Chapter 12

Capacity

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Definition 12.1. *A schedule is SINR$_\phi$-feasible if each transmission is assigned a slot such that the affectance of each link l_v caused by the set of concurrently scheduled links S is less than ϕ, i.e. $a_{l_v}(S) \leq \phi$. If $\phi = 1$ we say that a schedule is SINR-feasible.*

Theorem 12.2. *The physical model is robust against minor (constant) changes. In particular, given a SINR-feasible schedule, we can construct a schedule which is SINR$_\phi$-feasible that has an overhead that is bounded by $\lceil 2/\phi \rceil^2$.*

Proof. Here is a constructive way to get from a SINR-feasible schedule to a SINR$_\phi$-feasible schedule: For each slot S in the SINR-feasible schedule, process links of S in decreasing order of their length. For each link l_v, assign l_v to set S_j with minimum j such that $a_{l_v}(S_j) \leq \phi/2$. Then, the affectance on l_v by longer links is at most $\phi/2$. After doing so we have the sets $S_1, S_2, \ldots, S_m$. Now let us look at some link $l_v \in S_m$. Since l_v was not scheduled in any earlier set, we know that $a_{l_v}(S_i) > \phi/2$ for $i = 1, 2, \ldots, m-1$. If $m \geq 2/\phi + 1$, we have

$$\sum_{1 \leq i < m} a_{l_v}(S_i) > (m-1) \cdot \phi/2 = 1.$$

By additivity of affectance, i.e. $a_{l_v}(S) = \sum_{1 \leq i \leq m} a_{l_v}(S_i)$, we get $a_{l_v}(S) > 1$ which contradicts the original assumption that S was SINR-feasible. In other words, $m < 2/\phi + 1$, or simply $m \leq \lceil 2/\phi \rceil$.

For each of these sets S_j, do the process in reverse order (short links first), getting sets $S_{j1}, S_{j2}, \ldots, S_{jk}$. Now, the affectance on a link in such a refined set by shorter links is at most $\phi/2$. Thus, the total affectance is at most ϕ for each link, at most $\phi/2$ by shorter links and at most $\phi/2$ by longer links. Again, each set is partitioned at most into $\lceil 2/\phi \rceil$ sets. In total, each original set S is partitioned into at most $\lceil 2/\phi \rceil^2$ sets. □