Exercise 7
Sample Solution

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Question: Slotted Aloha

We use slotted Aloha and all machines would like to send in each slot

$$\Pr(success) = n \cdot p \cdot (1-p)^{(n-1)}$$

We do not know the exact number of n but

$$A \leq n \leq B$$

What p is worst case optimal in this scenario?

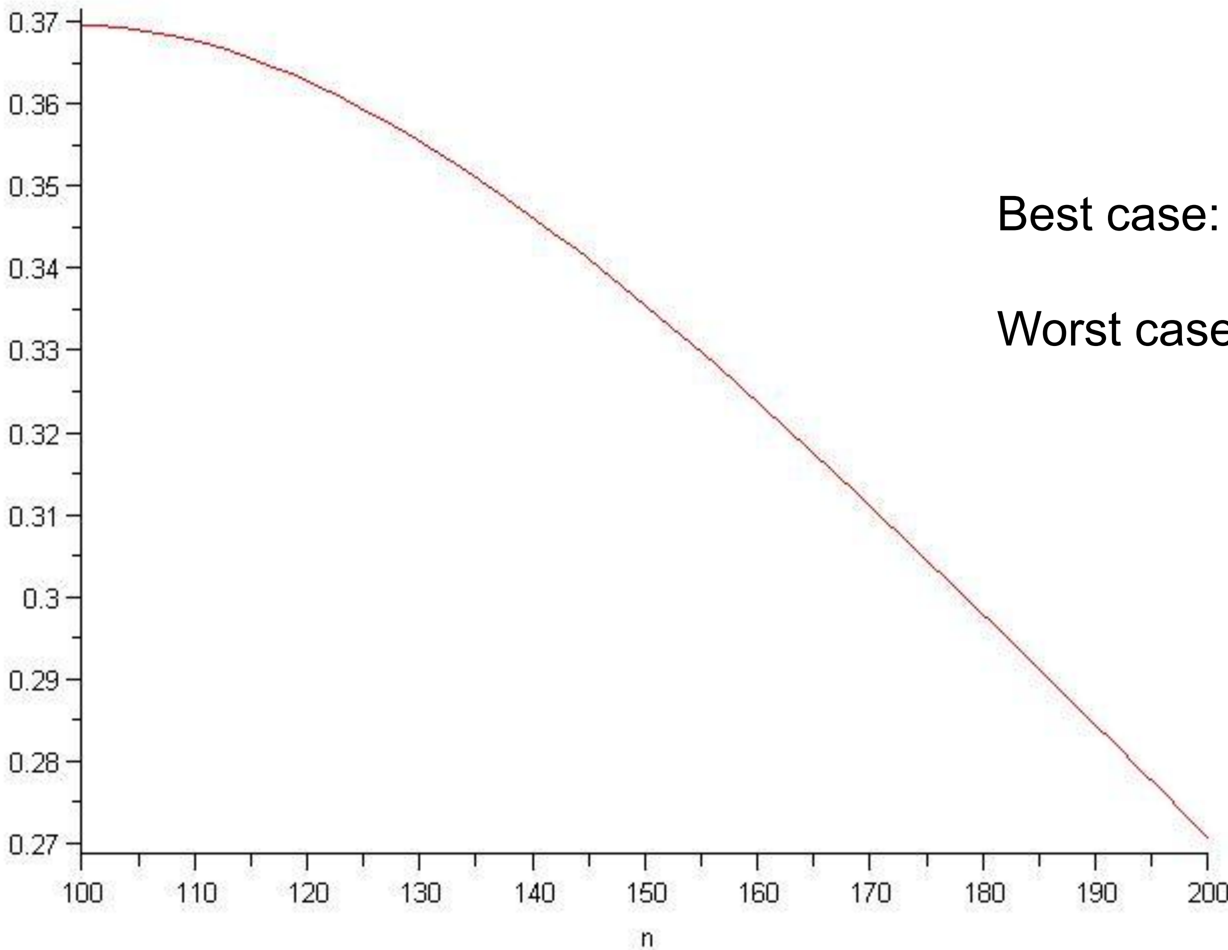
Worst Case Optimal?!?

1.) You select a transmission probability p between 0 and 1

2.) An evil adversary knows what p you have chosen and is now allowed to decide on the number of machines in the network.
(Bounded by A and B)

What p do you have to chose to get the maximal Pr(success)?

What happens for p=1/A?



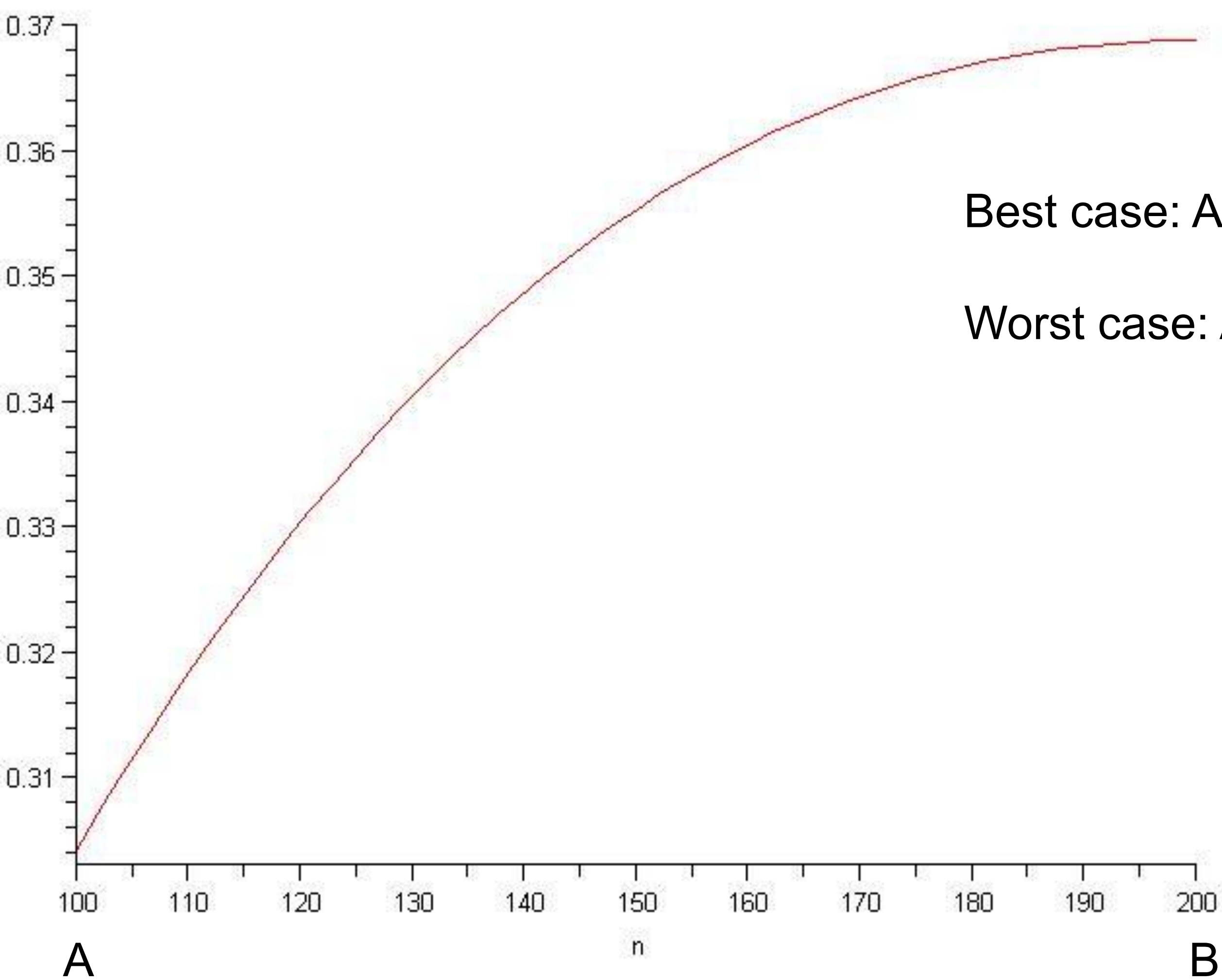
A

B

Best case: Adversary chooses n=A

Worst case: Adversary chooses n=B

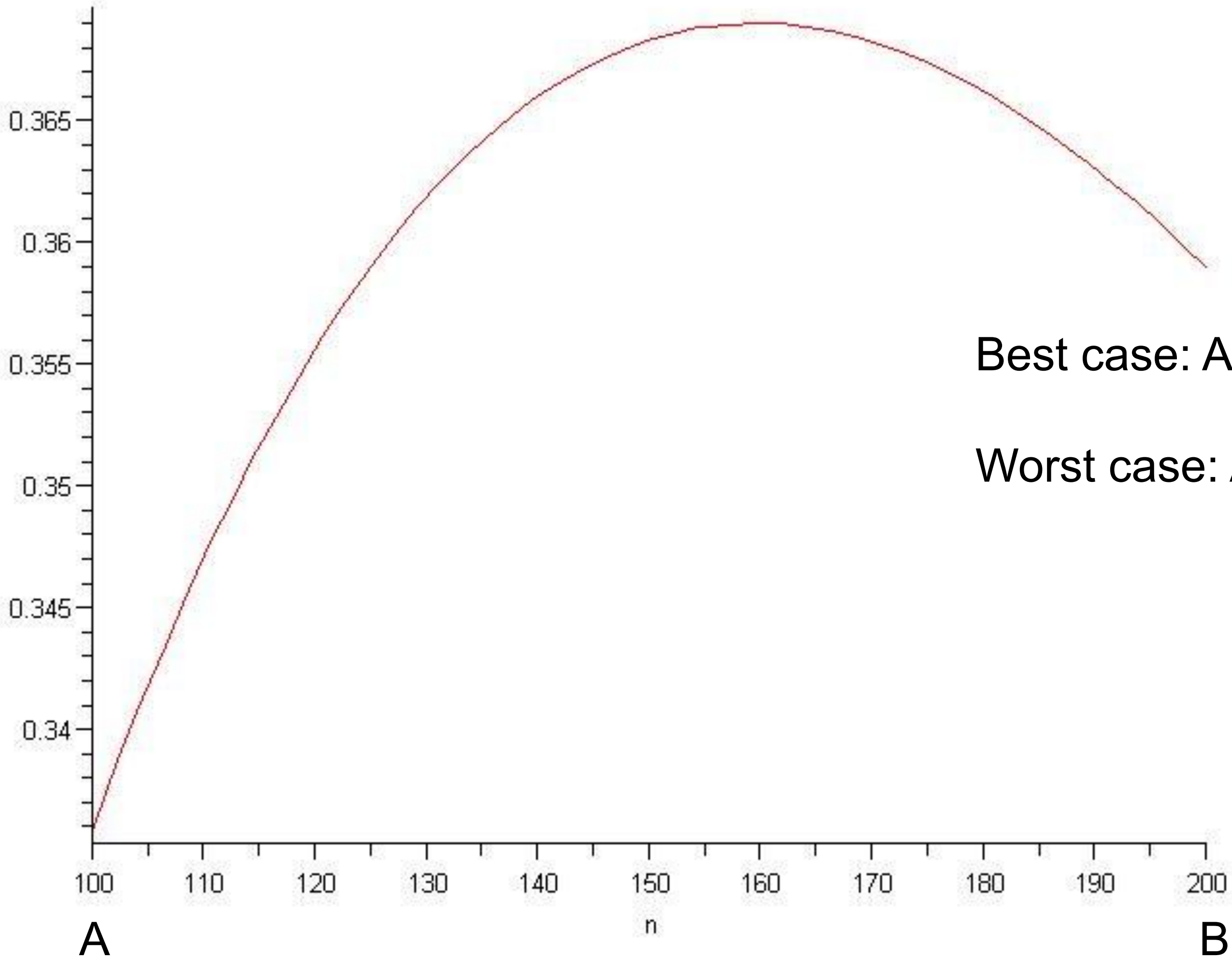
What happens for p=1/B?



Best case: Adversary chooses n=B

Worst case: Adversary chooses n=A

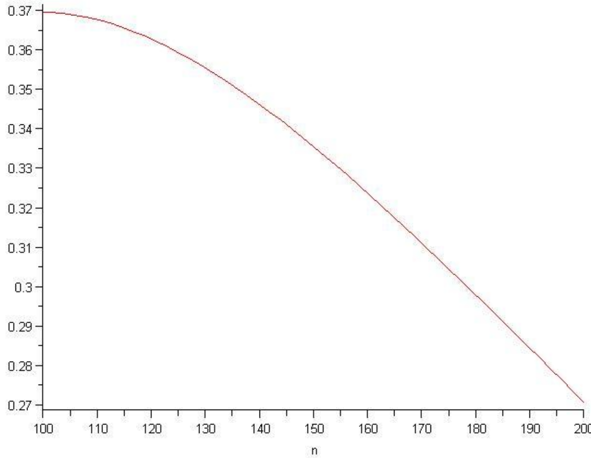
What happens for p=1/160?

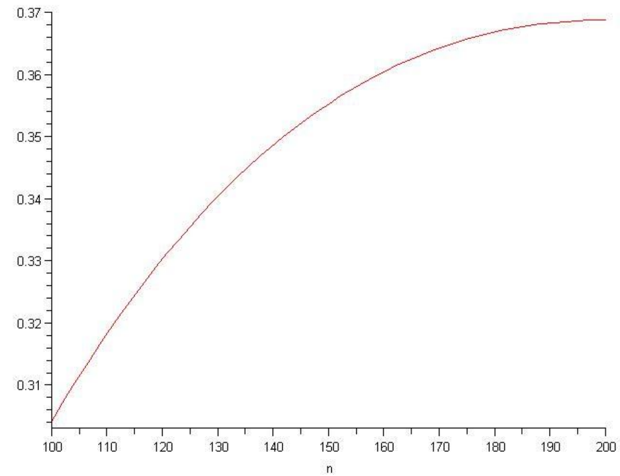


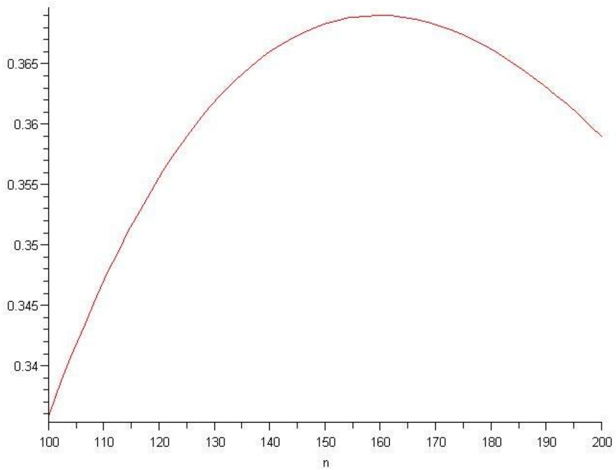
Best case: Adversary chooses n=160

Worst case: Adversary chooses n=A

Which n will the Adversary choose?

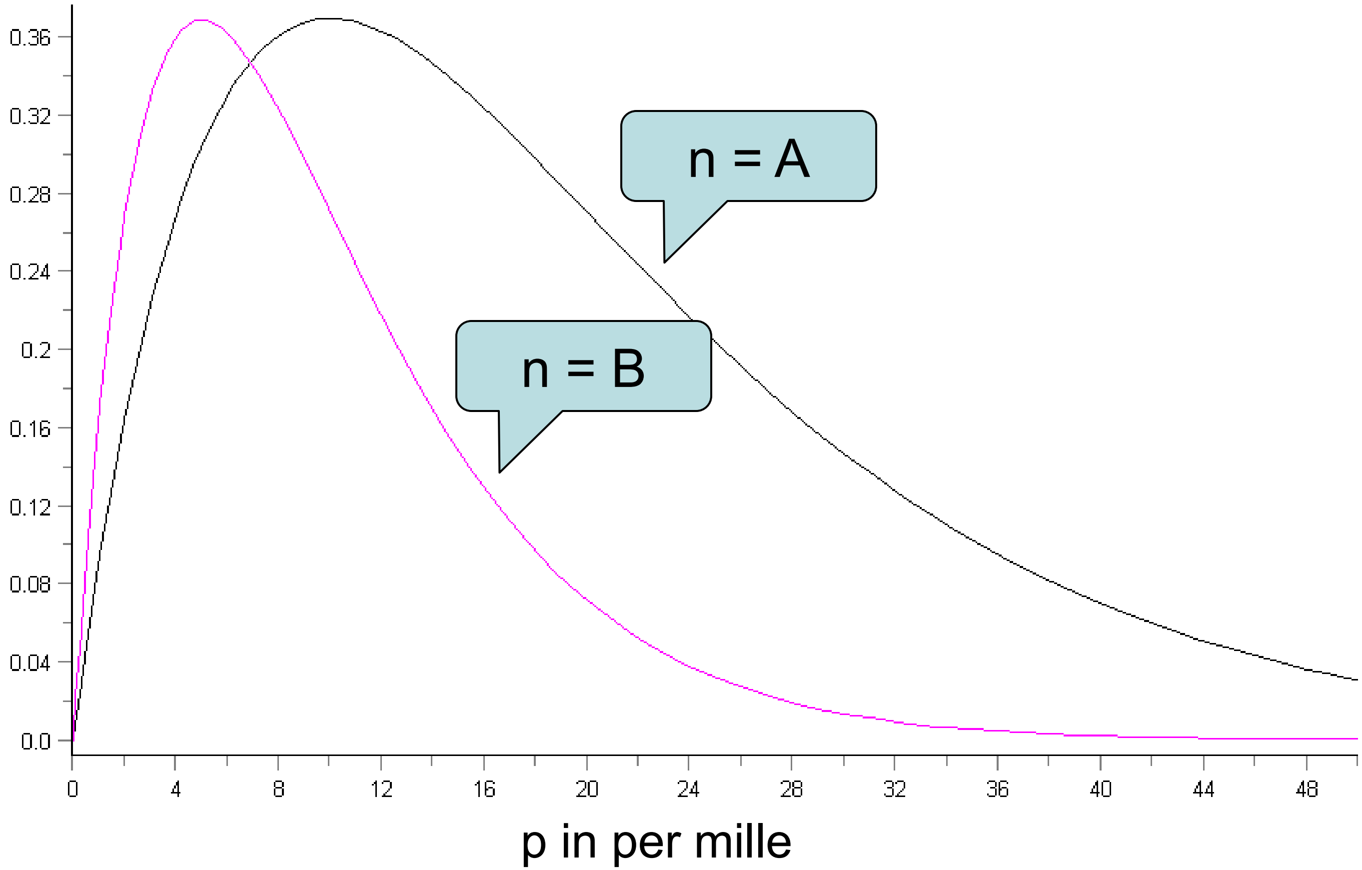






Optimizing for the Worst Case

Find p_{opt} where $\min\{Pr(A,p_{opt}), Pr(B,p_{opt})\}$ is maximized!



Optimizing for the Worst Case

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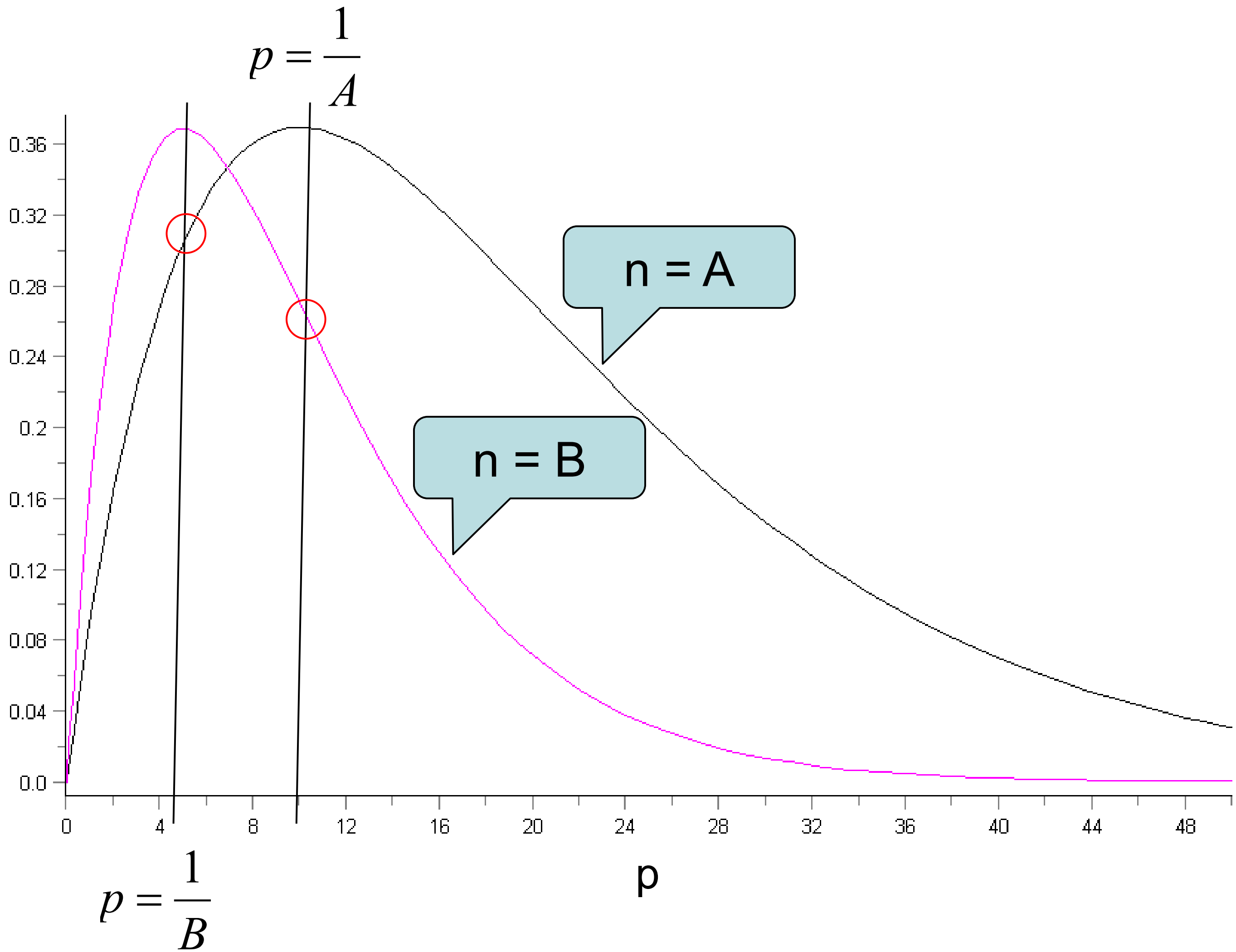
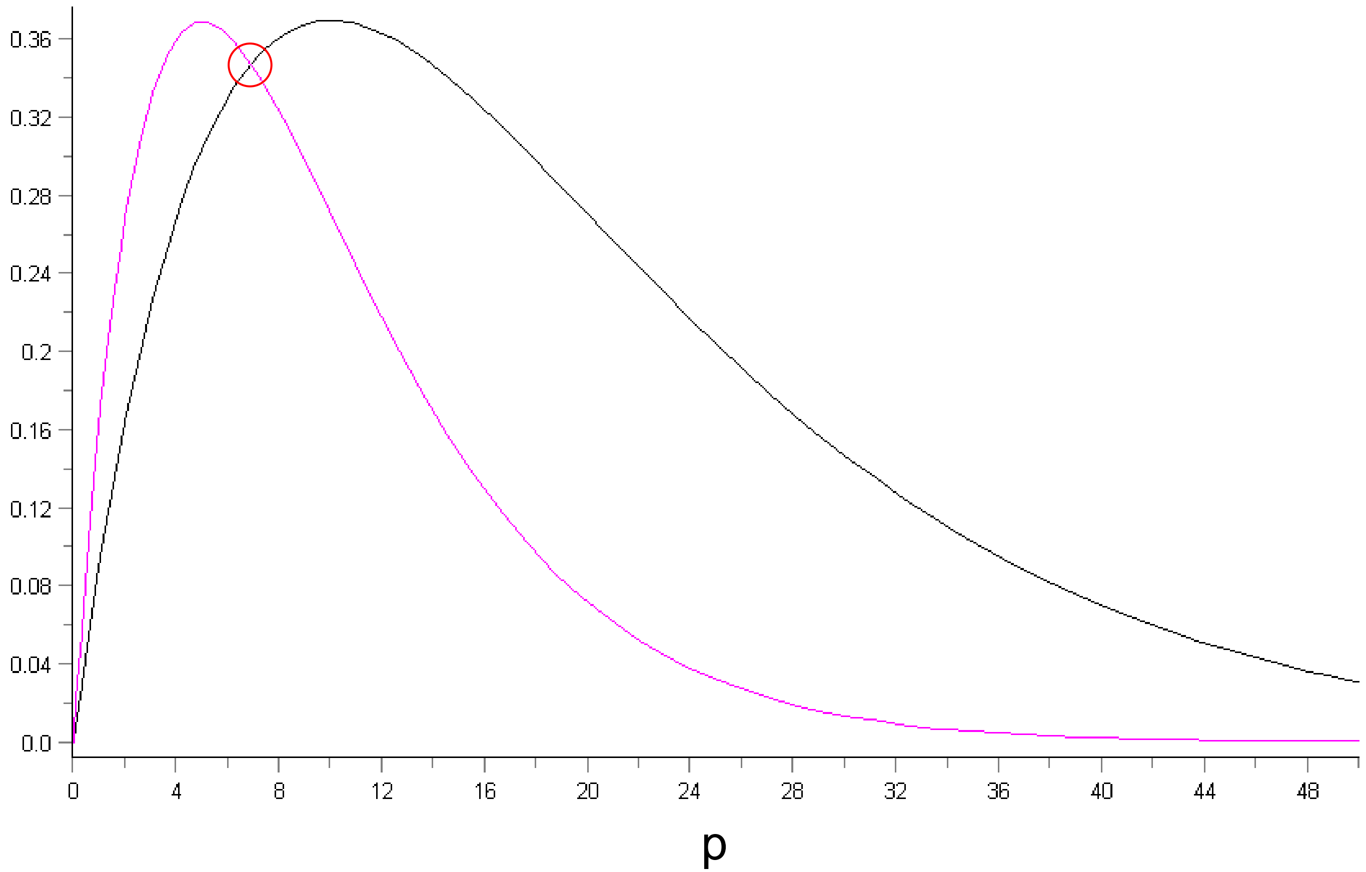


Optimizing for the Worst Case

p_{opt} is where the minimum of the two curves is maximized



Gory Mathematical Details

$$
\begin{aligned}
A p_{\mathrm{opt}} (1 - p_{\mathrm{opt}})^{A-1} &= B p_{\mathrm{opt}} (1 - p_{\mathrm{opt}})^{B-1} \\
\frac{A}{B} &= (1 - p_{\mathrm{opt}})^{B-1-(A-1)} = (1 - p_{\mathrm{opt}})^{B-A} \\
p_{\mathrm{opt}} &= 1 - \sqrt[B-A]{\frac{A}{B}}.
\end{aligned}
$$

For A = 100 and B = 200 we get

$p_{opt} = 0.006908$