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Distributed Computing Group

HS 2009
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Distributed Systems
Theory exercise 2

Assigned: November 13, 2009
Discussion: November 20, 2009

1 A store

Alex and Bea are working in a huge store. They would like to meet each other, but first they have to agree on a place to meet. Unfortunately, their boss does not like his workers to do anything else but work, so he carefully arranges schedules so that no two workers are ever in the same section.

a) In order to evade their boss's surveillance Alex and Bea came up with an idea. In the fruit corner of the store there is a big basket. They can insert or remove fruit from the basket without their boss noticing. Can they make a decision about a meeting place or not? Why?

b) What if all workers of the store want to meet?

c) The boss has noticed what is going on. Workers are no longer allowed to enter the fruit corner. Unfortunately the boss has no understanding of fruit, so he has to rely on his workers to tell him what to do. Workers are allowed to ask the boss how many fruits of some sort are in the basket, and the boss will replace old fruits with fresh fruits as soon as a worker asks him to do so. Do Alex and Bea have any chance to meet now? Why?

2 A proof

Why is it not possible for 6 processes to reach consensus if two of them are Byzantine? Sketch a proof.

3 A graph

In the lecture you learned how to reach consensus in a fully connected network where every process can communicate with every other process. We consider a network that is organized as a 2 dimensional grid such that every process has up to 4 neighbors. The width of the grid is w, the height is h. The grid is big, meaning that $w + h$ is much smaller than $w * h$. While there are faulty and correct processes in the network, we assume that two correct processes are always connected through at least one path of correct processes. In every round processes may send a message to each of its neighbors, the size of the message is not limited.

Hint: unlimited message size allows a process to merge several messages into one big message.

a) Assume there are no faulty processes. Write a protocol to reach consensus. Optimize your protocol for maximum speed.

b) How many rounds does your protocol from a) require?

c) Assume there are $w + h$ faulty processes. The faulty processes may die any time, but may not send wrong messages. In a worst case scenario, how many rounds does the protocol from a) require now?

d) Assume there are $w/2$ Byzantine failures. How could they sabotage your protocol from a)? Only a general idea is required, don not go into details.

A more realistic setting is a network organized as a hypercube. There are $n = 2^m$ processes, each process can communicate with m other processes.

e) Modify the king algorithm so that it works in a hypercube. Optimize the algorithm for maximum resilience. How many failures can the algorithm handle? Assume Byzantine processes can neither forge nor alter source or destination of a message.

f) How many rounds does the algorithm from e) require?

4 A riddle

The hangman summons his 100 prisoners, announcing that they may meet to plan a strategy, but will then be put in isolated cells, with no communication. He explains that he has set up a switch room that contains a single switch, which is either on or off. It is not known to the prisoners whether the switch initially is on or off. Also, the switch is not connected to anything, but a prisoner entering the room may see whether the switch is on or off (because the switch is up or down). Every once in a while, the hangman will let one arbitrary prisoner into the switch room. The prisoner may throw the switch (on to off, or vice versa), or leave the switch unchanged. Nobody but the prisoners will ever enter the switch room. The hangman promises to let any prisoner enter the room from time to time, arbitrarily often. That is, eventually, each prisoner has been in the room at least once, twice, a thousand times, any number you want. At any time, any prisoner may declare "We have all visited the switch room at least once". If the claim is correct, all prisoners will be released. If the claim is wrong, the hangman will execute his job (on all the prisoners). What's the strategy?

Hint: First try a simplified version of the game. What if the prisoners know the initial state of the switch? What if there are only three prisoners? What if there is a leader?