basic idea: S is spanner of graph G if S is a subgraph of G that has certain properties for all pairs of nodes, e.g.
  - Geometric spanner: $d_S(u,v) \leq c \cdot d_G(u,v)$
  - Power spanner: $d_S(u,v)^\alpha \leq c \cdot d_G(u,v)^\alpha$, for path loss exponent α
  - Weak spanner: path of S from u to v within disk of diameter $c \cdot d_G(u,v)$
  - Hop spanner: $d_S(u,v)^0 \leq c \cdot d_G(u,v)^0$
  - Additive hop spanner: $d_S(u,v)^0 \leq d_G(u,v)^0 + c$
  - (α, β) spanner: $d_S(u,v)^0 \leq \alpha \cdot d_G(u,v)^0 + \beta$
  - The stretch can be defined as maximum ratio d_S/d_G

Gabriel Graph

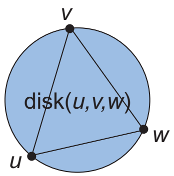
- Let disk(*u*,*v*) be a disk with diameter (*u*,*v*) that is determined by the two points *u*,*v*.
- The Gabriel Graph GG(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is an edge between two nodes *u*,*v* iff the disk(*u*,*v*) including boundary contains no other points.
- As we will see the Gabriel Graph has interesting properties.



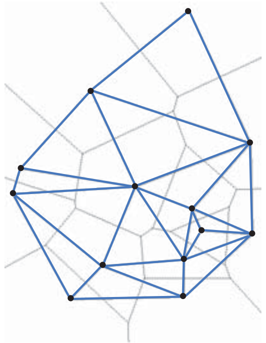


Delaunay Triangulation

- Let disk(*u,v,w*) be a disk defined by the three points *u,v,w*.
- The Delaunay Triangulation (Graph) DT(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is a triangle of edges between three nodes *u,v,w* iff the disk(*u,v,w*) contains no other points.

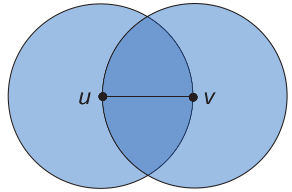


- The Delaunay Triangulation is the dual of the Voronoi diagram, and widely used in various CS areas
  - the DT is planar
  - the DT is a geometric spanner

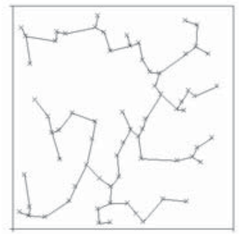


Other Proximity Graphs

- Relative Neighborhood Graph RNG(V)
  - An edge e = (u,v) is in the RNG(V) iff there is no node w in the “lune” of (u,v), i.e., no noe with with (u,w) < (u,v) and (v,w) < (u,v).



- Minimum Spanning Tree MST(V)
  - A subset of *E* of *G* of minimum weight which forms a tree on *V*.

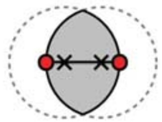


Properties of Proximity Graphs

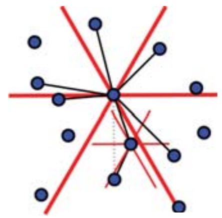
- Theorem 1:
  MST $\subseteq$ RNG $\subseteq$ GG $\subseteq$ DT
- Corollary:
  Since the MST is connected and the DT is planar, all the graphs in Theorem 1 are connected and planar.
- Theorem 2:
  The Gabriel Graph is a power spanner (for path loss exponent $\alpha \geq 2$).
  So is GG $\cap$ UDG.
- Remaining issue: either high degree (RNG and up), and/or no spanner (RNG and down). There is an extensive and ongoing search for “Swiss Army Knife” topology control algorithms.

More Proximity Graphs

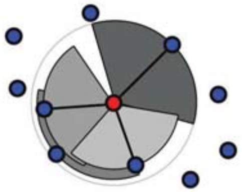
- β-Skeleton
  - Disk diameters are $\beta \cdot d(u,v)$, going through u resp. v
  - Generalizing GG (β = 1) and RNG (β = 2)



- Yao-Graph
  - Each node partitions directions in k cones and then connects to the closest node in each cone



- Cone-Based Graph
  - Dynamic version of the Yao Graph. Neighbors are visited in order of their distance, and used only if they cover not yet covered angle

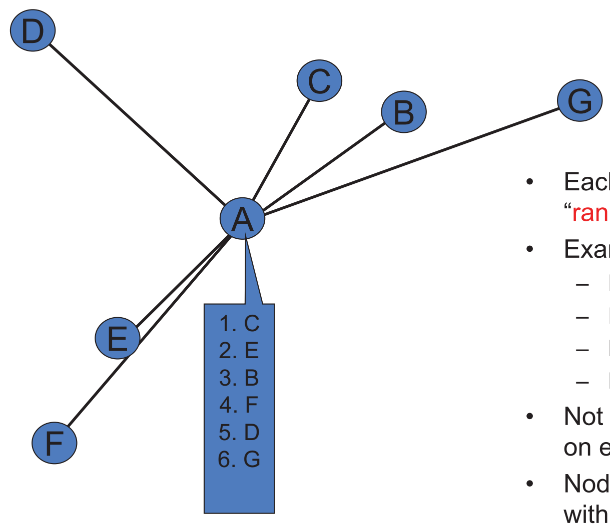


Lightweight Topology Control

- Topology Control commonly assumes that the node positions are known.

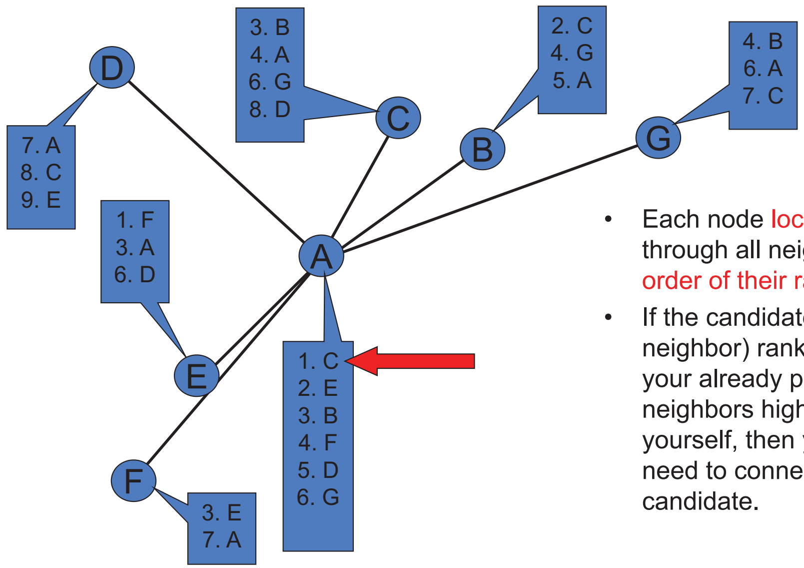
What if we do not have access to position information?

XTC: Lightweight Topology Control without Geometry



- Each node produces "ranking" of neighbors.
- Examples
  - Distance (closest)
  - Energy (lowest)
  - Link quality (best)
  - Must be symmetric!
- Not necessarily depending on explicit positions
- Nodes exchange rankings with neighbors

XTC Algorithm (Part 2)



- Each node locally goes through all neighbors in order of their ranking
- If the candidate (current neighbor) ranks any of your already processed neighbors higher than yourself, then you do not need to connect to the candidate.

XTC Analysis (Part 1)

- Symmetry: A node u wants a node v as a neighbor if and only if v wants u.

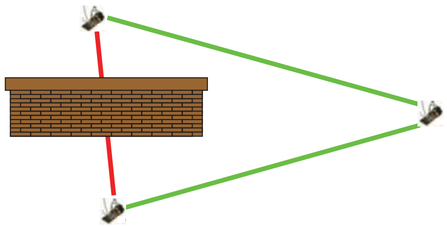
- Proof:
  - Assume 1) $u \to v$ and 2) $u \not\leftarrow v$
  - Assumption 2) $\Rightarrow \exists w$: (i) $w \prec_v u$ and (ii) $w \prec_u v$

In node *u*'s neighbor list, *w* is better than *v*

Contradicts Assumption 1)

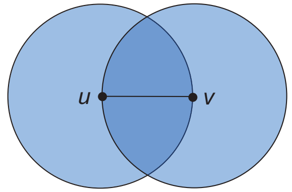
XTC Analysis (Part 1)

- Symmetry: A node u wants a node v as a neighbor if and only if v wants u.
- Connectivity: If two nodes are connected originally, they will stay so (easy to show if rankings are based on symmetric link-weights).
- If the ranking is energy or link quality based, then XTC will choose a topology that routes around walls and obstacles.

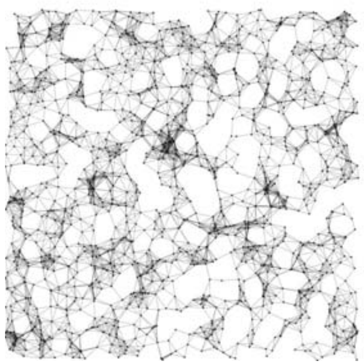


XTC Analysis (Part 2)

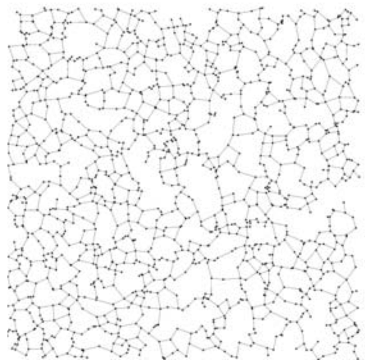
- If the given graph is a Unit Disk Graph (no obstacles, nodes homogeneous, but not necessarily uniformly distributed), then …
- The degree of each node is at most 6.
- The topology is planar.
- The graph is a subgraph of the RNG.
- Relative Neighborhood Graph RNG(V):
  - An edge e = (u,v) is in the RNG(V) iff there is no node w with (u,w) < (u,v) and (v,w) < (u,v).



XTC Average-Case

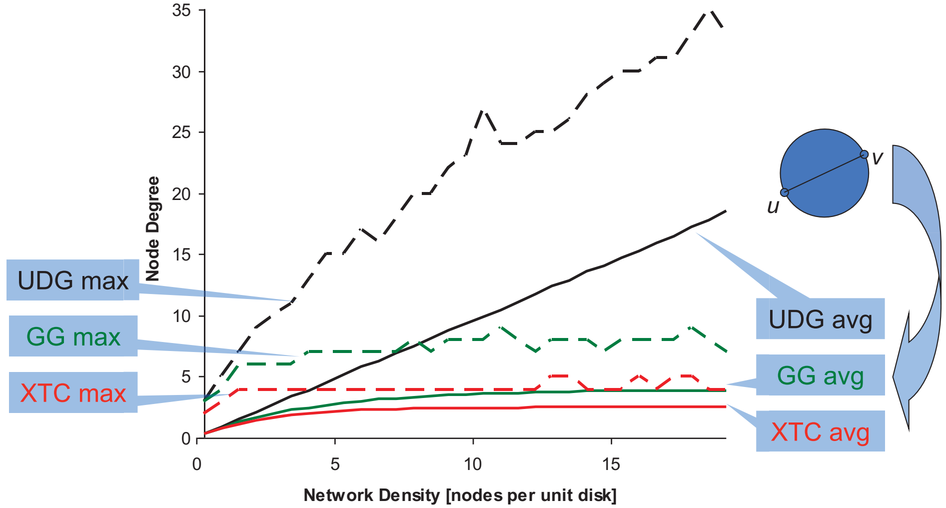


Unit Disk Graph

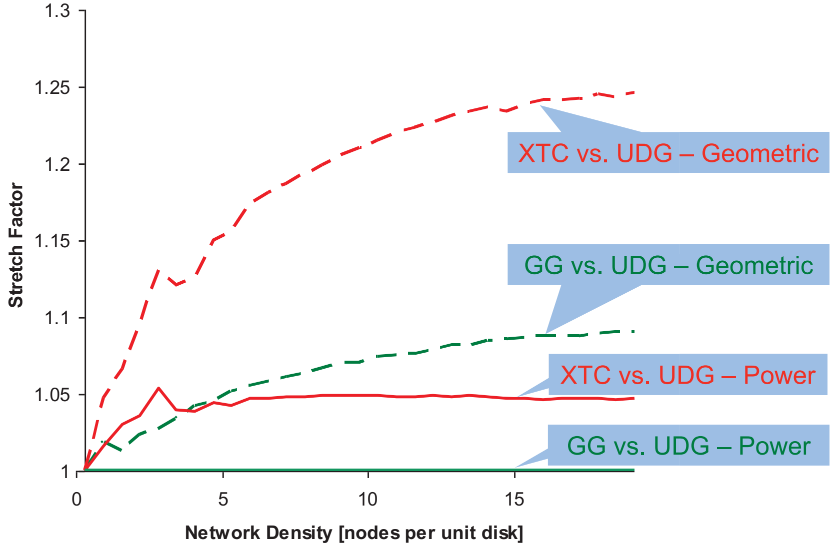


XTC

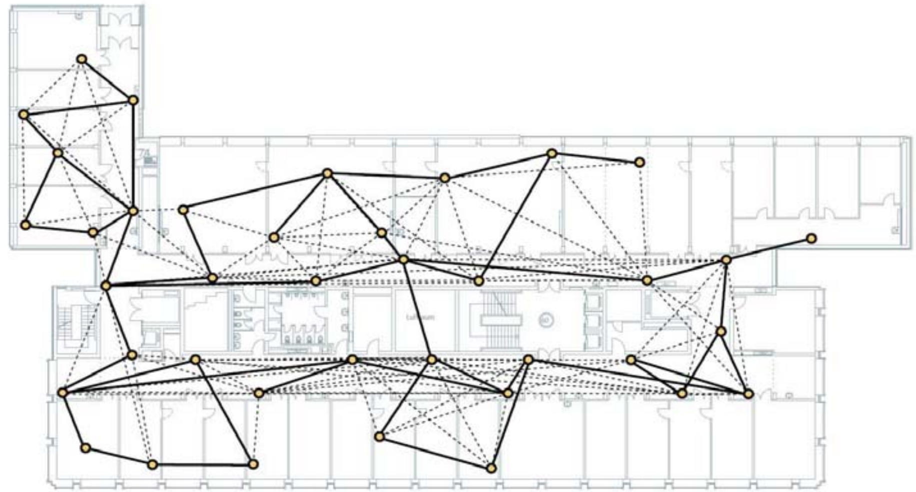
XTC Average-Case (Degrees)



XTC Average-Case (Stretch Factor)

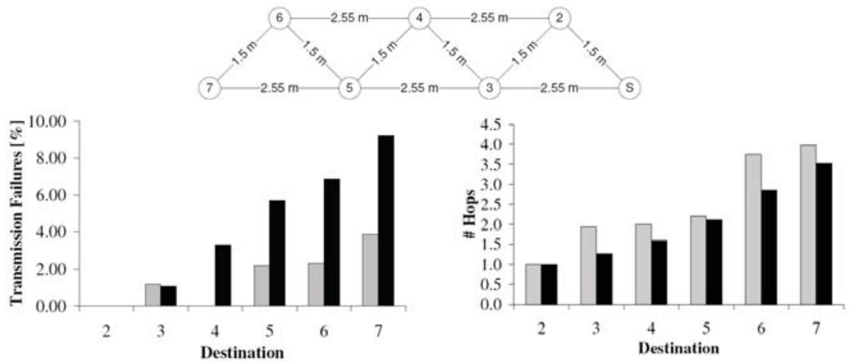


Implementing XTC, e.g. BTnodes v3

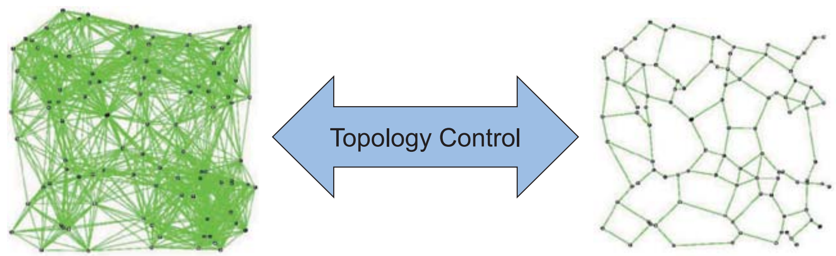


Implementing XTC, e.g. on mica2 motes

- Idea:
  - XTC chooses the reliable links
  - The quality measure is a moving average of the received packet ratio
  - Source routing: route discovery (flooding) over these reliable links only
  - (black: using all links, grey: with XTC)



Topology Control as a Trade-Off

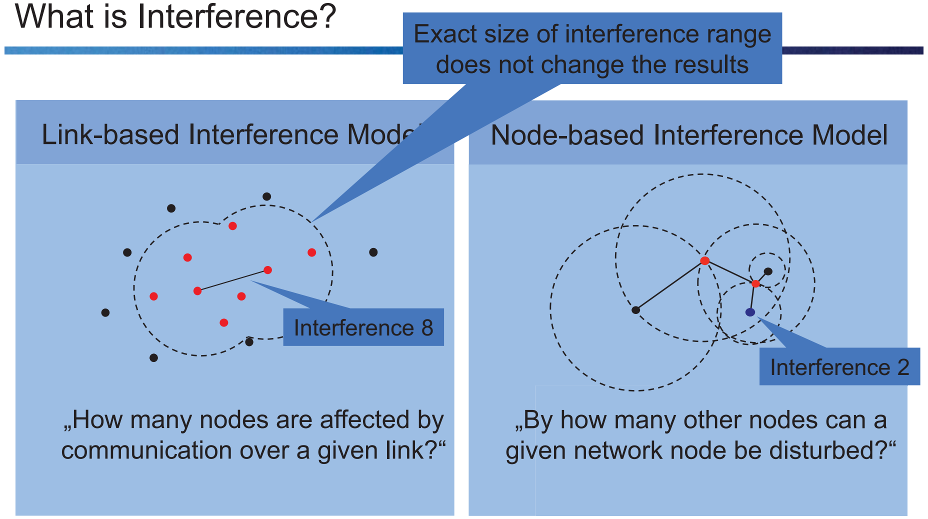


Network Connectivity
Spanner Property

Conserve Energy
Reduce Interference
Sparse Graph, Low Degree
Planarity
Symmetric Links
Less Dynamics

Really?!?

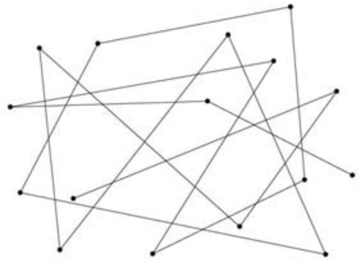
What is Interference?



- Problem statement
  - We want to minimize maximum interference
  - At the same time topology must be connected or spanner

Low Node Degree Topology Control?

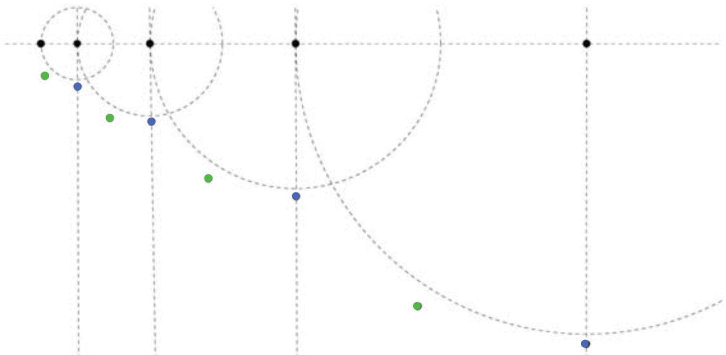
Low node degree does not necessarily imply low interference:



Very low node degree but huge interference

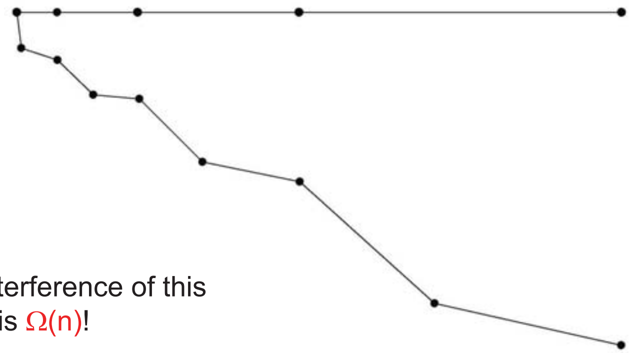
Let's Study the Following Topology!

…from a worst-case perspective



Topology Control Algorithms Produce…

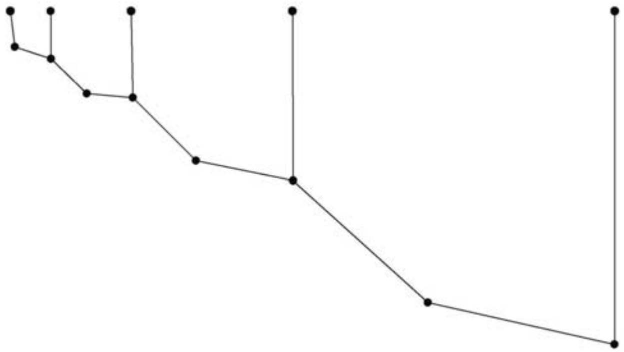
- All known topology control algorithms (with symmetric edges) include the nearest neighbor forest as a subgraph and produce something like this:



- The interference of this graph is $\Omega(n)$!

But Interference…

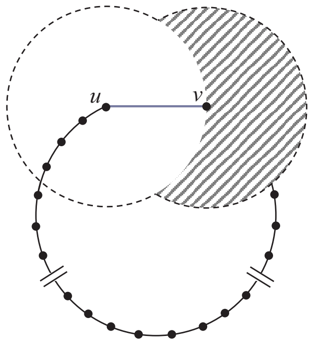
- Interference does not need to be high…



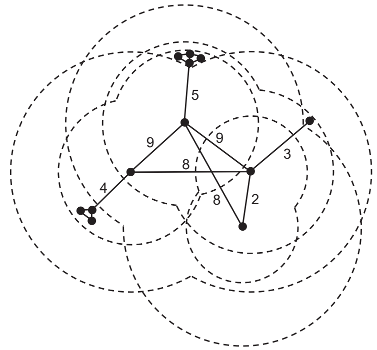
- This topology has interference O(1)!!

Link-based Interference Model

There is no local algorithm that can find a good interference topology



The optimal topology will not be planar



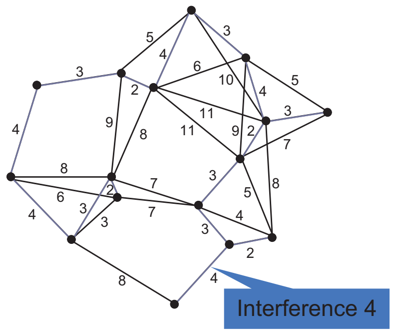
Link-based Interference Model

- LIFE (Low Interference Forest Establisher)
  - Preserves Graph Connectivity

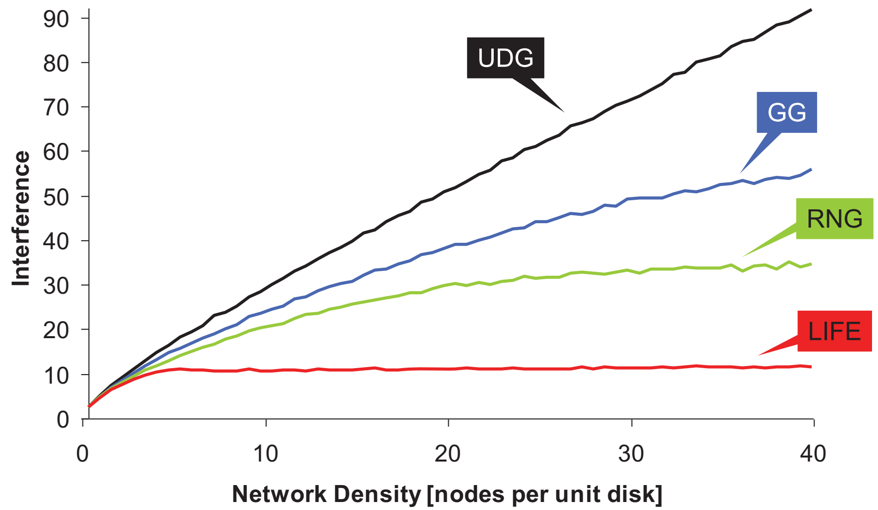
LIFE

- Attribute interference values as weights to edges
- Compute minimum spanning tree/forest (Kruskal's algorithm)

LIFE constructs a minimum- interference forest

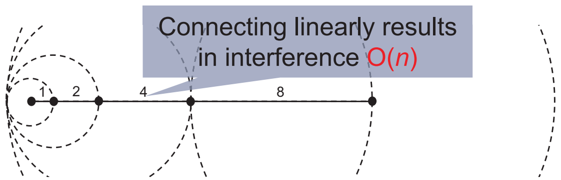


Average-Case Interference: Preserve Connectivity



Node-based Interference Model

- Already 1-dimensional node distributions seem to yield inherently high interference...



- ...but the exponential node chain can be connected in a better way

Node-based Interference Model

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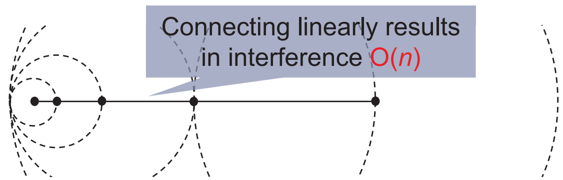


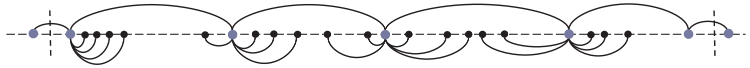
- ...but the exponential node chain can be connected in a better way

Interference $\in O(\sqrt{n})$

Matches an existing lower bound

Node-based Interference Model

- Arbitrary distributed nodes in one dimension
  - Approximation algorithm with approximation ratio in $O(\sqrt[4]{n})$



- Two-dimensional node distributions
  - Simple randomized algorithm resulting in interference $O(\sqrt{n \log n})$
  - Can be improved to $O(\sqrt{n})$

Open problem

- On the theory side there are quite a few open problems. Even the simplest questions of the node-based interference model are open:

- We are given n nodes (points) in the plane, in arbitrary (worst-case) position. You must connect the nodes by a spanning tree. The neighbors of a node are the direct neighbors in the spanning tree. Now draw a circle around each node, centered at the node, with the radius being the minimal radius such that all the nodes' neighbors are included in the circle. The interference of a node u is defined as the number of circles that include the node u. The interference of the graph is the maximum node interference. We are interested to construct the spanning tree in a way that minimizes the interference. Many questions are open: Is this problem in P, or is it NP-complete? Is there a good approximation algorithm? Etc.