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Discrete Event Systems
Exercise 7

1 Probability of Arrival

In the script, there is a lemma saying that the probability of arrival can be computed as

$$f_{ij} = p_{ij} + \sum_{k:k\neq j} p_{ik}f_{kj}.$$

Prove this lemma.

2 Basketball [Exam]

Mario, Luigi and Trudy meet to play basketball. To improve their scoring abilities, Mario suggests the following game: Each player has to score m times. After each miss, he has to perform 10 push-ups.

  a) Assume that Mario always scores with a constant probability p. How many push-ups does he do in expectation in his game?

  b) Luigi wants to show that he's better and wants to score m times in sequence. After each miss, he performs 10 push-ups as well, and then tries again to score m times in a row. How many push-ups does Luigi do in expectation, assuming he also scores with a constant probability p?

  c) Trudy accepts Luigi's game and tries to score m times in a row. But Trudy is a bit lazy and gives up as soon as she has missed two times in a row. Trudy scores with constant probability $p = 0.5$, and for her we know that $m = 3$.

   (i) What is the probability that Trudy scores $m = 3$ times in a row? What is the probability that she gives up?

   (ii) How many push-ups does Trudy do in expectation?

   Hint:

$$\sum_{i=1}^{\infty} i \cdot q^{i-1} = \frac{1}{(1-q)^2} \qquad \text{for } |q| < 1$$

3 Night Watch

In order to improve their financial situation, Raphael and Jasmin also work at nights. Their task is to guard a famous Swiss bank which, from an architectonic perspective, looks as follows:

Thus, there are 4x4 rooms, all connected by doors as indicated in the figure.

In a first scenario, Raphael and Jasmin always stay together. They start in the room on the upper left. Every minute, they change to the next room, which is chosen uniformly at random from all possible (adjacent) rooms.
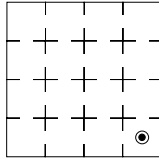
Figure 1: Offices of a Swiss bank.

a) Compute the probability (in the steady state) that Jasmin and Raphael are in the room where the thief enters the bank (indicated with ⊙)!

b) Since Jasmin and Raphael are very strong, they can easily catch a thief on their own. Thus, in a second scenario, they decide that it be smarter to patrol individually: After every minute, each of them chooses the next room *independently*. What is now the probability that at least one of them is in the room where the thief enters?

Note: This exercise is inspired by an exam question of the winter term 2004/2005.