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Discrete Event Systems
Exercise 4

1 Regular and Context-Free Languages

a) Consider the following context-free grammar $G : S \rightarrow SS|1S2|0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.

b) The regular languages are a subset of the context-free languages. Give the context-free grammar for a language L that is regular.

2 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

a) $L = \{w|$ the length of w is odd$\}$

b) $L = \{w|$ contains more 1s than 0s$\}$

3 Pushdown Automata

Consider the following context-free grammar G with non-terminals S and A, start symbol S, terminals '(', ')', and '0':

$$\begin{array}{rcl} S & \rightarrow & SA \,|\, \epsilon \\ A & \rightarrow & (S) \,|\, 0 \end{array}$$

a) What are the 4 shortest strings produced by G?

b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.

c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.

4 Pumping Lemma revisited

a) Determine whether the language $L = \{1^{n^2} | n \geq 1\}$ is regular.

b) Consider a regular language L and a pumping number p such that every word $u \in L$ can be written as $u = xyz$ with $|xy| \leq p$ and $|y| \geq 1$, and that $xy^iz \in L \; \forall i \geq 0$.

What can you say about the minimum number of states needed for the corresponding DFA? What about the minimum number of states of the corresponding the NFA?